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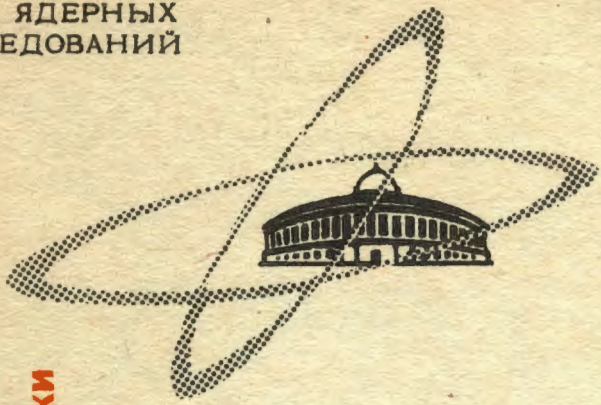

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PRELIMINARY CALCULATIONS OF THE GROUND STATE CHARACTERISTICS OF He^4 NUCLEUS

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PRELIMINARY CALCULATIONS OF THE GROUND STATE CHARACTERISTICS OF He^4 NUCLEUS

Бланк И., Улегла И., Кухтина И.Н. E4-3927

Предварительные расчёты характеристик основного состояния ^{4}He

Приведены некоторые результаты вычислений энергии связи, среднеквадратичного радиуса, распределения заряда и форм-фактора ^{4}He. Используется теория возмущений Голдстоуна до второго порядка в энергии и первого порядка в волновой функции. Расчёты проделаны с потенциалом Хамады-Джонстона.

Препринт Объединенного института ядерных исследований.
Дубна, 1968.

Blank J., Úlehla I, Kukhtina I.N. E4-3927

Preliminary Calculations of the Ground State Characteristics of ^{4}He Nucleus

Some results of the calculation of the binding energy, mean square radius, charge distribution and form factor of the ^{4}He nucleus are presented. The Goldstone perturbation theory is applied up to the second order in the energy and first order in the wave function. The Hamada-Johnston potential is used.

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Dubna, 1968**

Introduction

The purpose of these calculations is to check the applicability of the Goldstone perturbation theory to the many body problem, which we meet in the nucleus. The ^{4}He nucleus is chosen for its relative simplicity.

Mang and Wild[1] used a special method developed by Brenig[2] for the study of ^{4}He . Their method has some similar features to the Brueckner formulation. Although their calculation is very accurate, the two-nucleon potential considered by them is unrealistic. It contains only a central force with a hard core and an attractive part of the square-well form. It is queried , if the extremely good result - the binding energy of ^{4}He 27.3 MeV and the radius 1.75 f in comparison with the experimental value 28.3 MeV, 1.65 f resp. - is not incidental.

Later on, Kallio and Day[3] following Eden and Emery solved the Bethe-Goldstone equation with some improvement. They dealt also with the ^{4}He nucleus and obtained for it, again, a good value of

the binding energy. But only the S-state interaction with a hard core and an attractive exponential part was considered by them.

The method of the exact solution of the Bethe-Goldstone equation for the reaction matrix in finite nuclei described in detail in [4] has been applied to obtain the values for the binding energy and other ground-state characteristics of ^4He. Although our result is not final, we consider it as interesting, because it gives already some specific information. The calculation could not be performed till the end, since we used a too low cut-off in one case.

In the first part of this paper we give the basic definitions, in the second the method of calculation [5] is shortly described and in the third we present preliminary results.

In the calculation Hamada-Johnston potential [6] has been used; its matrix elements for the harmonic-oscillator wave functions are given in [7].

I. Basic Definitions

The Hamiltonian of the N-fermion system has in the representation of the second quantization and in the space of the harmonic-oscillator wave functions the form;

$$H = H_0 + H_1 ,$$

where

$$H_0 = \sum_{n_1} E_{n_1} b^*_{n_1} b_{n_1}$$

$$H_1 = \frac{1}{4} \sum_{n_1 n_2 n_3 n_4} (n_1 n_2 | v | n_3 n_4)_a \, b^*_{n_1} b^*_{n_2} b_{n_3} b_{n_4} .$$

The quantities $b^*_{n_i}$, b_{n_i} are the creation and annihilation operators of the particle in the state $|n_i\rangle$. The one-particle symbol $|n_i\rangle$ represents in fact five quantum numbers: $|n_i\rangle \equiv |p_i, \ell_i, m_i, s_i, r_i\rangle$, where p_i is the radial quantum number, ℓ_i and m_i belong to the orbital momentum and its projection, s_i and r_i are spin and isospin projections. The corresponding energy eigenvalue is

$$E_{n_i} = \hbar\omega \left(G_i + \frac{3}{2} \right) \qquad G_i = 2p_i + \ell_i .$$

The oscillator frequency ω is a parameter on which the results of the calculation may depend.

