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REACTIONS WITH TRANSFER OF k NUCLEONS AND NUCLEAR STRUCTURE

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Abstract

Assuming a pure transfer mechanism, the structure factor determining the excitation of the final nucleus in reactions of the type A + B → (A − k) + (B + k) with k = 2,3,4 is discussed in some details. All intermediate states of the group of the transferred particles are taken into account. The structure factor is written in a form analogous to the structure factor of reactions with one-nucleon transfer. The theory determining the dynamical factor of reactions with one-nucleon transfer can therefore be applied also to reactions with transfer of a group of nucleons.

It is shown that there is no factorization of the spectroscopic factor of the reaction into two factors one of which depends only on the properties of the nuclei A and A − k and the other depends only on the properties of the nuclei B and B + k. From this, it follows in most cases a reduction of the number of terms determining the spectroscopic factor of the reaction. The excitation spectra of the final nucleus observed in different reactions with transfer of k nucleons on the same target nuclei will be different from one another, as a rule, and different from the spectrum of the corresponding usual spectroscopic factors. Exception from this rule are discussed.

1. Introduction

In a preceding paper[1] (in the following designated as I) the mechanism of reactions with transfer of two particles was discussed. The structure factor which determines the probability for excitation of a level of the final nucleus in reactions between complex nuclei has been given there by taking into account all intermediate states which the transferred nucleons can form. Here, reactions with transfer of three or four nucleons of the type

$$A + B \rightarrow (A-k) + (B+k)$$

are considered in the approximation of a pure one-step process

$$\begin{array}{ccc} A & \longrightarrow & A-k \\ & \Big\downarrow k & \\ B & \longrightarrow & B+k \end{array}$$

($k = 2,3,4$). As in paper I, all intermediate states of the group of the transferred nucleons are taken into account.

There is not yet a theory describing both the structure properties and the dynamical properties of reactions induced by heavy ions in which transfer of several nucleons takes place. In this paper, the structure factors will be discussed in some details. They are written in a form analogous to the form of the structure factors of reactions with transfer of only one nucleon. To get the cross section of the reaction, one has to multiply the structure factors $A_{NL}^{N'L'}$

by the dynamical factors $B_{NL}^{N'L'}$ and to sum over all intermediate states. The $B_{NL}^{N'L'}$ show up like the dynamical factors in the theory of reactions with transfer of only one nucleon.

The structure factors are compared with the usual spectroscopic factors for nucleon groups as deuterons, He^3 or tritons, and α -particles. Most reactions with transfer of a nucleon group will lead to an excitation of the final nucleus which is not proportional to the usual spectroscopic factor for the corresponding nucleon group. It is shown from which type of reactions one can get the usual spectroscopic factors.

2. Formalisms

2.1. Transfer of a group of k particles

It has been shown in paper I that the structure factor for reactions $A + B \rightarrow (A-k) + (B+k)$ with transfer of a group of nucleons can be written in the form

$$A_{NL}^{N'L'} = R\,R' \left\{ \binom{A}{k} \binom{B+k}{k} \right\}^{1/2}$$

$$(T_1\; T_{1z} \mid T_2 T_{2z}, T_k T_{kz})(T_4 T_{4z} \mid T_3 T_{3z}, T_k\; T_{kz}) \tag{1}$$

$$\langle \chi_1 \mid \chi_2 \chi_k \rangle \langle \chi_4 \mid \chi_3\, \chi_k \rangle$$

$$\left(\frac{A}{A-k}\right)^{N+L/2} \left(\frac{B+k}{B}\right)^{N'+L'/2} K(n\ell, NL, L_k) K(n\ell, N'L', L'_k).$$

Here, the indices 1,2,3,4, correspond to the nuclei $A, A-k, B, B+k$. The factors $R\; R'$ contain all uncertainties coming from the radius dependence of the structure factor. For the main discussion here, they will be neglected $(R = R' = 1)$. The $(T_i\; T_{iz} \mid T_j\; T_{jz}, T_k T_{kz})$ are Clebsch-Gordan coefficients. The oscillator quantum numbers of the group of the k transferred nucleons are the following: $n\ell$

the inner quantum numbers, N, L, the quantum numbers of the centre-of-mass with respect to the nucleus $A-k$, and N', L', the quantum numbers of the centre-of-mass with respect to the nucleus B. L_k and L_k', are the orbital momenta of the group of the transferred nucleons with respect to the nucleus $A-k$, and B, respectively.

The main factors which the structure factor $A_{NL}^{N'L'}$ consists of are the overlap integrals $<\chi_1|\chi_2\chi_k> K(n\ell, NL, L_k)$ on the one hand and the overlap integrals $<\chi_4|\chi_3\chi_k> K(n\ell, N'L', L'_k)$ on the other hand. The integral $<\chi_1|\chi_2\chi_k>$ is the overlap integral of the shell-model wave function of the nucleus A with the shell-model wave function of the nucleus $A-k$ and that of the k separated nucleons (fractional parentage coefficient). It takes into account the structure of the nuclei in the initial and final states. The integral $K(n\ell, NL, L_k)$ is the overlap integral of the wave function of the k separated nucleons with the wave function of the group of the k nucleons in the intermediate state. It takes into account the formation of a nucleon group with the inner quantum numbers n, ℓ and the relative quantum numbers N, L from the k separated nucleons. The integral $K(n\ell, N'L', L'_k)$ is analogous to $K(n\ell, NL, L_k)$ and the integral $<\chi_4|\chi_3\chi_k>$ to $<\chi_1|\chi_2\chi_k>$.

The factors most important for the excitation of a level are the two fractional parentage coefficients $<\chi_1|\chi_2\chi_k>$ and $<\chi_4|\chi_3\chi_k>$. The properties of these coefficients are well known.

