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A.A.Korneichuk, L.A.Malov,
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1. The collective nonrotational states in even-even deformed nuclei are studied in a large number of papers. Recently the energies and the wave functions of the one-photon states have been studied on the basis of the microscopic approach of the superfluid nuclear model. In these calculations the interactions leading to the superconductive pairing correlations and the multipole-multipole interactions are taken into account. In papers[1-8] in the framework of the approximate second quantization method, the energies of the quadrupole one-phonon states and the B(E2) values are calculated for a number of even-even deformed nuclei. The mathematical methods for considering the one-phonon states and the experimental information about them are summed up in ref.[9]. The component composition of the one-phonon states is given in ref.[10] in the form of tables. The properties of the one-phonon states taking into account the spin-quadrupole interactions are investigated in refs.[11,12] The energies and the $B(E\lambda)$ values for the quadrupole and octupole states are calculated in paper[13] with the use of the surface delta interaction.

All the calculations of the one-phonon state characteristics are performed up to now with the use of the one-particle energies and the wave functions of the Nilsson potential[14]. In some cases the accuracy of calculations was restricted by a rough description of the one-particle energies and the wave functions of the average field. Besides, the available calculations are carried out for a restricted and therefore insufficient number of even-even deformed nuclei.

In the present paper the characteristics of the one-phonon states of the quadrupole type are carried out for a large number of even-even deformed nuclei in the region $150 \leq A \leq 174$. The calculations are made on the basis of the approximate second quantization method with the use of the one-particle energies and the wave functions of the Woods-Saxon potential calculated in refs.[15,16] as well as with the use of the Nilsson potential modified in ref.[17]. The results of calculations are compared with the corresponding experimental data.

2. The collective vibrational states are treated by means of the approximate second quantization method (random phase approximation). To describe these states we introduce the phonon operators $Q_i(\lambda\mu)$ of multipolarity $\lambda\mu$.

$$Q_i(\lambda\mu) = \tfrac{1}{2} \sum_{qq'} \{ \psi^{\lambda\mu i}_{qq'} A(q,q') - \phi^{\lambda\mu i}_{qq'} A^{+}(qq') \}, \qquad (1)$$

the operators

$$A(q,q') = \frac{1}{\sqrt{2}} \sum_{\sigma} \sigma a_{q\sigma} a_{q-\sigma}, \qquad \left(\text{or} = \frac{1}{\sqrt{2}} \sum_{\sigma} a_{q\sigma} a_{q'\sigma}\right), \quad (2)$$

$$B(q,q') = \sum_{\sigma} a^{+}_{q\sigma} a_{q'\sigma} \qquad \left(\text{or} = \sum_{\sigma} \sigma a^{+}_{q-\sigma} a_{q'\sigma}\right), \quad (3)$$

are expressed through the quasiparticle operators $a_{q\sigma}$, $q\sigma$ denote the quantum numbers of the average field levels, $\sigma = \pm 1$.

The wave function of the ground state of an even-even nucleus is defined as one containing no phonons

$$Q_i(\lambda\mu)\ \Psi = 0 \qquad (4)$$

and the excited states are treated as one-phonon

$$Q_i(\lambda\mu)^+ \Psi \ . \qquad (5)$$

Next, we calculate the average value of the Hamiltonian H_0 operator containing the average nuclear field, the interaction leading to pairing correlations and the multipole-multipole interactions over the state $Q_i^+(\lambda\mu)\ \Psi$. The energy $\omega_i^{\lambda\mu}$ of the state (5) and the functions $\psi^i_{qq'}$, $\phi^i_{qq'}$ are found by means of the variational principle (the excited states are lebelled by $i = 1,2,3 \ldots$ in the order of increase of the excitation energy)

$$\delta\{ < Q_i(\lambda\mu)\, H_0\, Q_i(\lambda\mu)^+ > - \frac{\omega_i^{\lambda\mu}}{2} (\sum_{qq'} (\psi^{\lambda\mu i}_{qq'})^2 - (\phi^{\lambda\mu i}_{qq'})^2 - 2) \} \ .$$

