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Mλ -TRANSITION PROBABILITIES AND MAGNETIC MOMENTS IN THE RARE EARTH NUCLEI CALCULATED WITHIN THE FRAMEWORK OF THE DEFORMED SAXON-WOODS POTENTIAL

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Mλ -TRANSITION PROBABILITIES AND MAGNETIC MOMENTS IN THE RARE EARTH NUCLEI CALCULATED WITHIN THE FRAMEWORK OF THE DEFORMED SAXON-WOODS POTENTIAL

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1. Introduction

One of the fruitful methods for testing the nuclear models is the comparison of experimentally determined magnetic moments and absolute gamma ray transition probabilities with the theoretical predictions, because the matrix elements in these cases are very sensitive to the structure of the wave functions of the states involved.

Nowadays, the deformed Saxon-Woods potential is applied more and more frequently for analysis of different nuclear properties. The method described in ref./1/ is very convenient for the calculation of single particle wave functions in this potential. These wave functions have been successfully used for the description of N-forbidden β-decays/2/ and $E\lambda$-transitions/3/ and for the calculation of the energies of the collective states in even nuclei/4/, as well.

We calculated the magnetic moments and the magnetic transition probabilities using the wave functions for the deformed Saxon-Woods potential/5/ and compared the results with the experimental data and the predictions of the Nilsson model/6/ (NM).

The pairing interaction and the Coriolis coupling were not included in our calculation, because the pairing interaction causes no essential modification of the magnetic transitions near the Fermi surface, and the Coriolis coupling is important only for certain nuclei, f. i. ^{155}Gd and ^{161}Dy .

2. $M\lambda$ -Transitions

We use the following definition of the hindrance factors F_{SW} and F_N

$$F_{SW} = \frac{T\ 1/2\ (\text{exp.})}{T\ 1/2\ (\text{Saxon-Woods})} \qquad F_N = \frac{T\ 1/2\ (\text{exp.})}{T\ 1/2\ (\text{Nilsson})} ,$$

where the symbol $T_{1/2}$ stands for the half-life. In our calculation we used the free nucleon g -factors. The rotational g_R -factors and the deformation parameters β have the same values as in ref./6/. In accordance with the different representation of the wave function in the SWM the Nilsson formula/7/ for the $M\lambda$ -transitions and the magnetic moments was slightly modified.

In general, we can expect an essential effect due to Saxon-Woods model in the following cases:

i) If the radial part of the SW-wave functions differs from that of the Nilsson wave functions, the latter seems to be important for $M\lambda$ -transitions with $\lambda \geq 2$.

ii) If the mixing of states with different principle quantum numbers is essential.

iii) Some effects we can expect for proton transitions, because in the SWM the Coulomb interaction is treated more exactly.

2.1. Magnetic Dipole Transitions

The hindrance factors F_{SW} and F_N for $M\lambda$- transitions are listed in Tables 1 and 2.

The hindrance factors of the neutron transition $\frac{5}{2}\frac{5}{2}^+[642] \rightarrow \frac{3}{2}\frac{3}{2}^+[651]$ in ^{155}Gd differ strongly in both models (factor $\approx$ 2000). This remarkable result is due to the $\Delta N = 2$ mixing in the $[651]$ -state.

For the neutron transition $\frac{3}{2}\frac{3}{2}^-[521] \to \frac{5}{2}\frac{5}{2}^-[523]$ in ^{167}Dy the F_{SW} is 4 times larger than F_N and depends weakly on the change of the deformation parameter. A more detailed discussion of this case can be found in the next section. For the proton transition $\frac{5}{2}\frac{5}{2}^+[402] \to \frac{7}{2}\frac{7}{2}^+[404]$ in ^{181}Ta the hindrance factor F_{SW} is 30 times smaller than the Nilsson's one, but this is not sufficient to explain the strong experimental retardation. Recently, S. Wahlborn/8/ suggested that the large hindrance factor may be accounted for by the use of modified magnetic dipole operator.

In the NM the hindrance factors for the M1 proton transitions in ^{153}Eu and ^{155}Eu change very rapidly with the deformation/6/, therefore, in these cases we can fit the experimental data well by a small variation of the deformation parameter. In the SWM the dependence of the transition matrix elements on the deformation parameter β is as a rule, weaker than in the NM, unless the admixture of states with different principle quantum numbers is essential. Especially for the proton transitions $\frac{5}{2}\frac{3}{2}^+[411] \to \frac{7}{2}\frac{5}{2}^+[413]$ in ^{153}Eu and $\frac{5}{2}\frac{3}{2}[411] \to \frac{5}{2}\frac{5}{2}^+[411]$ in ^{155}Eu we obtained too large hindrance factors.

2.2. Magnetic Quadrupole Transitions

For proton and neutron M2-transitions we obtained in both models nearly the same theoretical results. In all cases the dependence of the F_{SW} on the deformation is very weak.