The overlap integrals $K(n\ell, NL, L_k)$ and $K(n\ell, N'L', L'_k)$ contain the dependence on the quantum numbers N, L and N', L'. The two integrals are not independent of each other. That means,

$$\left(\frac{A}{A-k}\right)^{N+L/2} \left(\frac{B+k}{B}\right)^{N'+L'/2} K(n\ell, NL, L_k) K(n\ell, N'L' L'_k) \qquad (2)$$

is not separable into a part which depends only on the nuclei A and $A-k$ and a part which depends only on the nuclei B

and $B+k$. This follows from the assumption that the group of transferred nucleons has definite quantum numbers S_k, T_k and f_k (spin, isospin and symmetry) as well as n and l which are not changing in the transfer process[x/]. By this, (2) connects the two vertices of the reaction.

To get the cross section of the reaction, one has to multiply the structure factors by the corresponding dynamical factors and to sum over all intermediate states. The dynamical factors are the same as for reactions with transfer of only one nucleon keeping in mind the fact that the quantum numbers N, L and N', L' characterizing the motion of the centre-of-mass of the transferred nucleon group with respect to the nuclei A and $B+k$ correspond to the quantum munbers n, ℓ and n', ℓ' characterizing the motion of the transferred nucleon in the nuclei A and $B+1$.

2.2. $K(n\ell, NL, L_k)$ for transfer of two nucleons

The overlap integral $K(n\ell, NL, L_k)$ for transfer of two nucleons is given by a Moshinsky coefficient. If the two nucleons are identical $(n_1\ell_1 = n_2\ell_2)$, then it follows

$$K^{(2)}_s(n\ell, NL, L_k) = \langle n\ell, NL:L_k | 1,1 | n_1\ell_1, n_2\ell_2 : L_k \rangle . \qquad (3)$$

Here, the defirition of Smirnov[2] for the generalized Moshinsky coefficients was used. If the two nucleons are distinguishable $(n_1\ell_1 \neq n_2\ell_2)$, then

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[x/] This assumption is the simplest one. The change of the quantum numbers should be taken into account only in a higher order approximation. In such a case, however, one cannot neglect processes in which the nucleons are transferred after each other, i.e. processes in which the nucleons are not transferred as a group in a one-step process.

$$K^{(2)}(n\ell, NL, L_k) = \frac{1}{\sqrt{2}} \{ \langle n\ell, NL : L_k | 1,1 | n_1\ell_1, n_2\ell_2 : L_k \rangle$$

$$+ (-1)^{sy} \langle n\ell, NL : L_k | 1,1 | n_2\ell_2, n_1\ell_1 : L_k \rangle \}$$

$$= \sqrt{2}\, K_s^{(2)}(n\ell, NL, L_k)\, \delta_{L_k \cdot L \pm 2 + sy} \tag{4}$$

$$(i = 0, \; , 2, \ldots).$$

Here $sy = 0$ if the orbital wave function of the two nucleons is symmetrical and $sy = 1$ if the orbital wave function of the two nucleons is antisymmetrical.

2.3. $K(n\ell, NL, L_k)$ for transfer of three nucleons

The overlap integral $K(n\ell, NL, L_k)$ for the transfer of three nucleons can be calculated in the following manner. If the three particles are identical then

$$\chi_k((n_1\ell_1, n_2\ell_2, n_3\ell_3)^3 : L_k)$$

$$= \Sigma \langle \ell^3_{1,2,3} | \ell^2_{1,2} : \ell^0_a, \ell_3 \rangle$$

$$\langle n_0\ell_0, N_0L_0 : \ell_a^0 | 1,1 | n_1\ell_1, n_2\ell_2 : \ell_a^0 \rangle \tag{5}$$

$$U(L_0\ell_0\ell_3L_k, \ell_a^0\ell_a^{00}) \times \Psi$$

$$\Psi \equiv \{ \phi_{n_0 \ell_0}(\vec{r}_{12}) \{ \phi_{N_0 L_0}(\vec{R}_{12}) \phi_{n_3 \ell_3}(\vec{r}_3) \}_{\ell_a^{00}} \}_{L_k}$$

$$= \Sigma < n_{00} \ell_{00} NL : \ell_a^{00} | 2,1 | N_0 L_0, n_3 \ell_3 : \ell_a^{00} > \tag{6}$$

$$U(\ell_0, L\ \ell_0 L_k ; \ell_a^{00} \ell\ )$$

$$\{\{ \phi_{n_0 \ell_0}(r_{12}) \phi_{n_{00} \ell_{00}}(\vec{R}_{12} - \vec{r}_3) \}_\ell \Psi_{NL}(\vec{R}_3) \}_{L_k} \quad .$$

Here, one has to sum over all free values. $< \ell^3_{1,2,3} | \ell^2_{1,2} ; \ell^0_a , \ell_3 >$ is the orbital part of the fractional parentage coefficient of the three transferred particles (tables of fractional parentage coefficients in /3/). One gets from (5) and (6)

$$K_s^{(3)}(n\ \ell\ , NL, L_k) = \Sigma < \ell^3_{1,2,3} \ | \ \ell^2_{1,2} : \ell^0_a , \ell_3 >$$

$$< n_0 \ell_0, N_0 L_0 \ : \ell^0_a \ | 1,1 | n_1 \ell_1, n_2 \ell_2 : \ \ell^0_a >$$

$$< n_{00} \ell_{00}, NL : \ell_a^{00} | 2,1 | N_0 L_0, n_3 \ell_3 : \ \ell_a^{00} > \tag{7}$$

$$U(L_0 \ell_0 \ell_3 L_k ; \ell^0_a \ell^{00}_a) U(\ell_{00} L \ell_0 L_k ; \ell^{00}_a \ell\ ),$$

where

$$2 n_0 + \ell_0 + 2n_{00} + \ell_{00} = 2n + \ \ell \quad . \tag{8}$$

If the three nucleons are not identical but $n_1 \ell_1 = n_2 \ell_2 \neq n_3 \ell_3$ then for the case that the orbital wave function is symmetrical it follows:

$$K^{(3)}(n\ \ell, NL, L_k) = \sqrt{\frac{1}{3}}\{K_s^{(3)}(n\ell\ , NL, L_k) + + (1+(-1)^{\ell_a^0 + L_0})K_s^{3\leftrightarrow 1}(n\ \ell, NL, L_k)\}, \tag{9}$$

where $K_s^{3\leftrightarrow 1}$ follows from $K_s^{(3)}$ by exchange of n_3, ℓ_3 and n_1, ℓ_1.