When the diagonal matrix elements are absent (i.e. for gamma and octupole vibrations) the secular equation is of the form:

$$1 = 2\kappa^{(\lambda)} \sum_{qq'} \frac{(f^{\lambda\mu}(qq'))^2 U^2_{qq'} (\epsilon(q) + \epsilon(q'))}{(\epsilon(q) + \epsilon(q'))^2 - (\omega_i^{\lambda\mu})^2} \ . \qquad (7)$$

Here $\kappa^{(\lambda)}$ is the interaction constant of multipolarity λ , in this case it is assumed $\kappa^{(\lambda)}_{pp} = \kappa^{(\lambda)}_{nn} = \kappa^{(\lambda)}_{np} = \kappa^{(\lambda)}$, $f^{\lambda\mu}(qq')$ is the matrix element of the multipole moment operator $\lambda\mu$.

$$\epsilon(q) = \sqrt{C^2 + \{E(q) - \lambda\}^2}\,, \quad U_{qq'} = U_q V_{q'} + U_{q'} V_q\,,$$

in this case $U_q^2 + V_q^2 = 1$, $U_q^2 = \frac{1}{2}\{1 + \frac{E(q) - \lambda}{\epsilon(q)}\}$, where C is the correlation function, λ is the chemical potential, $E(q)$ are the energies of the average field one-particle levels.

The function $U_{qq'}$ defines the predominant interaction particle-hole as compared to the interactions particle-hole and hole-hole. Using the normalization condition it is easy to find

$$\psi^{\lambda\mu i}_{qq'} = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{Y^i(\lambda\mu)}} \frac{f^{\lambda\mu}(qq') U_{qq'}}{\epsilon(q) + \epsilon(q') - \omega_i^{\lambda\mu}}\,, \qquad (8)$$

$$\phi^{\lambda\mu i}_{qq'} = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{Y^i(\lambda\mu)}} \frac{f^{\lambda\mu}(qq') U_{qq'}}{\epsilon(q) + \epsilon(q') + \omega_i^{\lambda\mu}}\,, \qquad (9)$$

$$Y^i(\lambda\mu) = \sum_{qq'} \frac{(f^{\lambda\mu}(qq'))^2 U^2_{qq'} \omega_i^{\lambda\mu} (\epsilon(q) + \epsilon(q'))}{[(\epsilon(q) + \epsilon(q'))^2 - (\omega_i^{\lambda\mu})^2]^2}\,. \qquad (10)$$

In this treatment the collective nonrotational excited states are considered side by side with the two- quasiparticle excited states. The wave functions of the collective nonrotational states are a superposition of the two-quasiparticle states.

The collective $K^\pi = 0^+$ states contain both the nondiagonal and diagonal matrix elements of the quadrupole moment operator.

Since among the $K^{\pi} = 0^{+}$ states there is one redundant (spurious) state then the corresponding secular equation is found simultaneously with the exclusion of the spurious state. To this end the method suggested by Baranger[18] is used. The explicit form of these equations is given, e.g. in ref. [3], their particularities are studied in ref.[6].

To improve the accuracy of calculations in ref.[6] a simplified method of taking into account the blocking effect was suggested. It consists in the following: firstly, the chemical potentials λ_n and λ_p for the one-phonon states are determined from the conditions of conservation, on the average, of the number of photons and neutrons, which have the form

$$N = \langle Q_i(\lambda\mu) \sum_{s\sigma} a^{+}_{s\sigma} a_{s\sigma} Q^{+}_i(\lambda\mu) \rangle ,$$
$$Z = \langle Q_i(\lambda\mu) \sum_{\nu\sigma} a^{+}_{\nu\sigma} a_{\nu\sigma} Q^{+}_i(\lambda\mu) \rangle ; \qquad (11)$$

secondly, the quantities $\epsilon(q) + \epsilon(q')$ are replaced by $\mathcal{E}(qq') - \mathcal{E}_0$ i.e. by the difference of the energies of the two-quasiparticle and ground state calculated with the account of the blocking effect. This improvement is used in papers[6-10] in calculating the energies of the one-phonon states.

