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E4 - 4015

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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1968

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Submitted to Phys. Lett.

In the stripping reactions on deformed nuclei the excitation by incident and outgoing particles of the low-lying rotational levels in target- and daughter nuclei leads to the multi-step feature of the process. In a previous paper /1/ the differential cross section for $A(dp)B$ stripping was obtained, where, using the adiabatic approximation, rotational levels of nuclei A and B were taken into consideration

$$\sigma(\theta) = c^{2}\left[16\pi(2s_{n}+1)(1+\delta_{0K_{A}})\right]^{-1}\sum_{JL}(2L+1)^{-1}(J J_{A} K_{A}\Omega \mid J_{B} K_{B})^{2}$$

$$\sum_{m'}\Big|\sum_{\ell''\Lambda} a_{\ell''\Lambda}(L s_{n}\Lambda\Sigma \mid J\Omega)\sum_{\substack{\ell\ell' m\\ \bar\ell\bar\ell'\bar m}} I^{\bar\ell\bar\ell'\bar m}_{\ell\ell' m}(\ell'')\,\hat\ell\,\hat{\bar\ell}\,\hat\ell''^{-1}(\ell\bar\ell\, 0\, 0 \mid \ell'' 0)$$

$$(\ell\bar\ell\, m\,\bar m \mid \ell''\Lambda)\,(\ell'\bar\ell'\, m\,\bar m \mid L\Lambda)\,(\ell'\bar\ell'\, m' 0 \mid L m')\,Y^{*}_{\bar\ell' m'}(\theta,\phi)\Big|^{2}. \tag{1}$$

Here

$$I^{\bar\ell\bar\ell'\bar m}_{\ell\ell' m}(\ell'') = \int_{0}^{\infty} R^{\bar m}_{\bar\ell\bar\ell'}\, R_{\ell''}\, R^{m}_{\ell\ell'}\, r^{2}\, dr$$

$$(\hat\ell = \sqrt{2\ell+1}) \tag{2}$$

is the overlap integral of the radial functions of the relative motion and bound state $R_{\ell''} = \frac{e^{-\alpha r}}{\alpha r}$ (upperlined indexes refer to the deuteron channel). The relevant distorted waves Ψ_p and Ψ_d are for the deformed nuclear field. This leads immediately to coupled channel equations[2] for radial functions with the appropriate asymptotic form

$$R^{m}_{\ell\ell'} = 2\pi\,(-i)^{\ell'} e^{i\sigma_{\ell'}} (kr)^{-1}[(F_\ell + iG_\ell)\,\delta_{\ell\ell'} + S^{m}_{\ell\ell'}(F_\ell - iG_\ell)] \tag{3}$$

$$\bar{R}^{\bar{m}}_{\bar{\ell}\bar{\ell}'} = [\pi(2\bar{\ell}'+1)]^{1/2}\, i^{\bar{\ell}'} e^{i\sigma_{\bar{\ell}'}} (\bar{k}r)^{-1}[(\bar{F}_{\bar{\ell}} + i\bar{G}_{\bar{\ell}})\,\delta_{\bar{\ell}\bar{\ell}'} + \bar{S}^{\bar{m}}_{\bar{\ell}\bar{\ell}'}(\bar{F}_{\bar{\ell}} - i\bar{G}_{\bar{\ell}})].$$

Selection rules for the angular momentum transfer J ($\vec{J} = \vec{J}_B - \vec{J}_A$) and its projection on the nuclear simmetry axis $\Omega = K_B - K_A$ follow from eq. (1). In general the angular momentum transfer J does not coincide with the nonconserved momentum j of a neutron, captured on the Nilsson orbit ($\vec{j} = \vec{\ell}'' + \vec{s}_n$). The well known result of Satchler[3] (in our terminology it is one-step stripping) follows from eq. (1) if the central field distorted waves are used ($\ell = \ell'$; $\bar{\ell} = \bar{\ell}'$; $R^{m}_{\ell\ell'} = R_\ell$; $\bar{R}^{\bar{m}}_{\bar{\ell}\bar{\ell}'} = R_{\bar{\ell}}$). In this case the cross section consists of the kinematic part and an S-factor of the form

$$S_{JL} = \frac{2J_A+1}{2J_B+1}(1+\delta_{OK_A})^{-1}[(J_A J K_A \Omega \mid J_B K_B)\, C^{\Omega}_{JL}]^2 \tag{4}$$

where

$$C^{\Omega}_{JL} = \sum_{\Lambda} a_{L\Lambda}(L\, s_n \Lambda \Sigma \mid J\,\Omega). \tag{4'}$$

Now the momentum transfer J coincides with the neutron momentum j and we are forced to consider j as coserved.