2.4. $K(n\ \ell, NL, L_k)$ for transfer of four nucleons

In an analogous manner, one gets the following value for K in the case of four transferred particles which are identical

$$K_s^{(4)}(n\ \ell, NL, L_k) =$$

$$= \Sigma < \ell^4_{1,2,3,4} : L_k \mid \ell^2_{1,2} : \ell_a, \ell^2_{3,4} : \ell'_a > \tag{10}$$

$$< n_0\ell_0, N_0L_0 : \ell_a \mid 1,1 \mid n_1\ell_1, n_2\ell_2 : \ell_a >$$

$$< n'_0\ell'_0, N'_0L'_0 : \ell'_a \mid 1,1 \mid n_3\ell_3, n_4\ell_4 : \ell'_a >$$

$$< n_{00}\ell_{00} NL : L_{aa} \mid 2,2 \mid N_0 L_0, N'_0 L'_0 : L_a >$$

$$\{(2\ell_a+1)(2\ell'_a+1)(2\ell_{aa}+1)(2L_{aa}+1)\}^{1/2}$$

$$\begin{Bmatrix} \ell_0 & L_0 & \ell_a \\ \ell'_0 & L'_0 & \ell'_a \\ \ell_{aa} & L_{aa} & L_k \end{Bmatrix} U(\ell_{00}L\ \ell_{aa}L_k, L_{aa}\ell).$$

Here

$$2n_{00} + \ell_{00} + 2n_0 + \ell_0 + 2n'_0 + \ell'_0 = 2n + \ell \,. \tag{11}$$

The factor

$$\langle \ell^4_{1,2,3,4} : L_k \mid \ell^2_{1,2} : \ell_a \,, \ell^2_{3,4} : \ell'_a \rangle$$

is the orbital part of the fractional parentage coefficient for the four transferred particles (tables in /4/).

If the four nucleons are not identical but, for example, $n_1\ell_1 = n_2\ell_2 \neq n_3\ell_3 = n_4\ell_4$ then it follows

$$K^{(4)}(n\ell, NL, L_k) = \sqrt{\frac{1}{6}}\{K^{(4)}_s(n\ell, NL, L_k) + K_s^{1,2 \leftrightarrow 3,4}(n\ell, NL, L_k) + (1 - (-1)^{\ell_a + L_0} + (-1)^{\ell'_a + L'_0} + (-1)^{\ell_a + \ell'_a + L_0 + L'_0}) K_s^{2 \leftrightarrow 3}(n\ell, NL, L_k)\} \tag{12}$$

in the case that the orbital wave function of the four nucleons is symmetrical. $K_s^{1,2 \leftrightarrow 3,4}$ follows from $K^{(4)}_s$ by exchange of $n_1\ell_1$, $n_2\ell_2$ and $n_3\ell_3$, $n_4\ell_4$ whereas $K_s^{2 \leftrightarrow 3}$ follows from $K^{(4)}_s$ by exchange of $n_2\ell_2$ and $n_3\ell_3$.

2.5. Factorization of the amplitude

In paper /5/, factorization of the amplitude $A^{N'L'}_{NL}$ into two factors is assumed one of which depends only on the properties of the vertex $A \to (A-k)+k$ and the other depends only on the properties of the vertex $B+k \to (B+k)$. In such a theory, the product of the overlap integrals is

$$Y_0 = \sum_{n\ell NL} K(n\ell, NL, L_k) \sum_{n'\ell' N'L'} K(n'\ell', N'L', L'_1) \qquad (13)$$

(up to multiplication by the dynamical factors $B_{NL}^{N'L}$) where summation goes over n, ℓ and n', ℓ' independent of each other. That means, the group of particles is assumed to change their inner quantum numbers n, ℓ to all possible values $n'\ell'$ in the transfer process.

In section 3, the results of the theory assuming factorization of the amplitude into two parts each of which depends only on one vertex will be discussed and compared with the results of the theory without factorization of the amplitude.

2.6. Definition of the spectroscopic factor for a group of nucleons

The spectroscopic factor for a group of nucleons such as deuterons, He^3 or tritons, and α-particles is defined in the following manner/6/

$$S^{1/2} \sim \binom{A}{k}^{1/2} \left(\frac{A}{A-k}\right)^{N+L/2} \langle \chi_A | \chi_{A-k}, \chi_k \rangle \qquad (14)$$

$$K_0(n=0\ \ell=0, NL, L_k).$$

The spectroscopic factor is defined for a group of nucleons observed in the experiment. This group is in the lowest state, $n = \ell = 0$.

In general, Γ_0 differs from K. For example, one gets

$$\Gamma_0 = \sum_{NL} K_0(00, NL, L_k) = 0$$

but

$$Z_0 = \sum_{n\ell, NL, N'L'} K(n\ell, NL, L_k) K(n\ell, N'L', L_k) = 1$$

for the case of two particles with the orbital wave function x/ $(0p)^2[11]P$ in both the initial nucleus A and the final nucleus $B+k$. An analogous result follows, for example, for the case of three particles with the orbital wave function $(0p)^3[21]\,P$ in both nuclei:

$$\Gamma_0 = \sum_{NL} K_0(00, NL, L_k) = 0$$

$$Z_0 = \sum_{n\ell, NL, N'L'} K(n\ell, NL, L_k) K(n\ell, N'L', L_k') = \frac{1}{3}.$$

In some cases, the K reduce to K_0. If $n_1\ell_1 = n_2\ell_2 = 0s$ for two-nucleon transfer then

$$K(n\ell, NL, L_k) = 1,$$

$$K(n\ell, N'L', L'_k) = K_0(00, N'L', L'_k)$$

and

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x/ Here, following Brody and Moshinsky /7/, the energy levels of the harmonic oscillator are numbered as follows: $0s, 0p, 1s, 0d, 1p, 0f, \ldots$.

$$Z_0 = \sum_{n\ell, NL, N'L'} K(n\ell, NL, L_k)\, K(n\ell, N'L', L'_k)$$

$$= \sum_{N'L'} K_0(00, N'L', L'_k) = T_0 .$$

Analogous results follow for transfer of three and four particles. That means, if the transferred nucleons come from the 0s -shell of the initial nucleus A then the structure factors determining the excitation of the levels of the final nucleus are proportional to the corresponding usual spectroscopic factors.