We note the particularities of the solution of the secular equation (7). When an one-phonon state is a strongly collectivized one, the first root $\omega_1^{\lambda\mu}$ is lowered much below the first pole in (7). In this case a small change of $\kappa^{(\lambda)}$ leads to a noticeable change of the collective states energy. To improve the accuracy of calculations in this case it is necessary to go beyond the framework of the quasiboson approximation. If the value of the root is close to that of the pole then the state becomes close to the two-quasipar-

ticle one and the accuracy of calculations is mainly restricted by the rough description of the average field levels.

3. The calculations of the characteristics of the one-phonon states are performed with the following two schemes of the one-particle energies and the corresponding wave functions: the modified Nilsson potential and the Woods-Saxon potential.

In ref.[17] an additional term proportional to $\langle \vec{l}^2 \rangle = \frac{N(N+3)}{2}$ was introduced in the Nilsson potential. The account of this term permitted to construct the one-particle level scheme for nuclei in the actinite region with the same values of the parameters μ and κ and to decrease as compared to the popular scheme given in [19] the number of shifts of the subshells and the number of parameters for nuclei in the region $150 < A < 190$. Some improvement of the parameters of the modified Nilsson potential is made in ref.[20]. As a result, a rather good description of the one-particle average field levels is obtained.

In ref.[15] an approximate method of finding the eigenvalues of the energies and the wave function of the Woods-Saxon potential was suggested which turned out to be very effective. We have used the one-particle energies and the wave functions calculated by this method in [16]. However it was necessary to improve slightly the Woods-Saxon potential parameters. In the scheme of neutron levels given in ref.[16] the sequence of the one-particle $633\uparrow$ and $521\downarrow$ levels is not correct. To eliminate this shortcoming in the spherical basis the following subshells were shifted up as follows: $2f_{7/2}$ by 0.07 MeV, $1h_{9/2}$ by 0.13 MeV and $3p_{1/2}$ by 0.5 MeV. This shift can be made by the two ways: either by changing the spin-orbital interaction constant by 10% or by changing the parameter r_0 by 0.01 (in units 10^{-13}cm).

In the calculations based on the Woods-Saxon potential the region of deformed nuclei $150 < A < 190$ was divided into three zones: A=155, A=165 and A=181 and the calculations of the one-particle energies and the wave function in ref.[16] were performed for these values of A. The results of calculations in the zone A=181 are published separately[21]. In the present calculations for the neutron system an additional zone with A=173 is introduced.

The calculations in the zones A=155 and A=165 are performed for the deformations $\beta_0 = 0.31$ and in the zone A=173 for $\beta_0 = 0.26$.

The levels of the lowest shells in the neutron system up to $N = 3$, in the proton one up to $N = 2$ were not taken into account. Thus, in solving the secular equations 43 neutron and 40 proton levels were calculated. All the matrix elements of the quadrupole moment operators between the one-particle states were taken into account. To this end the program of calculations with the electronic computer BESM-4 was improved, as compared with the calculations in [6-10], so that it became possible to solve secular equations containing up to 500 different matrix elements. The matrix elements of the multipole moment operators by the wave functions of the Woods-Saxon potential were calculated using an ALGOL program.

Owing to the absence of common rules it is very difficult to compare the matrix elements of the operators $r^\lambda Y_{\lambda\mu}$ calculated with the wave functions of the Woods-Saxon and Nilsson potentials. It may be said that the matrix elements allowed by the asymptotic quantum number selection rules are rather close to one another and the remaining ones may differ by a factor of several times. However the following rough relations between the matrix elements

calculated with the wave functions of the Woods-Saxon and Nilsson potentials may be given : a) matrix elements of the type

$$\langle N, n_z, \Lambda | r^{\lambda} Y_{\lambda\mu} | N \pm 2, n'_z, \Lambda' \rangle$$

are close to one another if $n_z = n'_z$; if n_z differs strongly from n'_z the Nilsson matrix elements are very small they are by two orders smaller than those calculated with the wave functions of the Woods-Saxon potential; b) matrix elements of the type $\langle N, n_z, \Lambda | r^{\lambda} Y_{\lambda 0} | N, n'_z, \Lambda \rangle$ are close to one another if $n_z = n'_z$ and if $n_z \neq n'_z$ they may differ by a factor of several times, c) matrix elements of the type $\langle N, n_z \Lambda | r^{\lambda} Y_{\lambda\mu} | N', n'_z, \Lambda' \rangle$ are close to one another for $n_z = n'_z$, in the remaining cases the difference fluctuates strongly.