The evaluation of the terms $\ell' \neq L$ in the cross section (1) which are due to nuclear deformation is rather difficult because the coupled channel equations must be solved. Therefore for qualitative (and sometimes for quantitative analysis) one can transform eq. (1) using the diffraction approximation. In this approximation we assume "blackness" for the region inside the nuclei A and B ($\ell, \ell' \ldots < \ell_0 = kR \gg 1$) and that transferred angular are small as compared with the relative motion momenta

$$\ell, \ell' \ldots \gtrsim \ell_0 \gg L, \ell'' \ldots \tag{5}$$

Then in computing the radial integral (2) and eq. (1) the asymptotic (3) and the approximate formulas for certain Clebsh-Gordan coefficients can be used. In this approximation the scattering matrix for the deformed nuclear field $S^{m}_{\ell\ell'}$ can be simply expressed[4] through the central field S_ℓ-matrix. In this way the differential cross section for diffraction stripping can be obtained[5]

$$\sigma(\theta) = \frac{2J_B+1}{2J_A+1}\sum_{JL}\frac{2J_A+1}{2J_B+1}(1+\delta_{OK_A})^{-1}(J_A J K_A \Omega \mid J_B K_B)^2 \Big|\sum_{n=1} C^{\Omega}_{JL(n)}\, t^{(n)}_L(\theta)\Big|^2 \tag{6}$$

$$C^{\Omega}_{JL(n)} = (4n-3)\hat{L}\sum_{\ell''\Lambda} a_{\ell''\Lambda}\,\hat{\ell}''^{-1}(L\, s_n \Lambda \Sigma \mid J\Omega)(L\, 2n-2\, \Lambda 0 \mid \ell''\Lambda)(L\, 2n-2\, 0 0 \mid \ell'' 0) \tag{7}$$

$$t^{(n)}_L(\theta) = C\sum_{\ell}\hat{\ell}\, e^{i(\sigma_\ell - \bar{\sigma}_\ell)} B^{(n)}_\ell\, Y_{\ell L}(\theta, 0) \tag{8}$$

$$B^{(n)}_\ell = \frac{1}{4}\int_{-1}^{1} dx\,[A_\ell\, \bar{S}_\ell(x) + \bar{A}_\ell\, S_\ell(x)]\, P_{2n-2}(x) \tag{9}$$

$$A_\ell = \int (F_\ell - iG_\ell)\frac{e^{-ar}}{ar}(\bar{F}_\ell + i\bar{G}_\ell)\,dr\,;\quad \bar{A}_\ell = \int(\bar{F}_\ell - i\bar{G}_\ell)\frac{e^{-ar}}{ar}(F_\ell + iG_\ell)\,dr. \qquad (10)$$

It may be seen from eq. (1) that the amplitude is a superposition of many "step" stripping amplitudes (8). Therefore the spectroscopic S -factor cannot be extracted from the cross section. It is obtainable only for a particular one-step stripping. In this case $t^{(1)}_L(\theta)$ coincides formally with Dar's amplitude of the diffraction stripping on a spherical nucleus/6/ and $C^{\Omega}_{JL(1)}$ is the usual factor(4').