Such a result does not follow from a theory in which factorization of the amplitude $A_{NL}^{N'L'}$ into two parts is assumed each of which depends only on one vertex. One has

$$Y_0 = \sum_{n\ell, NL} K(n\ell, NL, L_k) \sum_{n'\ell', N'L'} K(n'\ell', N'L', L'_k)$$

$$= \sum_{n'\ell', N'L'} K(n'\ell', N'L', L'_k)$$

in the case of two-particle transfer with $n_1\ell_1 = n_2\ell_2 = 0s$, Y_0 is not proportional to T_0. For example, it is[x]

$$Y_0 = \sum_{n'\ell', N'L'} K(n'\ell', N'L', L'_k) = \begin{cases} 0 & \text{for } L'_k = 0,2 \\ 1 & \text{for } L'_k = 1 \end{cases}$$

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[x] In the theory of Glendenning /8/ and also in the theory of El-Batanoni et.al. /8/, $\ell = \ell'$ is assumed. For the case of two-particle transfer into the 0p -shell of the final nucleus, it follows

$$\sum_{n', N'L'} K(n'0, N'L', L'_k) = \begin{cases} 0 & \text{for } L'_k = 0 \\ 0.707 & \text{for } L'_k = 2 \\ 0 & \text{for } L'_k = 1 \end{cases}$$

for transfer of two particles into the $0p$ -shell $(n'_1 = n'_2 = 0,\ \ell'_1 = \ell'_2 = 1)$, whereas

$$T_0 = \sum_{N'L'} K_0(00, N'L', L'_k) = \begin{cases} 0{,}707 & \text{for} \quad L'_k = 0,2 \,. \\ 0 & \text{for} \quad L'_k = 1 \,. \end{cases}$$

In reality, one has to multiply the components depending on N' and L' by the dynamical factor and a correction factor like $(B/(B+k))^{N'+L'/2}$. This will change the result $Y_0 = 0$ for $L'_k = 0,2$ into a value nonvanishing but depending strongly on B and the dynamical features.

3. Discussion

Here, two points will be discussed: firstly, the quantum numbers NL and $N'L'$ which are important for the multiplication with the dynamical factors $B^{N'L'}_{NL}$ and, secondly, the cases in which the usual spectroscopic factors for nucleon groups are proportional to the structure factors $A^{N'L'}_{NL}$ determining the cross section of reactions with transfer of a nucleon group. To this end, the structure factors $A^{N'L'}_{NL}$ will be summed up over the oscillator quantum numbers $n\ell$, NL and $N'L'$ without multiplication with the dynamical factors $B^{N'L'}_{NL}$.

The overlap integrals $K(n\ell, NL, L_k)\,K(n\ell, N'L', L'_k)$ contain all the dependence on the quantum numbers NL and $N'L'$ characterizing the motion of the centre-of-mass of the group of the transferred particles in the intermediate state with respect to the nucleus A and with respect to the nucleus $B+k$. These quantum numbers correspond to the quantum numbers $n\ell$ and $n'\ell'$ characrerizing the motion of the transferred nucleon in the nucleus A and in the nucleus $B+1$ in the case of one-nucleon transfer. The theory of the dynamical part $B^{N'L'}_{NL}$ is, therefore, the same as in the case of one-nucleon transfer.

To get the excitation probability for the levels of the final nucleus in reactions with transfer of several nucleons one has, naturally, to multiply the structure factors by the dynamical factors (see also [8]).

$$S \sim \sum A_{NL}^{N'L'} B_{NL}^{N'L'} \tag{15}$$

where summation goes over all intermediate states of the nucleon group. Here, it will be shown that $A_{NL}^{N'L'}$ is proportional to the usual spectroscopic factor only for a small class of reactions.

3.1. Reactions with transfer of two particles

First of all, one has to do some remarks about the theory assuming factorization of the amplitude $A_{NL}^{N'L'}$ into two parts each of which depends only on one vertex. One expects that the overlap integrals and the amplitude $A_{NL}^{N'L'}$ are large in the case in which the group of the transferred particles has the same quantum numbers in the initial nucleus A as in final nucleus $B+k$ ($n_1\ell_1 = n'_1\ell'_1$, $n_2\ell_2 = n'_2\ell'_2$). But the theory assuming factorization of the amplitude into two parts $A_{NL}^{N'L'} = a_{NL}\, a_{N'L'}$ does not give such a result.

In table 1, the values

$$Z_0 = \sum_{n\ell,NL,N'L'} K(n\ell, NL, L_k)K(n\ell, N'L', L'_k), \tag{16}$$

$$Z = \sum_{n\ell,NL,N'L'} \left(\frac{A}{A-k}\right)^{N+L/2} \left(\frac{B+k}{B}\right)^{N'+L'/2} K(n\ell, NL, L_k)K(n\ell, N'L', L'_k) \tag{17}$$

characterizing the theory without factorization of the amplitude $A_{NL}^{N'L'}$ and the values

$$Y_0 = \sum_{\substack{n\ell,NL \\ n'\ell',N'L'}} K(n\ell, NL, L_k)\, K(n'\ell', N'L', L'_k), \tag{18}$$

$$Y = \sum_{\substack{n\ell,NL \\ n'\ell',N'L'}} \left(\frac{A}{A-k}\right)^{N+L/2} \left(\frac{B+k}{B}\right)^{N'+L'/2} K(n\ell,NL,L_k) K(n'\ell',N'L',L'_k) \tag{19}$$

characterizing the theory with factorization of the amplitude $A_{NL}^{N'L'}$ are given for two particles transferred from the 0p-shell of the initial nucleus ($A = 9$) into the 0p -shell of the final nucleus ($A = 7$). As one expects, Z_0 and Z are maximal if the two separated nucleons have the same quantum numbers in the initial nucleus as in the final nucleus. Z_0 and Z are vanishing if the Young scheme of the two particles is changed in the transfer process. The Y_0 and Y show another behaviour. Y_0 vanishes not only in the cases in which the Young scheme of the transferred particles is changed in the transfer process, but also in the cases where $[f_k] = [f'_k] = [2]$ and $L_k = L'_k$. This result is not expected. The Y are non-vanishing in all cases, but they strongly depend on A and B in the cases for which the corresponding Y_0 are vanishing.