The pairing interaction constants G_N and G_Z were chosen so that to obtain correct values of the pairing energies. In this case the correlation functions turned out to be close to those given in ref.[20]. The quadrupole-quadrupole interaction constant were chosen so that to describe in a best manner the first $K^{\pi} = 2^+$ state energies. It is taken to be equal to

$$\kappa^{(2)} = (2,5 - 2,6)\ A^{-4/3}\ \frac{\text{MeV}}{\text{cm}^4} \qquad (12)$$

in calculations with the Woods-Saxon potential and equal to

$$\kappa^{(2)} = 6,7\ A^{-4/3}\ \hbar\omega_0^0 \qquad (12')$$

in calculations with the Nilsson potential.

4. We have calculated the energies and the wave functions of the first and second $K^{\pi} = 0^+$ and 2^+ states and the corresponding B(E2) for even-even nuclei in the region $150 \leq A \leq 174$.

A large amount of new information on the energies of the first $K^{\pi} = 2^+$ states and on the B(E2) for them has been recently

obtained[22,23] . In the region of nuclei considered by us this information is complete enough. On the basis of the experimental data the $\kappa^{(2)}_{exp}$ values are calculated which correspond to the energies of the first $K^\pi = 2^+$ states measured experimentally. Fig.1 gives the $\kappa^{(2)}_{exp}$ values for nuclei in the region $150 \leq A \leq 186$ including the results calculated in ref.[21]. These calculations were performed with the energies and the wave functions of the Woods-Saxon potential. It is seen from this figure that $\kappa^{(2)}_{exp} A^{4/3}$ value is about constant; it is practically constant for nuclei from ^{150}Nd to ^{168}Er then it fluctuates weakly and for nuclei from ^{176}Yb to ^{186}Os is practically constant which is by (10-15%) smaller than the corresponding value in the beginning of the deformation region. It should be noted that the introduction of a new zone A=173 leads to a noticeable decreases of the fluctuation of $\kappa^{(2)}_{exp} A^{4/3}$ for nuclei in the region A=168-172. In calculating the first $K^\pi = 2^+$ state energies with the Nilsson potential wave functions the obtained $\kappa^{(2)}_{exp} A^{5/3}$ values deflect strongly from the constant as compared to the results of Fig.1.

According to[24] $\kappa^{(2)}$ is proportional to $A^{-7/3}$. In the calculations with the Nilsson potential wave functions the dimensionless matrix elements are proportional to $A^{-1/3}$. Therefore $\kappa^{(2)}$ (in MeV) is proportional to $A^{-5/3}$. In the calculations with the Woods-Saxon potential wave functions the matrix elements are independent of A. Fig.1 gives the $\kappa^{(2)} A^{7/3}$ values which fluctuate about the constant. Apparently the decrease of $\kappa^{(2)}$ as $A^{-7/3}$ is too large and the dependence $A^{-4/3}$ seems to be preferable. However, in the region $150 < A < 190$ both dependences $\kappa^{(2)}$ on A do not contradict the experimental data.

A small part of the obtained results is given in Table 1. The energies of the first $K^\pi = 2^+$ states and the B(E2) are calculated with the one-particle values of the energies and the wave functions

of the Woods-Saxon and Nilsson potential for the same values of the constants $\kappa^{(2)}$. All B(E2) in one-particle units $B_{s.p.}(E2) = 3.A^{4/3}e^2 10^{-53}\ cm^4$, are calculated with the effective charge value $e^{(2)}_{eff} = 0,2$, i.e. for protons $e_p = 1,2$, for neutrons $e_n = 0,2$. It is seen from Table 1 that the calculated first $K^\pi = 2^+$ state energies and the corresponding B(E2) calculated for the same $\kappa^{(2)}A^{4/3}$ and $e^{(2)}_{eff}$ values are in rather good agreement with experimental data. It should be noted that a part of the obtained results is given in the report by Vogel /25/.