Now we estimate the order of the quantity involved and reveal the typical angular dependence of the step-amplitudes contained in the stripping cross section (6). For simplicity we consider only one term $\bar{A}_\ell S_\ell(x)$ in eq. (9) i.e. we suppose the major contribution is given by one (d or p) channel. Then with $\bar{A}_\ell \approx \hat{\ell}^{-1}\exp(-\frac{a}{k}\ell)$ /6/ and $S_\ell = \rho(\ell)\, e^{2i\sigma_\ell}/[1+\exp\frac{\ell_0+\Delta(x)-\ell}{\lambda}]$, where $\Delta(x) = \ell_0\sqrt{\frac{5}{4\pi}}\,\beta\, P_2(x)$ and using the poles of S_ℓ /7/ $\ell_\pm = \ell_0 + i\pi\lambda + \Delta(x)$ we obtain for $\theta \gtrsim 1$:

$$t^{(n)}_L(\theta) = \mathrm{const}\,(-1)^L\frac{e^{-\pi\lambda(\theta)}}{\sqrt{\sin\theta}}\int_0^1 dx\, P_{2n-2}(x)\, e^{i\Delta(x)\theta}\cos[(\ell_0+\tfrac{1}{2})\theta-(2L+1)\frac{\pi}{4}+\Delta(x)\,\theta+\theta_0]$$

$$(\theta_0 = \phi - \frac{a}{k}\pi\lambda + i\pi\lambda\,\theta_0) \qquad (11)$$

(when both terms in (9) are taken into account, the amplitude (8) is the sum of the two terms of type (11)). The parameters θ_0 and ϕ are given by $\sigma(\ell_\pm) = \sigma(\ell_0) + \frac{1}{2}\theta_0\,[\Delta(x) + i\pi\lambda]$; $\rho(\ell_\pm) = \rho(\ell_0)\exp(\pm i\phi)$. However, they can be considered as arbitrary ones/8/.

The integrand (11) contains the deformation parameter β and under the condition $\beta\ell_0 \ll 1$ can be expanded in power series in β. Then

$$t^{(n)}_L(\theta) = \mathrm{const}\,(-1)^L\frac{(-\beta\ell_0\sqrt{\frac{5}{4\pi}})^{n-1}}{(n-1)!}\; C^n_{n-1}(\theta)^{n-1}\;\frac{e^{-\pi\lambda\theta}}{\sqrt{\sin\theta}}$$

$$\cos[(\ell_0+\tfrac{1}{2})\theta - i(n-1)\frac{\theta_0}{\theta} - (2L+1)\frac{\pi}{4} - (n-1)\frac{\pi}{2} + \theta_0] \qquad (12)$$

$$(\,C^n_{n-1} = \int_0^1 dx\; P_{2n-2}(x)\,[P_2(x)]^{n-1}\,)$$

whence the phase rule follows: for the n -th stripping step the phase shift is $(L+n-1)\frac{\pi}{2}$ where L is the angular momentum transfer.

As an example we consider the reaction $^{24}Mg(dp)\,^{25}Mg$ with transition to the rotational levels $5/2^+$ (g.s.) and $7/2^+$ (1.61 MeV) on the orbit /202/ 5/2 $^+$. In this case the Nilsson coefficients do not vanish only for $\ell'' = 2$ therefore it follows from eq. (7) that the usual one-step stripping cannot be realized to the level $7/2^+$. In the first order of perturbation theory (eq. (12)) one-step stripping($n = 1$) is possible to the $5/2^+(L=2)$ level but two-step ($n = 2$) takes place to the $7/2^+$ ($L = 4$) level. In fig. 1 these cross sections are given by the dotted lines and, as is seen, they are out of phase. The dash-dotted lines are for the case when only two levels in each channel are taken into account (eq. (11) $n = 1,2$). (The same approximation has been used elsewhere/9/). The solid lines represent the exact calculations (eq. (11)) in which all the stripping steps are taken into account. It is clear that the exact calculations violate the phase rules. Moreover, the strong difference in shape and in absolute value between the exact and approximate cross sections indicated that the use of the deformation expansion and of the restriction to the nearest stripping steps/9/ seems to be incorrect.

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Received by Publishing Department
on August 1, 1968.