In the Schrödinger representation the one-particle state $|n_i\rangle$ is given by

$$(\vec{x} \,|\, n_i) = \left(\frac{m\omega}{\hbar} \right)^{3/4} R_{p_i \ell_i}(r) Y_{\ell_i m_i}(\theta,\phi) \chi_{s_i, r_i} , \quad (1.1)$$

where

$$R_{p\ell}(r) = \sqrt{\left(\frac{2p!}{\Gamma(p+\ell+3/2)} \right)} r^{\ell} e^{-\frac{r^2}{2}} L_p^{(\ell+\frac{1}{2})}(r^2)$$

and r is related to the actual distance x by

$$r = \sqrt{\frac{m\omega}{\hbar}}\, x .$$

The matrix element in the perturbation term is antisymmetrized:

$$(n_1 n_2 | v | n_3 n_4)_a = (n_1 n_2 | v | n_3 n_4) - (n_1 n_2 | v | n_4 n_3) .$$

The perturbation term H_1 consists of four parts. The first term is the realistic nucleon-nucleon interaction given by Hamada and Johnston. The second term is represented by the Coulomb interaction. The third part compensates the potential energy operator of the unperturbed state of the N-fermion system. The fourth part is formed by the harmonic oscillator potential, which acts on the center of the mass of the nucleus. Then, according to Lipkin[8], the energy of the CM-motion can be simply excluded and the binding energy E_{int} of the nucleus calculated from

$$E_{int} = E - \frac{3}{2} \hbar\omega ,$$

where E is the energy of the system, the center of the mass of which is put in the harmonic-oscillator well.

It can easily be shown that the potential v has now the form

$$v_{ij} = V_{ij} - \frac{1}{2N} m \omega^2 (\vec{x}_i - \vec{x}_j)^2 ,$$

where m is the nucleon mass, $\vec{x}_i$ is the position vector of the i-th particle and V_{ij} is the realistic two-nucleon potential with the Coulomb term.

Following Goldstone we define the reaction matrix

$$(n_1 n_2 | t(s) | n_3 n_4)_a = (n_1 n_2 | v | n_3 n_4)_a + \\ + \frac{1}{2} \sum_{n_5, n_6} \frac{(n_1 n_2 | v | n_5 n_6)_a (n_5 n_6 | t(s) | n_3 n_4)_a}{s - E_{n_5} - E_{n_6}} , \qquad (1,2)$$

where $s = E_{n_3} + E_{n_4} + \delta E$ and the sum extends only over such states n_5, n_6 which are unoccupied in the unperturbed system of nucleons. The quantity ∂E is the excitation energy of the unperturbed system, the rule for the explicit form of which is given in [9].

With the help of the reaction matrix, the energy E of the system can be written in the form

$$E = E_0 + \sum_{i=1}^{\infty} \Delta E^{(i)},$$

where E_0 is the ground-state energy of the unperturbed system and

$$\Delta E^{(1)} = \frac{1}{2} \sum_{n_1^0, n_2^0} (n_1^0 n_2^0 \,|\, t(E_{n_1^0} + E_{n_2^0}) \,|\, n_1^0 n_2^0)_a ,$$

$$\Delta E^{(2)} = \sum_{n_1^0, n_2^0, n_3^0, n_4^0} \frac{(n_3^0 n_2^0 \,|\, t(E_{n_2^0} + E_{n_3^0}) \,|\, n_3^0 n_4)_a \, (n_1^0 n_4 \,|\, t(E_{n_1^0} - E_{n_2^0}) n_1^0 n_2^0)_a}{E_{n_2^0} - E_{n_4}}$$

etc.

In these expressions for $\Delta E^{(1)}$ and $\Delta E^{(2)}$, as well as in those for $\rho_{lab}^{(0)}$ and $\rho_{lab}^{(1)}$ (see below), the states denoted by the upper index 0 are occupied, while the remaining states are unoccupied. We see that $\delta E = 0$ in all the t-matrix elements appearing in $\Delta E^{(1)}$ and $\Delta E^{(2)}$.