The Z_0 and Z show some regularity: if the quantum numbers of the two nucleons in the initial nucleus A and in the final nucleus $B+2$ are the same, then Z and Z_0 are maximal. If the symmetry of the wave function must be changed in the transfer process ($[f_k] \neq [f'_k]$) then $Z = Z_0 = 0$. There is some dependence on L_k and L'_k. In general, Z and Z_0 are maximal if L_k and L'_k are maximal. Such a dependence on L_k

and L'_k is known also for the values

$$T_0 = \sum_{N'L'} K(00, N'L', L'_k)$$

and the spectroscopic factors.

The transfer of two 1s-nucleons of the initial nucleus into the 0d-shell of the final nucleus is of some interest. For $L_k = L'_k = 0$ it follows

$$Z_0 = 0.000,$$

$$Z = 0,013,$$

$$T_0 = 0.298 .$$

In this case, Z shows a strong dependence on A and B.

From this discussion, it follows that the N', L' and N, L are not independent of each other. The connection of N, L and N', L' caused by the condition $n\ell = n'\ell'$ leads in the most cases to a limitation of the possible values of N, L or N', L' and therefore to a simplification of the sum (15).

The usual spectroscopic factors are proportional to

$$T = \sum_{N'L'} \left(\frac{B+k}{B}\right)^{N'+L'/2} K(00, N'L', L'_k). \tag{20}$$

In reactions with transfer of two particles coming from the 0s-shell of the initial nucleus A, the structure factors determining the excitation of the levels of the final nucleus $B+2$ are proportional to $Z = T$ (up to multiplication by the dynamical factors $B_{00}^{N'L'}$). Because of $n = \ell = 0$, one has in the most cases only one term in (20). That means, one can get the usual spectroscopic factors from a study of these reactions. Moreover, because of their simplicity they are suitable also for a study of the contributions of the different components mixed into the wave function .

Reactions induced by lithium ions will also lead to an excitation spectrum of the final nucleus which is similar to the spectrum of the corresponding usual spectroscopic factors. The wave function of ^6Li is $[42]^{13}S$. Therefore, the two transferred nucleons have the same configuration as a deuteron. In table 2, the values Z and T are given for the reaction $^7Li(^6Li,\alpha)\,^9Be$. Deviations of the wave function of $^6Li_{g.s}$ from the shell-model wave function $[42]^{13}S$ in favour of the cluster-model wave function $\alpha + d$ which exist in reality will make the ^6Li nucleus only more suitable for a study of the spectroscopic factors for deuterons.

The excitation spectrum of the final nucleus which one gets in reactions such as $(^{11}B, ^9Be)$ or $(^9Be, ^7Li)$ will, as a rule, differ from the corresponding spectrum of spectroscopic factors. The fractional parentage coefficients $^{11}B \rightarrow {^9Be}+[11]P$ and $^9Be \rightarrow {^7Li}+[11]P$ are not small. In table 3 the values Z and T are given for the reaction $^{12}C(^9Be, ^7Li)\,^{14}N$ and in table 4 for the reaction $^{12}C(^{11}B, ^9Be)\,^{14}N$. For the cases $[11]P \rightarrow [11]P$, the Z are not small. Therefore, the two nucleons can be transferred also in a state the orbital wave function of which is antisymmetric. To get the cross section, one has to sum over the intermediate states of the pair of the transferred nucleons with the symmetry [2] as well as over the intermediate states with the symmetry [11].

In table 5, the values Z and T are given for the case that the transferred nucleons were in the initial nucleus in a shell different from that of the final nucleus. The results are similar to those of $(0p)^2 \rightarrow (0p)^2$ (tables 3,4).

From this, it follows that even the excitation spectra of the final nucleus which one gets in $(^{11}B, ^9Be)$ and $(^9Be, ^7Li)$ reactions on the same target nucleus must not be similar to each other and will differ, in general, from the excitation spectrum of the final nucleus which one gets in the $(^3He, p)$ reaction on the same target nucleus. For a more detailed discussion of these interference effects see paper /9/.

3.2. Reactions with transfer of three particles

As in the case of two-particle transfer, the theory without factorization of the amplitude $A^{N'L'}_{NL}$ will be compared with the theory factorizating the amplitude $A^{N'L'}_{NL}$ into two parts a_{NL} and $a_{N'L'}$ each of which depends only on one vertex.

In table 6, the values Z_0 and Z characterizing the theory without factorization of the amplitude and the values Y_0 and Y characterizing the theory with factorization of the amplitude are given for three-particle transfer $(0p)^3 \to (0p)^3$. As in the case of two-particle transfer, the Z_0 and Z are maximal if the quantum numbers of the transferred particles are the same in the initial nucleus as in the final nucleus, whereas Y_0 and Y do not show such a regularity.

The Z_0 and Z are vanishing for the case in which the Young scheme of the two transferred nucleons is changed in the transfer process and $L_k = L'_k$. As in the case of two-particle transfer, Z_0 and Z show some dependence on L_k and L'_k: they are maximal if L_k and L'_k are maximal.

From the results of table 6, one can draw the conclusion that the condition $n\ell = n'\ell'$ is a reasonable one. This condition leads in the most cases to a simplification of the sum (15), i.e. to a limitation of the number of terms with different $N\ L$ or N', L'. This simplification is of some importance for the analysis of a concrete reaction.