2.3. Magnetic Octupole Transitions

For the proton transition the hindrance factor is only 4 times larger than the F_N and, therefore, can still not explain this fast transition. If one identifies the state in ^{179}W at 221 KeV as $\frac{1}{2}\frac{1}{2}^-[521]$ instead of $\frac{1}{2}\frac{1}{2}^-[510]$ the M3- neutron transition to the ground states becomes even more delayed. The hindrance factor in this case is $F_{SW} \approx 500$.

3. g_k -Values

The magnetic properties of deformed nuclei can be expressed in terms of two parameters g_k and g_R . A comparison of the experimental g_k - values with the theoretically predicted ones in the NM shows deviations of 50 to 100%. Such deviations have been also obtained for the magnetic moments of ground state/10/. Latter on de Boer and Rogers suggested to use the effective factors in place of free nucleon g_s -factor to account for the effect of the spin interaction of the odd nucleon with the even core/11/.

We calculated the effective spin g -factors g_s^{z} so that the g_k -values obtained from Nilsson's formula

$$g_k = \frac{1}{k} (g_s^{z} < s_z > + g_\ell < \ell_z >) \qquad (1)$$

agree with the experimentally derived values. The expectation values $< s_z >$ and $<\ell_z>$ have been calculated using SW -wave functions.

The results of the calculations and the predictions of the NM are given in Tables 7 and 8.

For $^{155/157}Gd$ and ^{163}Dy we obtained values larger than 1.5 for the ratio g_s^{z} / g_s This confusing result seems to be in contradiction with the usual belief that $g_s^{z} / g_s < 1$ must be valid. With wave functions of ref./12/ we also obtained in those cases nearly the same deviations of the g_k -values from the experimental ones. These results are likely due to the large spacing of the $1h^{9/2}$ and $2f^{7/2}$ states in the spherical Saxon-Woods potential.

If we only include in the deformed Saxon-Woods potential besides the usual quadrupole deformation β the hexadecupole deformation β_4/13/ and leave the spherical SW -well unchanged and vary β_4 at fixed β , we obtain no better results. We established that the ratio g_s^{z}/g_s tended to unity as β_4 was increased, but a more reasonable value for the effective g_s -factors would have been achieved at unreasonable values of β_4 ($\beta_4 > 0.1$).

For proton states we got nearly the same g_k -values as in the NM.

For states with $k = 1/2$ one can calculate two different g_s -factors, which can be estimated from the magnetic parameters g_k and b_0 /14/.

$$b_0 (g_k - g_R) = - a (g_\ell - g_R^0) - (g_s^+ - g_\ell) < s_+ > , \qquad (2)$$

where $<s_+>$ is the expectation value of the spin operator $s_+ = s_x + i s_y$ while the constant a is the decoupling parameter and g_R^0 is the rotational g -factor for the even core.

For the nuclei ^{182}W, ^{171}Yb, ^{171}Tm and ^{169}Tm we calculated the effective g_s -factors with various deformation parameters β_4 at fixed β .

The results are listed in Table 5. The most reasonable effective g_s -factors are obtained for those deformation parameters β_4 , which agree with the estimate given in ref./15/.

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Table 1. Mλ-transitions of odd-neutron nuclei in the rare earth region.

Nucleus level in KeV	Transition initial state	final state	Transition energy in KeV	Multipolarity	Deformation	F_N	F_{sw}
^{155}Gd 105.32	5/2 $5/2^+$[642] →	3/2 $3/2^+$[651]	18.7	M1	0.33	300	0.18
^{161}Dy 74.5	3/2 $3/2^+$[521] →	5/2 $5/2^-$[523]	48.5	M1	0.32	1.4	5.2
^{173}Yb 351.2	7/2 $7/2^+$[633] →	5/2 $5/2^-$[512]	351.2	M2	0.30	$\gtrsim$0.3	$\gtrsim$0.56
^{173}Yb 351.2	7/2 $7/2^+$[633] →	7/2 $5/2^-$[512]	272.4	M2	0.30	0.16	0.31
^{177}Hf 321.4	9/2 $9/2^+$[624] →	7/2 $7/2^-$[514]	321.4	M2	0.29	0.6	0.27
^{177}Hf 321.4	9/2 $9/2^+$[624] →	9/2 $7/2^-$[514]	208.4	M2	0.29	0.1	0.05
^{181}W 365.5	5/2 $5/2^-$[512] →	9/2 $9/2^+$[624]	365.5	M2	0.23	41	53.1
^{175}Yb 500	1/2 $1/2^-$[510] →	7/2 $7/2^-$[514]	500	M3	0.30	0.9	3.15
^{177}Yb 322	1/2 $1/2^-$[510] →	7/2 $7/2^-$[514]	288	M3	0.30	2.0	6.75
^{179}Hf 375	1/2 $1/2^-$[510] →	7/2 7/2[514]	161	M3	0.30	2.6	6.0
^{179}W 221.8	1/2 $1/2^-$[510] →	7/2 $7/2^-$[514]	221.8	M3	0.22	180	200

Table2. Mλ -transitions of odd-proton nuclei in the rare earth region.