The other physical quantities can also be expressed with the help of the reaction matrix. Since the charge distribution (normalized to unity) - $\rho_{CM}(\vec{r})$ in the coordinate system related to the center of mass of the nucleus is determined as the expectation value of the N-particle operator

$$\frac{1}{Z} \sum_{i=1}^{N} \delta_{\frac{1}{2}\tau_i} \delta\left(\vec{r} + \frac{1}{N} \sum_{j=1}^{N} \vec{r}_j - \vec{r}_i\right),$$

it is easier to calculate first the quantity ρ_{lab}, which is the expectation value of the operator

$$\frac{1}{Z} \sum_{i=1}^{N} \delta_{\frac{1}{2}\tau_i} \delta(\vec{r} - \vec{r}_i)$$

where Z is the number of protons. When ρ_{lab} is given, ρ_{CM} can be obtained from

$$\rho_{lab}(\vec{r}) = \int |f(\vec{R})|^2 \rho_{CM}(\vec{r} - \vec{R}) d^3\vec{R} \qquad (1.3)$$

where $f(\vec{R})$ is the normalized function of the center-of-mass motion. The mean square radius is defined by

$$(r.m.s.)^2_{CM} = \int \rho_{CM}(\vec{r}) r^2 d^3\vec{r}.$$

If we put

$$(r.m.s.)^2_{lab} = \int \rho_{lab}(\vec{r}) r^2 d^3\vec{r}$$

we obtain

$$(r.m.s.)^2_{CM} = (r.m.s.)^2_{lab} - \int |f(\vec{R})|^2 R^2 d^3\vec{R}.$$

The explicit expression for ρ_{lab} can easily be derived. If we limit ourselves to the zero-th and first order term in t, we get

$$\rho_{lab} = \rho^{(0)}_{lab} + \rho^{(1)}_{lab}$$

$$\rho^{(0)}_{lab}(\vec{x}) = \frac{1}{Z} \sum_{n_i^{(0)}} (n_i^0 | \vec{x}) (\vec{x} | n_i^{(0)}) \delta_{\frac{1}{2} r_i} \tag{1.4}$$

$$\rho^{(1)}_{lab}(\vec{x}) = 2\mathrm{Re} \left\{ \frac{1}{Z} \sum_{n_1^\bullet, n_2^\circ, n_3} \delta_{s_2 s_3} \delta_{\frac{1}{2} r_2} \delta_{\frac{1}{2} r_3} \times \right.$$

$$\left. \times \frac{(n_2^\circ | \vec{x})(\vec{x} | n_3^\circ)(n_1^\circ n_2^\circ | t (E_{n_1^\circ} + E_{n_2^\circ}) | n_1^\circ n_3)_a}{E_{n_2^\circ} - E_{n_3}} \right\} .$$

Here again $\partial E = 0$.

II. The Method of the Solution

First it is necessary to simplify the equation for the reaction matrix. For the quantities, we are going to calculate, we need only such solutions of eq.(1.2), in which both n_3 and n_4 are occupied states.

From one-particle states in the equation (1.2) we pass to the LST-coupling, in which the spins and isospins are coupled to the total spin S and isospin T and the angular momenta of both particles are coupled to

$$\vec{\ell}_{ij} = \vec{\ell}_i + \vec{\ell}_j \,, \qquad m_{ij} = m_i + m_j \,.$$

We denote the unperturbed coupled two-particle states in this representation by

$$|n_i\, n_j\rangle = |p_i \ell_i\, p_j \ell_j \ell_{ij}\, m_{ij}\, S S_z T T_z\rangle .$$

Since the potential v depends only on the relative coordinates, it is convenient to use the center-of-mass system. The transformation reads

$$\vec{r} = \frac{\vec{r}_1 - \vec{r}_2}{\sqrt{2}} \,, \qquad \vec{R} = \frac{\vec{r}_1 + \vec{r}_2}{\sqrt{2}} \,.$$

Denoting by $\vec{\ell}$ the angular mometum of the relative motion and by $\vec{L}$ the angular momentum of the center-of-mass motion of the nucleon pair, we are led to the following coupling:

$$\vec{\ell} + \vec{L} = \vec{\lambda} = \vec{\ell}_{ij} \,, \qquad m + M = \mu = m_{ij} \,.$$

