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Z.Oziewicz, A.Pikulski

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Submitted to Yadernaya Fizika

Recently many papers have appeared in which various aspects of the nuclear structure influence and small corrections to muon-proton interaction Hamiltonian on muon capture processes were studied /1/. These corrections are substantial in order to determine the exact size of various muon capture coupling constants. They may be of special importance in partial transitions because they may be distributed in various way among various transitions and they can give larger contribution to partial transitions than to the total capture rate.

In this letter we study the influence of all corrections of order αZ on the partial transition probability in muon capture taking as an example the process $^{12}C(0^+) \xrightarrow{\mu^-} {}^{12}B\ (1^+)$. The main advantages of this partial capture rate are as follows: its rather accurate experimental determination, possibility of avoiding nuclear physics by considering its ratio to the beta decay rate of ^{12}B back to ^{12}C. It is also interesting from the point of view of comparing the axial coupling constant in muon capture and beta decay.

These corrections were taken into account partially in some papers /2,3,4/. They are caused:

i) by large component of muon wave function G on the K-shell, i.e.

$$G(r) \propto 1 - \alpha Z m r \tag{1}$$

ii) by small wave function component of muon on the K-shell [3]

$$F(r) \propto -\frac{1}{2}\alpha Z \tag{2}$$

iii) by the correct account of the effect of bound muon state, i.e. we consider all terms of the type

$$\frac{1}{q}\frac{d}{dr}G(r) \propto -\alpha Z \frac{m}{q}, \tag{3}$$

where m – the reduced muon mass and q – energy of a neutrino.

We want to notice that in the paper of Morita and Fujii the muon has treated in the Hamiltonian as a free particle. Therefore we must do the following change in formula (11) in [2]

$$C_P L(\beta\gamma_5) \to C_P[(1-\frac{q}{m})L(\gamma_5) + \frac{\vec{P}}{m}L(\vec{\sigma})] \tag{4}$$

This fact gives us new corrections of the type iii) not taken into account in all other papers. We want to stress that the corrections of the type i) were calculated in the above mentioned paper. Therefore it was necessary to calculate all αZ corrections.

We assume as usual the V–A theory, cvc hypothesis etc.[2]. The transition probability $^{12}C(0^+) \xrightarrow{\mu^-} {}^{12}B(1^+)$ is

$$W = \frac{2}{3} q^2 [2 \mathcal{M}^2 + \mathcal{P}^2] \qquad (5)$$

where, in notation of paper /2 /, we have

$$\mathcal{M} = \langle {}^{12}B \mid G(r) \hat{M} \mid {}^{12}C \rangle \qquad (6)$$

$$\mathcal{P} = \langle {}^{12}B \mid G(r) \hat{P} \mid {}^{12}C \rangle \qquad (7)$$

$$\hat{M} = C_A (a_1 - a_{10}) - \frac{q}{2M} [C_V \mu a_2 - (C_A + C_P) a_3] - (\frac{3}{2})^{1/2} C_V a_8$$

$$\hat{P} = C_A (a_4 + a_9) - \frac{q}{2M} [C_V \mu a_5 - C_A a_6 + C_P a_7] + (\frac{2}{3})^{1/2} C_V a_{11}$$

$$a_1 = (j_0 + \frac{1}{4} j_2 - \frac{3}{8} \alpha Z j_1) T^M_{101} (\hat{r}, \vec{\sigma})$$

$$a_2 = [j_0 + \frac{1}{4} j_2 + \frac{3}{4} \alpha Z (j_{-1} - (\frac{m}{q} - \frac{1}{2}) j_1)] T^M_{101} (\hat{r}, \vec{\sigma})$$

$$a_3 = \frac{1}{20} \alpha Z (5 j_{-1} + 3 j_1 - 2 j_3) T^M_{101} (\hat{r}, \vec{\sigma})$$

$$a_4 = (j_0 - \frac{1}{2} j_2 + \frac{3}{4} \alpha Z j_1) T^M_{101} (\hat{r}, \vec{\sigma})$$

$$a_5 = \frac{3}{4} \alpha Z (j_{-1} + j_1) T^M_{101} (\hat{r}, \vec{\sigma})$$

$$a_6 = [j_0 - \frac{1}{2} j_2 - \frac{1}{2} \alpha Z \left(\frac{1}{2} j_{-1} + 3(\frac{m}{q} - \frac{1}{5}) j_1 + \frac{2}{5} j_3 \right)] T^M_{101} (\hat{r}, \vec{\sigma})$$