The usual spectroscopic factors for ^3He or tritons having the wave function $[3]^{22}S$ are proportional to T (eq. 20). For reactions with transfer of three particles coming from the $0s$-shell of the initial nucleus A, the structure factors $A^{N'L'}_{NL}$

determining the probability for excitation of the levels of the final nucleus $B+3$ are proportional to $Z \cdot T$ (up to the dynamical factors $B_{NL}^{N'L'}$). That means, the excitation spectrum of the final nucleus in reactions as (α, p) will be proportional to the spectrum of the corresponding spectroscopic factors if the reaction mechanism is the one-step process with transfer of three particles.

The wave function of ^{7}Li is in a good approximation /10/ $[43]^{22}P$ and the wave function /11/ of ^{19}F is $(0d^{2}, 1s)^{3}$ ^{22}S. Therefore, the three nucleons transferred in $(^{7}Li, \alpha)$ reactions and in $(^{19}F, ^{16}O)$ reactions x/ have the configuration [3] as He^{3} or t. The values Z and T for the reaction $^{12}C(^{7}Li, \alpha)$ ^{15}N are given in table 7, for the reaction $^{12}C(^{19}F, ^{16}O)$ ^{15}N in table 8, and for the reaction $^{16}O(^{7}Li, \alpha)$ ^{19}F in table 9. Comparison of the results for $(^{7}Li, \alpha)$ and for $(^{19}F, ^{16}O)$ on the same target nucleus shows that, in general, the values of $N'L'$ differ in the two reactions. Nevertheless, the excitation spectrum of the final nucleus taken in the $(^{7}Li, \alpha)$ reaction must not be different from the excitation spectrum of the final nucleus taken in the $(^{19}F, ^{16}O)$ reaction on the same target nucleus because the excitation spectrum is determined in the main by the fractional parentage coefficients which are independent on $N'L'$. Moreover, the excitation spectrum of the final nucleus taken in a $(^{7}Li, \alpha)$ reaction or in a $(^{19}F, ^{16}O)$ reaction will be, as a rule, similar to the spectrum of the corresponding usual spectroscopic factors.

If one considers, for example, the reaction $(^{10}B, ^{7}Li)$ then the three nucleons can be transferred not only in an intermediate state the orbital wave function of which is symmetric $([f_k] = [f'_k] = [3])$ but also in an intermediate state with the symmetry $[f_k] = [f'_k] = [21]$.

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x/ The wave function of $^{16}O_{g.s}$ is known only in an approximation not taking into account $(0p)^{-n}$ $(1s, 0d)^{n}$ admixtures $(n=3,4)$ which are important for the calculation of the structure factors of $(^{19}F, ^{16}O)$ reactions. But the main results discussed here will not be influenced by taking into account such admixtures to the wave function of $^{16}O_{g.s}$.

The fractional parentage coefficients $^{10}B \to {}^{7}Li + [21$ are not smaller than the farctional parentage coefficients $^{10}B \to {}^{7}Li + [3]$. The values Z and T for the reaction $^{7}Li(^{10}B, {}^{7}Li)\ {}^{0}B$ are given in table 10. Analogous results follow if the three transferred nucleons are in the initial nucleus in a shell different from that of the final nucleus (for example$(0p)^{3} \to (0d, 1s)^{3}$) From these results, it follows that the excitation spectrum of the final nucleus observed in reactions like (^{10}B, ^{7}Li) will, as a rule, be different from the excitation spectrum of the same final nucleus observed in (α, n) or (α, p) reactions or in other reactions with three-particle transfer as ($^{7}Li, \alpha$), (^{14}N, ^{11}B), (^{15}N, ^{12}C),(^{19}F ,^{16}O). It must be remarked here, that this result follows from the assumption that in all considered cases the reaction machanism is the direct transfer of three particles from the initial nucleus to the final nucleus.

3.3. Reactions with transfer of four particles

In table 11, the values Z_0 and Z of the theory without factorization of the amplitude $A_{NL}^{N'L'}$ and the values Y and Y_0 of the theory with factorization of the amplitude $A_{NL}^{N'L'}$ into two parts a_{NL} and $a_{N'L'}$ are given for four-particle transfer $(1p)^{4} \to$ $\to (1p)^{4}$. The Z_0 and Z are maximal, as a rule, if the quantum numbers of the transferred nucleons in the initial nucleus are the same as the quantum numbers of the transferred nucleons in the final nucleus. The Y_0 and Y do not show such a regularity. This result is analogous to the results which one obtains in the case of two- and three-particle transfer.

That means, the condition $n\ell = n'.\ell'$ seems to be justifiable also in the case of four-particle transfer. It leads in the most cases to a limitation of the number of terms depending on N', L' in the sum (15) what is important for the analysis of a concrete reaction.

The spectroscopic factors for α -particles having the wave function $[4]\ {}^{11}S$ are proportional to T. The structure factors $A_{NL}^{N'L'}$ determining the excitation of the levels of the final nucleus $B+4$ in reactions with transfer of four particles are proportional to $K(n\ell,NL,L_k)K(n\ell,N'L',L'_k) = K_0(00,N'L',L'_k)$ only in that case in which the particles come from the $0s$-shell of the initial nucleus A.

The wave function of ^6Li is $[42]\ {}^{13}S$ and that of ^7Li is $[43]\ {}^{22}P$ in the framework of the shell model /10/. The fractional parentage coefficients are, in general, small for a group of nucleons which consists from nucleons of different shells. Moreover, the nuclei $^6Li_{g.s.}$ and $^7Li_{g.s.}$ show some deviations from the shell-model description in favour of a cluster-model description as $\alpha + d$ and $\alpha + t$. Therefore, the four particles transferred in $(^6Li, d)$ reactions or in $(^7Li, t)$ reactions come from the $0s$-shell of the lithium nuclei, in the main. Such a result is confirmed also by experiments /12/. That means, the excitation spectrum of the final nucleus which one gets in $(^6Li, d)$ and $(^7Li, t)$ reactions must correspond to the spectrum of the usual spectroscopic factors for α-particles.