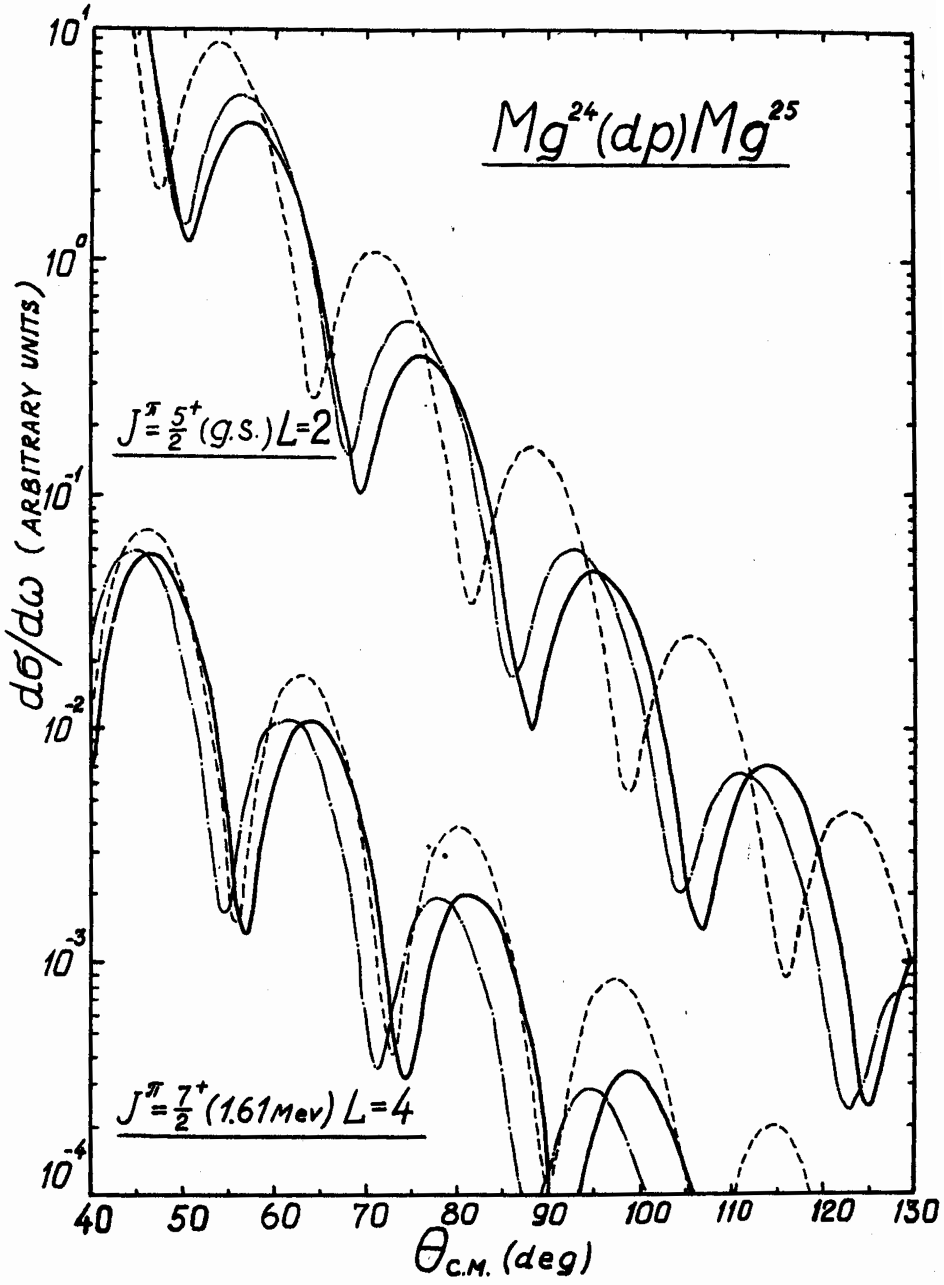

Fig. 1. Differential cross sections of the diffraction stripping on a deformed nucleus. Exact calculations (solid lines), calculations by the perturbation theory (dotted lines) and by the two-level approximation (dash-dotted lines) are shown. Parameters are $\ell_0 = 10$, $\lambda = 1.0$, $\theta_c = 0.1$, $\theta_0 = 1\cdot 0{,}314$, $\beta = 0.3$.