$$a_7 = \{ j_0 - \frac{1}{2} j_2 - \frac{1}{2} \alpha Z [(\frac{q}{m} - \frac{1}{2}) j_{-1} + 3(\frac{m}{q} + \frac{1}{5} + \frac{q}{5m}) j_1 -$$

$$- \frac{2}{5} (1 + \frac{q}{m}) j_3] \} T^M_{101} (\hat{r}, \vec{\sigma})$$

$$a_8 = \frac{i}{M} [j_1 - \frac{1}{3} \alpha Z (j_0 - \frac{1}{2} j_2)] T^M_{111} (\hat{r}, \vec{\nabla})$$

$$a_9 = \frac{1}{M} [3 j_1 - \frac{1}{2} \alpha Z (j_0 - 2 j_2)] T^M_{011} (\hat{r}) \vec{\sigma} \cdot \vec{\nabla}$$

$$a_{10} = \frac{1}{2M} \alpha Z (j_0 + j_2) T^M_{011} (\hat{r}) \vec{\sigma} \cdot \vec{\nabla}$$

$$a_{11} = \frac{1}{2M} \alpha Z (j_0 + j_2) T^M_{111} (\hat{r}, \vec{\nabla}) \qquad (8)$$

We abbreviated $j_w(qr)$ by j_w and $\mu \equiv 1 + \mu_p - \mu_n$ to save the space. The spherical tensors are

$$T^M_{kwu}(\hat{r}, \vec{a}) = \sum C^{uM}_{km_1 wm_2} Y^{m_2}_w(\hat{r}) \mathcal{Y}^{m_1}_k(\vec{a})$$

and $\mathcal{Y}^m_k(\vec{a})$ are the ordinary solid harmonics. In formulas (8) we used the ratio of nuclear matrix elements

$$\frac{\langle {}^{12}B \,|\, f(r)\, T^M_{121}(\hat{r}, \vec{\sigma}) \,|\, {}^{12}C \rangle}{\langle {}^{12}B \,|\, f(r)\, T^M_{101}(\hat{r}, \vec{\sigma}) \,|\, {}^{12}C \rangle} = -\frac{1}{2\sqrt{2}}$$

which can be easily obtained in a single particle model with j-j coupling /2/ .We want to notice, that this ratio varies slightly when the shell wave functions are varied and in fact does not influence our result /4,5/ .

The corrections of the order $(\alpha Z)^2$ are neglected in /8/. In the calculations of nuclear matrix elements we used a single-particle shell model with the pure j-j coupling and radial wave function of harmonic oscillator potential /2/

$$R_{1p}(r) = \left(\frac{8}{3}\right)^{1/2} \pi^{-1/4}\, r_0^{-5/2}\, r \exp\left[-\frac{1}{2}\left(\frac{r}{r_0}\right)^2\right].$$

We use the following numerical values

$$r_0 = 1.6 \text{ fm}.$$

$$q = 91.4 \text{ Mev}$$

$$C_V = -0.804\; C_A$$

$$\mu_p - \mu_n = 3.706$$

The results of the numerical calculations are the following. With all αZ corrections

$$\mathfrak{M} = 0.165\,(C_A + 4.3 \cdot 10^{-4}\,C_P)$$

$$\mathcal{P} = 6.42\,(19.1\,C_A - C_P)\,10^{-8} \quad . \tag{9}$$

Without αZ corrections

$$\mathfrak{M} = 0.174\,C_A$$

$$\mathcal{P} = 6.74\,(18.8\,C_A - C_P)\,10^{-8} \quad . \tag{10}$$

For comparison we give the result taken from the Morita and Fujii paper

$$\mathfrak{M} = 0.164\,C_A$$

$$\mathcal{P} = 6.17\,(19.5\,C_A - C_P)\,10^{-8} \quad . \tag{11}$$

The difference between our formulas (11) and (77) in /2/ is caused by using for transition probability the formula (46) instead of (56) in /2/.

We obtain the following results for the ratios of the transition probabilities for $C_P = 8\,C_A$

$$\frac{W(\text{with all } \alpha Z \text{ corrections})}{W(\text{Morita and Fujii})} = 1.02$$

$$\frac{W(\text{with all } \alpha Z \text{ corrections})}{W(\text{without } \alpha Z \text{ corrections})} = 0.9$$

We may conclude that although the corrections of the order αZ are, in principle, not large, however, they can play an important role in exact calculations of the muon-proton interaction constants. These corrections are of order of relativistic matrix elements and then they should be considered always together.

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