On the other hand, the most convenient representation for the potential v is that, in which $\vec{\ell}$ and $\vec{S}$ are coupled, i.e.

$$\vec{\ell} + \vec{S} = \vec{j} \,, \qquad m + S_z = j_z \,.$$

For connecting these two coupling, we introduce the "total" angular momentum $\vec{J}$ and its projection J_z by

$$\vec{J} = \vec{\ell} + \vec{L} + \vec{S} \,, \qquad J_z = m + M + S_z \,.$$

The corresponding two-particle states are denoted by

$$|(p\ell, PL)\lambda, S, JJ_z, T T_z\rangle .$$

With the help of Clebsh-Gordan coefficients and Moshinsky brackets /10/

$$< p\,\ell\ ,\ PL\ ,\ \lambda \mid p_1\,\ell_1\ ,\ p_2\,\ell_2\ ,\ \lambda >$$

for the transition between the laboratory and the CM-system, we obtain

$$|n_i\, n_j) = \sum_{\substack{J,J_z \\ -\ell P \ell}} (\ell_{ij}\, S\, m_{ij}\ S_z \mid J\, J_z) < p\,\ell, PL, \ell_{ij} \mid p_i\,\ell_i\ ,\ p_j\,\ell_j,\ \ell_{ij} > \times \tag{2.1}$$

$$\times\ \delta_{m_{ij}+S_z,\ J_z}\ \delta_{n+N,\ \mathcal{C}_i+\mathcal{C}_j}\ |(p\ell, PL)\,\ell_{ij}\, S,\ JJ_z\ ,\ TT_z\ ),$$

where

$$n = 2p + \ell, \qquad N = 2P + L\ , \mathcal{C}_i = 2p_i + \ell_i\ , \quad \mathcal{C}_j = 2p_j + \ell_j\ .$$

It is obvious that the vector $(n_1 n_2|$ in eq.(1.2) can be replaced by the vector $((p\ell\ ,PL)\,\ell_{12}, S\ ,\ JJ_z\ ,\ TT_z|$. For the kernel, and otherwise if necessary, we have to use the expansion (2.1). The inhomogeneous term in eq. (1.2) then becomes:

$$((p\ell\ ,PL)\ell_{12}\ S\ ,JJ_z\ ,TT_z|\,v\,|\ n_3^0\, n_4^0) =$$

$$= \delta_{SS^0}\,\delta_{T\,T^0}\delta_{T_z T_z^*}\,\delta_{JJ^0}\delta_{J_z J_z^0}(\ell_{34}^0\, S^0\, m_{34}^0\ S_z^0 \mid JJ_z)\times \tag{2.2}$$

$$\times \sum_{p'\ell'} \delta_{n'+N,\ \mathcal{C}_3^0+\mathcal{C}_4^0}\ p'\ell',\ PL,\ \ell_{34}^0\ \mid p_3^0\,\ell_3^0\ ,\ p_4^0\,\ell_4^0,\ \ell_{34}^0 > (p\,\ell\ \ell_{12}|\,v\,|p'\,\ell'\,\ell_{34}^0).$$

Hence, the system (1,2) has a non-trivial solution only if

$$\delta_{SS^\circ}\,\delta_{TT^\circ}\delta_{T_z T_z^\circ}\,\delta_{JJ^\circ}\,\delta_{J_z J_z^\circ}\,(\,\ell^0_{34}\;S^0\;m^0_{34}\;S^0_z\mid J J_z\,)\neq 0\,. \qquad (2.3)$$

The matrix element for v in (2.2) is given by

$$(p\,\ell\,\ell_{12}\mid v\mid p'\ell'\,\ell^0_{34}) = (-1)^{\ell_{12}+\ell^\circ_{34}} \sum_{j=|\ell-s|}^{\ell+s} (2j+1)\sqrt{(2\ell_{12}+1)}\,\sqrt{(2\ell^0_{34}+1)}\;\times$$

$$\times \begin{Bmatrix} L & \ell & \ell \\ S & J & j \end{Bmatrix} \begin{Bmatrix} L & \ell' & \ell^0_{34} \\ S & J & j \end{Bmatrix} \int_p^{+\infty} R_{p\ell}(r)\,v_{js\,\ell\ell^\circ}(r)\,R_{p'\ell'}(r)\,r^2\,dr\,,$$

where the 6- j symbols and the notation of eq. (1.1) are used.