Nucleus level in KeV	Transition initial state	final state	Transition energy in KeV	Multipolarity	Deformation	F_N	F_{SW}
^{153}Eu 172.85	5/2 $3/2^+[411]$	→ 7/2 $5/2^+[413]$	89.48	M1	0.26	0.26	1.4
^{153}Eu 103.2	3/2 $3/2^+[411]$	→ 5/2 $5/2^+[413]$	103.2	M1	0.26	0.24	1.34
^{153}Eu 172.85	5/2 $3/2^+[411]$	→ 5/2 $5/2^+[413]$	172.85	M1	0.26	3.2	17.5
^{155}Eu 246	5/2 $3/2^+[411]$	→ 5/2 $5/2^+[413]$	246	M1	0.26	1.8	10.2
^{175}Lu 343.4	5/2 $5/2^+[402]$	→ 7/2 $7/2^+[404]$	343.4	M1	0.24	2.5	0.56
^{181}Ta 482	5/2 $5/2^+[402]$	→ 7/2 $7/2^+[404]$	482	M1	0.2	8100	270
^{165}Ho 211.1	3/2 $3/2^+[411]$	→ 7/2 $7/2^-[523]$	211.1	M2	0.26	0.24	0.32
^{175}Lu 396.3	9/2 $9/2^-[514]$	→ 7/2 $7/2^+[404]$	396.3	M2	0.24	0.44	0.3
^{175}Lu 396.3	9/2 $9/2^-[514]$	→ 9/2 $7/2^+[404]$	282.6	M2	0.24	1.10	0.74
^{177}Lu 147	9/2 $9/2^-[514]$	→ 7/2 $7/2^+[404]$	147	M2	0.22	0.37	0.24
^{181}Ta 482	5/2 $5/2^+[402]$	→ 9/2 $9/2^-[514]$	476	M2	0.2	1.8	1.0
^{181}Ta	1/2 $1/2^+[411]$	→ 7/2 $7/2^+[404]$	615	M3	0.2	$1.9.10^{-2}$	8.10^{-2}

Table 3. The effective g -factors g_s^z/g_s for odd-neutron nuclei in the rare earth region (Experimental data from ref./19/

Nucleus	state	$g_{k\,exp}$	$g_s^z/g_{s\,SWM}$	$g_s^z/g_{s\,NM}$
^{155}Gd	$3/2^-[511]$	-0.476	1.64	0.60
^{157}Gd	$3/2^-[521]$	-0.539	1.82	0.68
^{161}Dy	$5/2^+[642]$	-0.34 -0.30	0.85 0.75	0.71
^{163}Dy	$5/2^-[523]$	0.24	1.6	0.50
^{165}Dy	$5/2^+[633]$	-0.24	0.64	0.63
^{167}Er	$5/2^+[633]$	-0.249	0.66	0.61
^{173}Yb	$5/2^-[512]$	-0.483 -0.491	0.99 1.00	0.79
^{177}Yb	$7/2^-[514]$	0.211	0.64	0.51
^{179}Hf	$9/2^+[624]$	-0.186	0.54	0.52

Table 4. The effective g-factors g^z_s/g_s for odd-proton nuclei in the rare earth region (Experimental data from ref./19/)

Nucleus	state	$g_{k\ exp}$	$g^z_s/g_{s\ SWM}$	$g^z_s/g_{s\ NM}$
^{153}Eu	$5/2^+$ [413]	0.654	0.56	0.69
^{159}Tb	$3/2^+$ [411]	1.788	0.70	0.71
^{165}Ho	$7/2^-$ [523]	1.329	0.70	0.70
^{175}Lu	$7/2^+$ [404]	0.716	0.58	0.58
^{177}Lu	$7/2^+$ [404]	0.722	0.57	0.57
^{181}Lu	$7/2^+$ [404]	0.771	0.51	0.51
^{185}Re	$5/2^+$ [402]	1.61	0.76	0.75
^{187}Re	$5/2^+$ [402]	1.86	o.78	0.77

Table 5. The effective g-factors g^π_s/g_s and g^+_s/g_s for odd-proton and odd-neutron nuclei in the rare earth region.

Nucleus	state	$g_{k\ exp}$	g_R	g^0_R	b_0	a	β	g^π_s/g_s			g^+_s/g_s			Ref.
								$\beta_4=0$	$\beta_4=-0.04$	$\beta_4=-0.08$	$\beta_4=0$	$\beta_4=-0.04$	$\beta_4=-0.08$	
^{169}Tm	1/2 [411]	-1.62	0.403	0.331	-0.142	-0.78	0.30	0.80	0.81	0.82	0.51	0.49	0.475	16
		-1.60	0.406		-0.147			0.79	0.80	0.81	0.50	0.48	0.465	
^{171}Tm	1/2 [411]	-1.42	0.432	0.329	-0.22	-0.86	0.30	0.75	0.76	0.78	0.42	0.40	0.38	16
		-1.46	0.391		-0.18			0.76	0.77	0.79	0.53	0.505	0.48	
^{171}Yb	1/2 [521]	-1.43	0.28	0.34	-0.42	0.87	0.30	0.94	1.10	1.44	0.68	0.625	0.56	17
^{183}W	1/2 [510]	-1.87	0.28	0.23	0.49	0.19	0.20	1.0	0.92	0.73	0.385	0.37	0.34	18