As in ref. /6/ for a given $\kappa^{(2)}A^{4/3}$ the first $K^\pi = 2^+$ states in ^{172}Yb and ^{174}Hf are actually two-quasiparticle ones and the second $K^\pi = 2^+$ states collective ones. The energies of the second $K^\pi = 2^+$ states calculated in ^{172}Yb and ^{174}Hf are 1,8 and 1,9 the B(E2) are 1,8 and 1,6 respectively. A small increase of the constant $\kappa^{(2)}$ results in that for these nuclei the first solutions of (7) become collective, and the second - two-quasiparticle ones. As is shown in ref. /26/ the same effect takes place when one introduces the spin-quadrupole interaction. In other cases the spin-quadrupole interaction effect on the first $K^\pi = 2^+$ states is small and on the $K^\pi = 0^+$ states it is fairly essential.

The calculated characteristics of the second $K^\pi = 2^+$ states point out that they are collectivized more weakly than the first one. The quantities $Y_2(22)$ showing the degree of collectivization are by about two-three orders larger than $Y_1(22)$. The B(E2) for the second $K^\pi = 2^+$ states assume the values 0.02-0.03 one-particle units.

The behaviour of the obtained energies of the first 0^+ states reproduces to a large extent the results given in refs. /6,9/. So the energies of the first 0^+ states are lowered rather strongly in the

beginning of the region of deformed nuclei (0.7 MeV for nuclei with N=90) then they increase with increasing A. In nuclei $^{160-164}Dy$, $^{162-168}Er$ the energies of the first 0^+ states exceed the energies of the first 2^+ states. Most of the first beta-vibrational states are collective ones.

Since the excited $K^\pi=0^+$ states have different structure (among them the two-phonon states are possible) it is difficult to compare the calculated energies and the B(E2) for excited $I^\pi K=2^+0$ states with the corresponding experimental data. If we assume that the experimentally observed first 0^+ states are beta-vibrational and calculate $\kappa^{(2)}_{exp}$ then the value of $\kappa^{(2)}_{exp}A^{4/3}$ differs noticeably from the constant for the nuclei of the considered region. The value of $\kappa^{(2)}_{exp}A^{4/3}$ is somewhat smaller than the corresponding values in Fig.1. in the region of Sm and Gd isotopes and increases noticeably in the region of the Er and Yb isotopes. To describe correctly some lowest $K^\pi=0^+$ excited states further investigations are needed.

5. In the microscopic treatment the collective non- rotational states are considered side by side with the two-quasiparticle ones. The wave functions of the one-phonon states are a superposition of the two-quasiparticle states. The investigations performed showed that the majority of the first excited states with $K^\pi=0^+$ and 2^+ have clearly expressed collective properties and a large number of the two-quasiparticle states contribute to their wave functions.

Information on the component composition of the one-phonon states can be obtained from some experimental data. They include beta decays to one-phonon states, cross sections for direct one-nucleon transfer reactions with the excitation of these states and gamma transitions from higher states to one-phonon states. The

experimental methods of determination of the one-phonon state components are summed up in ref.[27].

In ref.[28] one has attempted to determine the quantities $\psi^{221}_{qq'}$ from the (dp) reaction cross sections normalized to the cross sections for the two-quasiparticle state excitation. Table 2 gives the $\psi^{221}_{qq'}$ calculated with the Nilsson potential wave functions as well as calculated in[10] and in the present paper. In ref.[28] the cross sections calculated with the $\psi^{221}_{qq'}$ values given in Table 2 are compared with the corresponding experimental data. The results of calculations are in good agreement with experiment where $\psi^{221}_{qq'}$ are large and the cross sections are also large. Where the cross sections are not large the calculations give somewhat underestimated values of $\psi^{221}_{qq'}$ if it is assumed that the contribution to the cross section is given by only one two-quasiparticle component. In the latter case it is necessary to analyse the contribution of other two-quasiparticle components.