The wave function of $^{20}Ne_{g.s.}$ is

$$\Psi = 0.667\,[4]\,d^4\ {}^{11}S + 0.342\,[4]\,d^3 s\ {}^{11}S +$$

$$+ 0.548\,[4]\,d^2 s^2\ {}^{11}S + 0.227\,[4]\,s^4\ {}^{11}S + \dots$$

The wave function of $^{16}O_{g.s.}$ contains some admixtures of $(0p)^{-4}(1s,0d)^4$ components to the closed $0p$-core. These admixtures are important for the calculation of the fractional parentage coefficients $^{20}Ne_{g.s.} \to$ $\to\ {}^{16}O_{g.s.} + 4$ nucleons, but they are not yet known. Therefore, one can say only some words about $(^{20}Ne, {}^{16}O)$ reactions. The four nucleons transferred in $(^{20}Ne, {}^{16}O)$ reactions have the configuration $[4]$ as an α-particle, in the main. In all probability, there are no interference effects which lead to an excitation spectrum of the final nucleus observed in (^{20}Ne, ^{16}O) reactions which is not similar to the spectrum of the spectroscopic factors for α-particles.

As to reactions of the type (^{10}B, ^{6}Li) there is no reason for it that the excitation spectrum of the final nucleus must be similar to the spectrum of the spectroscopic factors for α-particles. The fractional parentage coefficients $^{10}B_{g.s.} \to {}^{6}Li_{g.s.} + \alpha$ are small /14/. They are much smaller than the fractional parentage coefficients $^{10}B_{g.s.} \to {}^{6}Li^{*} + \alpha$. The Z_0 for $(0p)^4 \to (0p)^4$ are given in table 12. They are nonvanishing also if the four transferred nucleons have a symmetry other than [4] in the intermediate state. Therefore, the four particles can be transferred not only in an intermediate state with the wave function of an α-particle but also in states with some other wave function. The excitation spectrum of the final nucleus taken in (^{10}B, ^{6}Li) reactions can be similar to the spectrum of the spectroscopic factors for α-particles only by chance.

Analogous results follow for the reaction (^{15}N, ^{11}B) and other reactions of such a type.

As to the ($^{16}O_{g.s.}$, $^{12}C_{g.s.}$) reaction, one can say only some words. The fractional parentage coefficients for the main configuration $(0p)^{12} \to (0p)^{8} + (0p)^{4}$ [4] are small /14/: they lead to a calculated spectroscopic factor for α-particles which is only 5% of the sum of spectroscopic factors for α-particles with respect to all excited states of ^{12}C (calculated for the pure configurations $(0p)^{12}$ and $(0p)^{8}$). Therefore, admixtures of the type $(0p)^{-4}\,(1s, 0d)^{4}$ to the wave functions of $^{16}O_{g.s.}$ and $^{12}C_{g.s.}$ as well as transfer of the four particles with a symmetry different from the symmetry of an α-particle can play here an important role. Especially, they can lead to interference effects.

4. Conclusions

In this paper, the structure factors $A_{NL}^{N'L'}$ determining the probability for excitation of the levels of the final nucleus in reactions with transfer of k nucleons

$$A + B \rightarrow (A-k) + (B+k)$$

($k = 2,3,4$) are discussed in more details. It is shown that there is no factorization of the spectroscopic factor of the reaction

$$S \sim \sum A_{NL}^{N'L'} B_{NL}^{N'L'} \tag{21}$$

into two factors each of which depends only on the properties of one pair of nuclei, either of A and $A-k$ or of B and $B+k$. The number of terms with different N',L' depends not only on the nuclei B and $B+k$, but also on A and $A-k$. That means, one has to calculate every reaction individually what seems to be a very complicated procedure. In reality, however, the connection of N,L and N',L' leads in most cases to a reduction of the number of terms in (21), in no one case, however, to an enlargement. That means, many reactions become simple and accessible to an analysis only by the connection between N,L and N',L'.

The reactions in which the transferred nucleons come from the $0s$-shell of the nucleus A are of some importance. In these cases, the connection between N,L and N',L' leads to a strong reduction of terms with different N',L'. Moreover, the structure factors $A_{NL}^{N'L'}$ determining the probability of excitation of the final nucleus are proportional to the usual spectroscopic factors. Therefore, these reactions are very suitable for an analysis of the nuclear structure.

The excitation spectra of the final nucleus observed in different reactions with transfer of k nucleons on the same target nucleus will be different from one another, as a rule, and different from the spectrum of the corresponding usual spectroscopic factors for k nucleons. There are only a few exceptions from this rule. For example, the ($^{6}Li, \alpha$) reactions will lead to an excitation spectrum of the final nucleus similar to the spectrum of reduced deuteron widths. Attention should be attracted to the fact that this result follows from the assumption that the reaction mechanism is the direct transfer of a nucleon group from the initial nucleus to the final nucleus. No effects like excitation of A or B before transfer or successive transfer of single nucleons are taken into account.

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Table 1

Comparison of Z and Y for a reaction with two-particle transfer ($A=9, B=7, (0p)^2 \to (0p)^2$):

$[f_k]\,L_k$	$[f_k']\,L_k'$	Z_0	Z	Y_0	Y
[2] S	[2] S	1.000	1.327	0	0.041
[2] S	[2] D	0.500	0.827	0	0.041
[2] D	[2] D	1.000	1.327	0	0.041
[11] P	[11] P	1.000	1.286	1.000	1.284
[2] S	[11] P	0	0	0	0.229
[2] D	[11] P	0	0	0	0.229

Table 2

Comparison of Z and T for the reaction $^7Li(^6Li,\alpha)^9Be$.

$[f_k]\,L_k$	$[f_k']\,L_k'$	Z	T
[2] S	[2] S	1.464	0.964
	[2] D	0.964	0.964
	[11] P	0	0

Table 3

Comparison of Z and T for the reaction $^{12}C(^9Be,\ ^7Li)^{14}N$.

$[f_k]\,L_k$	$[f_k']\,L_k'$	Z	T
[2] S	[2] S	1.250	0.745
[2] D	[2] S	0.745	0.745
[2] S	[2] D	0.745	0.745
[2] D	[2] D	1.250	0.745
[11] P	[11] P	1.225	0

Table 4

Comparison of Z and T for the reaction $^{12}C(^{11}B, ^{9}Be)^{14}N$.