The quantities $\psi^{221}_{qq'}$ calculated on the basis of the Woods-Saxon potential are somewhat larger than those obtained earlier. However, they do not contradict the available data on the cross sections. From the analysis given in Table 2 it follows that the calculations based on the approximate second quantization method with the use of the multipole-multipole interaction describe correctly the largest components of the first $K^{\pi}=2^{+}$ states.

6. The following conclusions may be drawn on the basis of the performed investigations·

1) In calculating the first $K^{\pi}=2^{+}$ one-phonon states the most important part (defining the properties of those states) of the nuclear forces is taken into account. Therefore for the overwhelming majority of nuclei the experimental energies and the B(E2) of the first $K^{\pi}=2^{+}$ states with the one value of the constant $\kappa^{(2)}A^{4/3}$

and with the one effective charge value $e_{eff}^{(2)}$ are well described.

2) The calculated energies ω_1^{22} and the B(E2) which are the integral characterictics of the one-phonon states depend weakly on the details of the radial dependence of nuclear residual interactions and on the radial parts of the wave functions describing the average field one- particle states. So, the calculated ω_1^{22} and B(E2) with quadrupole-quadrupole forces and surface delta interactions (for the same energies and the wave functions of the Nilsson potential) equally well agree with the correponding experimental data. Further if the sequence of the one-particles states is chosen to be identical then the calculations of ω_1^{22} and B(E2) with the wave function of the Nilsson and Woods-Saxon potentials give comparatively close results. Thus, the energies of the first $K^{\pi}=2^{+}$ states and B(E2) are defined by the general properties of residual interactions and the average field potentials. These quantities depend mainly on the sum of the average values of the corresponding matrix elements.

3) The calculations in which the eigenvalues of the energies and the wave functions of the Woods-Saxon potential are used have some advantages as compared to the calculations based on the Nilsson potential. The Woods-Saxon potential describes more correctly the average nuclear field than the Nilsson potential: it is finite, takes into account the diffusness of the nuclear boundaries, and changes in the behaviour of the energies and the wave functions depending on the mass number A and so on. All this leads to more correct description of the matrix elements. Besides, in order to get a correct sequence of the Nilsson potential levels it is necessary to make a number of arbitrary shifts of the subshells and in some cases of some levels. Therefore to decrease

the arbitrariness in the calculations[6-10] the identical values of the one-particle energies were used and thereby changes in the equilibrium deformations of different nuclei were not taken into account. In the calculations based on the Woods-Saxon potential the correct sequence of the one-particles levels is provided by making the potential parameters more accurate. In this case the calculation can be performed at deformations corresponding to the equilibrium form of each nucleus.

4). The obtained energies and the wave functions of the one-phonon states allow to make the following calculations of the deformed nuclear properties: a) to study non-rotational states in odd-mass deformed nuclei taking into account the interactions of quasi-particles with phonons; b) to find the two-phonon state admixture to the one-phonon states; c) to study the complication of the structure of the deformed nuclear states with increasing excitation energy (this is seen from examples in ref.[29]); d) to take into account the relationship of the internal motion with rotation and the connection of different types of oscillations among them.

Thus, the calculations of the one-particle energies and the wave functions of the Woods-Saxon potential, the account of the superconducting pairing correlations and the calculations of the characteristics of the one-phonon states serve as a basis for further study of the structure of deformed nuclei.

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References

1. E.R.Marshalek, J.O.Rasmussen. Nucl. Phys., 43, 438 (1963).
2. D.Bes. Nucl. Phys., 49, 544 (1963).
3. Лю Юань, В.Г. Соловьев, А.А. Корнейчук. ЖЭТФ, 47, 252 (1964).
4. В.Г. Соловьев. ДАН СССР, 159, 310 (1964).
5. D.R.Bés, P.Federman, E.Magueda, A.Zuker. Nucl. Phys., 65, 1 (1965).
6. V.G.Soloviev. Nucl. Phys., 69, 1 (1965).
7. П.Фогель. ЯФ, 1, 752 (1965).
8. М.К. Калпажиу, П. Фогель. Известия АН сер. физ., 30, 2025 (1966).
9. V.G.Soloviev. Atom. Energ. Rev., 3, 117 (1965).
10. К.М.Железнова, А.А. Корнейчук, В.Г. Соловьев, П. Фогель, Г. Юнг-клауссен. Препринт ОИЯИ Д-2157, 1965.
11. К.М. Железнова, Н.И. Пятов, М.Н. Черней. Изв. АН Сер. физ., 31, 550 (167). Н.И. Пятов, М.Н. Черней. Изв. АН сер.физ.,31,1689 (1967).
12. A.A.Kuliev, N.I.Pyatov. Nucl. Phys., A106, 689(1968).
13. A.Faessler, A.Plastino. Phys. Rev., 156, 1072 (1967); Z. f. Phys., 203, 333 (1967).
14. S.G.Nilsson. Mat. Fys. Medd. Dan. Vid. Selsk., 29, No 16(1955).
15. Ф.А.Гареев, С.П. Иванова, Б.Н. Калинкин, С.К. Слепнев, М.Г. Гинзбург. Препринт ОИЯИ Р4-3607 (1967).
16. Ф.А.Гареев, С.П. Иванова, Б.Н. Калинкин, С.К. Слепнев, М.Г. Гинзбург. Препринт ОИЯИ Р4-3607 (1967).
17. G.Gistafson, I.L.Lamm, S.Nilsson. Arkiv für Fysik, 36, 613 (1967).
18. M.Baranger. Phys. Rev., 120, 957 (1960).
19. B.R.Mottelson, S.G.Nilsson, Mat. Fys. Skr. Dan. Vid. Selsk.1 No 8 (1 959).

20. Л.А. Малов, В.Г. Соловьев, И.Д. Христов. Ядерная физика, 6, 1186 (1967).

21. Л.А. Малов, В.Г. Соловьев, У.М. Файнер. Proc. Int. Symp. on Nucl. Structure, Dubna, 1968.

22. B.Elbek, T.Grotdal, K.Nybo, P.O.Tjom, E.Veje. Int. Con. on Nucl. Structure , Tokyo, 1967, paper 4.90.

23. R.Bloch, B.Elbek, P.O.Tjom. Nucl. Phys., A91, 576 (1967); P.O.Fjom, B.Elbek. Nucl. Phys., A107, 385 (1968); E.Veje, B.Elbek, B.Herskind, M.C.Olesen. Nucl. Phys., A109, 489 (1968). T.Grotdal, K.Nybo, T.Thorsteinsen, B.Elbek. Nucl. Phys., A110, 385 (1968); R.I.Keddy, Y.Yoshisawa, B.Elbek, B.Herskind, M.C.Olesen. Nucl. Phys. . A113, 676 (1968); D.G.Burke, B.Elbek. Mat. Fys. Medd. Dan. Vid. Selsk., 36, 6 (1967).

24. S.T.Belyaev. Mat. Fys. Medd. Dan. Vid. Selsk., 31, No.11(1959).

25. P.Vogel. Proc. Int. Symp. on Nucl. Structure, Dubna, 1968.

26. П.И. Пятов, М.И. Черней. Известия АН сер. физ., 31, 1689 (1967).

27. R.K.Sheline, Proc. Int. Symp. on Nucl. Structure, Dubna, 1968.

28. J.Kern, O.Mikoshiba, R.Sheline, T.Udagawa, S.Yoshida. Nucl. Phys., A104, 642 (1967).

29. V.G.Soloviev. Proc. Int. Sympl. on Nucl. Structure, Dubna, 1968. Preprint P4-3903 , Dubna (1968).

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Table I. Energies of the first $K^{\pi} = 2^{+}$ states (in MeV) and the $B(E2)/B(E2)_{sp.}$ values. For nuclei in the region $150 \leq A \leq 174$