$[f_k]L_k$	$[f_k']L_k'$	Z	T
[2] S	[2] S	1.213	0.713
[2] D	[2] S	0.713	0.713
[2] S	[2] D	0.713	0.713
[2] D	[2] D	1.213	0.713
[11] P	[11] P	1.194	0

Table 5

Comparison of Z and T for the reaction $(^{14}N, ^{12}C)$ on ^{16}O.

$(n_i l_i)^2 [f_k] L_k$	$(n_i' l_i')^2 [f_k'] L_k'$	Z	T
$(0p)^2$ [2] S	$(1s)^2$ [2] S	0.344	0.477
$(0p)^2$ [2] D	[2] S	1.069	0.477
$(0p)^2$ [2] S	$(0p)^2$ [2] S	1.019	0.426
	[2] D	0.652	0.301
	[2] G	0.639	0.639
$(0p)^2$ [2] D	$(0d)^2$ [2] S	0.161	0.426
	[2] D	0.122	0.301
	[2] G	1.037	0.639
$(0p)^2$ [11] P	$(0p)^2$ [11] P	0.911	0
	[11] F	0.911	0
$(0p)^2$ [2] S	(1s,0d) [2] D	0.658	0.564
$(0p)^2$ [2] D	[2] D	1.154	0.564
$(0p)^2$ [11] P	[11] D	0.263	0

Table 6

Comparison of Z and Y for a reaction with three-particle transfer
(A = 15, B = 12, $(0p)^3 \to (0p)^3$).

$[f_k]$	L_k	$[f_k']$	L_k'	Z_c	Z	Y_0	Y
[3]	P	[3]	P	1.000	1.338	0.040	0.022
[3]	P	[3]	F	0.646	0.959	0.019	0.001
[3]	F	[3]	F	1.000	1.398	0.015	0.000
[21]	P	[21]	P	1.000	1.250	0.006	0.009
[21]	P	[21]	D	−0.745	−0.934	−0.078	−0.099
[21]	D	[21]	D	1.000	1.250	1.000	1.250
[111]	S	[111]	S	0	0	0	0

Table 7

Comparison of Z and T for the reaction $^{12}C(^{7}Li, \alpha)^{1}N$
with $(0p)^3 \to (0p)^3$.

$[f_k]$	L_k	$[f_k']$	L_k'	Z	T
[3]	P	[3]	P	1.725	0.641
		[3]	F	1.334	0.697

Table (8 .

Comparison of Z and T for the reaction
$^{12}C\ (^{19}F, ^{16}O)\ ^{15}N$ with $(0d^2, 1s) \to (0p)^3$.

$[f_k]$	L_k	$[f_k']$	L_k'	Z	--
[3]	S	[3]	P	−0.986	−0.314
		[3]	F	−0.638	−0.342

Table 9

Comparison of Z and T for the reaction ($^7Li, \alpha$) on ^{16}O .

$(n_i l_i)^3[f_k]\,L_k$	$(n_i' l_i')^3[f_k']\,L_k'$	Z	T
$(0p)^3$ [3] P	$(1s)^3$ [3] S	-0.747	-0.374
	$(0d^2,1s)$[3] S	-0.521	-1.301
	D	-0.353	-1.066
	G	-1.318	-0.509
$(0p)^3$ [3] P	$((d)^3$ [3] S	0.884	0.315
	D	0.794	0.313
	F	-0.358	0
	G	0.820	0.350
	I	1.160	0.624
$(0p)^3$ [3] P	$(1s^2,0d)$[3] D	-1.375	-0.443

Table 10

Comparison of Z and T for the reaction $^7Li(^{10}B, ^7Li)^{10}B$.

$[f_k]\,L_k$	$[f_k']\,L_k'$	Z	T
[3] P	[3] P	1.635	0.578
	[3] F	1.244	0.628
[3] F	[3] P	1.244	0.628
	[3] F	1.750	0.684
[21] P	[21] P	1.429	0
	[21] D	-1.065	0
[21] D	[21] P	-1.065	0
	[21] D	1.429	0
[111] S	[111] S	0	0

Table 11

Comparison of Z, Y and T for the reaction $^{12}C(^{16}O,\,^{12}C)\,^{16}O$.

$[f_k]\,L_k$	$[f_k']\,L_k'$	Z_0	Z	Y_0	Y	T_0	T
[4] S	[4] S	1.000	1.855	0.266	0.420	0.408	0.395
	[4] D	1.021	1.990	0.403	0.650	0.408	0.395
	[4] G	0.625	1.282	0.082	0.112	0.408	0.395
[4] D	[4] S	1.021	1.990	0.403	0.650	0.408	0.395
	[4] D	1.441	2.668	0.612	1.005	0.408	0.395
	[4] G	0.946	1.856	0.124	0.173	0.408	0.395
[4] G	[4] S	0.625	1.282	0.082	0.112	0.408	0.395
	[4] D	0.946	1.856	0.124	0.173	0.408	0.395
	[4] G	1.000	1.855	0.025	0.030	0.408	0.395
[31] P	[31] P	1.000	1.490	0.057	0.008	0	0
	[31] D	−0.026	0.018	−0.073	−0.031	0	0
	[31] F	0.044	0.156	0.175	0.068	0	0
[31] D	[31] P	−0.026	0.018	−0.073	−0.031	0	0
	[31] D	0.149	0.213	0.094	0.114	0	0
	[31] F	0.081	0.152	−0.226	−0.251	0	0
[31] F	[31] P	0.044	0.156	0.175	0.068	0	0
	[31] D	0.081	0.152	−0.226	−0.251	0	0
	[31] F	1.000	1.533	0.544	0.554	0	0
[22] S	[22] S	1.000	1.600	0.399	0.552	0	0
	[22] D	−0.676	−1.101	−0.234	−0.369	0	0
[22] D	[22] S	−0.676	−1.101	−0.234	−0.369	0	0
	[22] D	0.515	0.826	0.138	0.244	0	0
[211] P	[211] P	1.000	1.331	1.000	1.331	0	0