Nuclei	Energies of $K^{\pi} = 2^{+}$ states (in MeV)			$B(E2)/B(E2)_{sp.}$		
	Exper.	Calculby Woods-Saxon scheme	Calcul.by Nilsson Scheme	Exp.	Calcul.by Woods-Saxon scheme	Calcul.by Nilsson scheme
^{150}Nd	1.060	1.0	1.1	-	4.7	9.2
^{152}Sm	1.088	1.1	1.1	3.4	4.1	9.7
^{154}Sm	1.437	1.1	1.4	2.7	3.9	7.8
^{156}Sm	-	1.0	1.4	-	3.8	6.8
^{154}Gd	0.998	1.1	0.9	4.7	4.4	12.7
^{156}Gd	1.155	1.1	1.2	4	4.2	9.8
^{158}Gd	1.185	1.0	1.2	2.7	4.1	8.6
^{160}Gd	0.988	0.95	1.2	2.8	3.6	8.0
^{156}Dy	0.890	0.98	-	8.7	5.0	-
^{158}Dy	0.948	0.87	0.84	6.3	4.7	13.6
^{160}Dy	0.966	0.94	0.87	5.7	4.9	12.1
^{162}Dy	0.890	0.78	0.83	4.8	4.7	11.3
^{164}Dy	0.761	0.78	0.67	4.4	3.5	11.9
^{160}Er	-	0.95	0.96	-	4.4	12.7
^{162}Er	0.897	0.95	0.98	7.1	4.2	11.3
^{164}Er	0.858	0.80	0.96	7.1	4.1	10.5
^{166}Er	0.786	0.78	0.79	5.8	3.3	10.6
^{168}Er	0.821	1.0	1.1	5.8	3.8	8.3
^{170}Er	0.931	1.3	1.4	4.0	2.9	5.8
^{166}Yb	-	1.3	-	-	1.5	-
^{168}Yb	0.986	0.96	-	4.5	2.9	-
^{170}Yb	1.140	1.5	-	2.8	2.0	-
^{172}Yb	1.486	1.5	-	1.3	-	-
^{174}Yb	1.630	1.5	-	1.4	1.6	-
^{176}Yb	1.254	1.0	-	2.1	1.8	-
^{170}Hf	-	1.0	-	-	2.5	-
^{172}Hf	-	1.5-	-	-	1,8	-
^{174}Hf	-	1.5	-	-	-	-

Table 2. Calculated /28/ cross sections for 65° excitations of $K^{\pi} = 2^{+}$ states in (dp) reactions and the calculated $\Psi_{22'}^{22i}$ values

Final nuclei	$d\sigma/d\Omega$ (μb/sr)	Initial states	Configuration	Calculated $\lvert\Psi_{22'}^{22i}\rvert$ in ref. /28/	in ref. /10/	In the present paper
^{156}Gd	21.6	521	521 + 521	0.41	0.44	0.70
^{158}Gd	32.5	521	521 + 521	0.52	0.60	0,82
^{162}Dy	1.8	642	642 - 640	0.10	0.11	-
			642 - 660		0.28	0.31
^{164}Dy	18.1	523	523 - 521	0.52	0.71	0.80
^{168}Er	0.5	633	633 - 631	0.12	-	-
			633 - 651	-	0.16	0.34
^{174}Yb	35.6	512	512 - 510	0.56	1.1	1.0

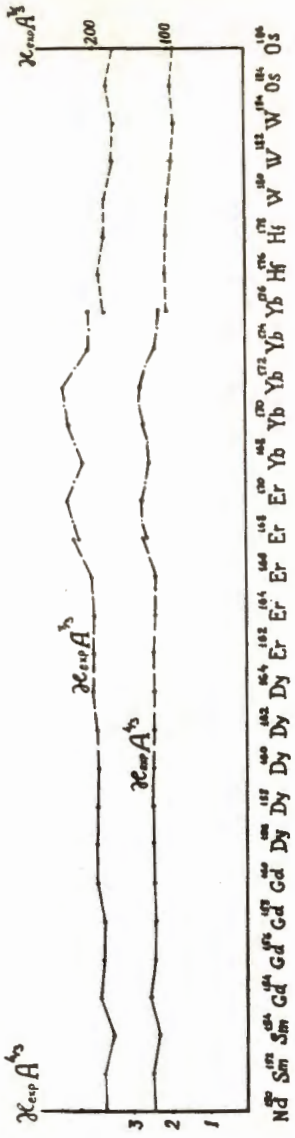

Fig.1. Constants of quadrupole-quadrupole interaction for the corresponding experimental energies. Notations:
— - scheme A=155 — · — scheme A=165
— · — · — scheme A=173 — - - — scheme A=181.