The kernel of the system (1.2)

$$\sum_{n_5,\,n_6} \frac{|\,n_5\,n_6\,)(\,n_5\,n_6\,|}{E_{n^\circ_3}+E_{n^\circ_4}-E_{n_5}-E_{n_6}}$$

can be transformed to the form

$$\sum_{\nu=2\mathcal{G}^*}^{\infty} \frac{1}{E_{n^\circ_3}+E_{n^\circ_4}-\hbar\omega\,(\nu+3)} \Big\{ \sum_{n+N=\nu} \sum_{\lambda\mu} |\,p\,\ell\;PL\,\lambda\mu\,)(p\,\ell\;PL\lambda\mu\,| -$$

$$\sum_{\substack{n+N=\nu \\ n'+N'=\nu}} \sum_{\lambda\mu} |\,p\,\ell\;PL\lambda\mu\,)(p'\ell'\;P'L'\,\lambda\mu\,|\; f^{(\nu,\,\mathcal{G}^*)}_{p\ell PL,\,p'\ell' P'L',\,\lambda} \Big\} \qquad (2.4)$$

$$f^{(\nu,\,\mathcal{G}^*)}_{p\ell\;PL,\,p'\ell'\;P'L',\,\lambda} =$$

$$= [\,1+(-1)^{L+L'}\,] \sum_{\nu'=0}^{\mathcal{G}^*-1} \sum_{\substack{n_5=\nu-\nu' \\ n_6=\nu'}} \langle p\ell\,,PL,\lambda|\;p_5\ell_5,p_6\ell_6,\lambda\rangle\langle p,\ell',P'L',\lambda|\,p_5\ell_5,p_6\ell_6,\lambda\rangle$$

and ϵ^* denotes the first unoccupied energy shell in the unperturbed system.

Note that

$$\sum_{\substack{JJ_z \\ S\lambda}} |\lambda S J J_z)(\lambda S J J_z| = \sum_{\substack{SS_z \\ \lambda\mu}} |\lambda \mu S S_z)(\lambda \mu S S_z| = \sum_{\lambda\mu} |\lambda\mu)(\lambda\mu| .$$

From the expression (2.2) condition (2.3) and the form of the kernel (2.4) we can conclude that the system (2.1) is decoupled with respect to T, T_z, S, J, J_z. Therefore it is possible to define the reduced matrix element for t by

$$((p\ell, PL)\ell_{12}\ S, JJ_z, TT_z\ |t|\ n_3^0 n_4^0) =$$

$$= \delta_{TT^0}\delta_{T_zT_z^0}\delta_{SS^0}\delta_{JJ^0}\delta_{J_zJ_z^0} \times$$

$$\times (\ell_{34}^0 S^0 m_{34}^0 S_z^0 | JJ_z)(p\ell, PL, \ell_{12}|\ t_{red}\ |\ p_3^0\ell_3^0, p_4^0\ell_4^0, \ell_{34}^0).$$

With the help of this definition we eliminate the dependence of t_{red} on J_z and for a given set of S, J, T, T_z we obtain the following system:

$$(p\ell, PL, \ell_{12}|t_{red}\ |\ p_3^0\ell_3^0, p_4^0\ell_4^0, \ell_{34}^0) =$$

(2.5)

$$= \sum_{p'\ell'} \delta_{n'+N, \epsilon_3^0+\epsilon_4^0} \langle p'\ell', PL, \ell_{34}^0|\ p_3^0\ell_3^0, p_4^0\ell_4^0, \ell_{34}^0\rangle (p\ell\ell_{12}|v|p'\ell'\ell_{34}^0) +$$

$$+ \sum_{\nu=2\mathcal{G}^*}^{\infty} \frac{1}{E_{n_3^0} + E_{n_4^0} - \hbar\omega(\nu+3)} \Big\{ \sum_{\substack{n'+N=\nu \\ \lambda'}} (p\,\ell\ell_{12} \,|\, v \,|\, p'\ell'\,\lambda')(p'\ell', PL, \lambda' | t_{red} | p_3^0 \ell_3^0,$$

$$p_4^0 \ell_4^0, \ell_{34}^0) - \sum_{\substack{n'+N=\nu \\ n''+N''=\nu \\ \lambda''}}^{(\nu, \mathcal{G}^*)} (p'\ell' PL, p''\ell'' P''L'', \lambda'' \Big\} (p\,\ell\ell_{12} \,|\, v \,|\, p'\ell'\,\lambda'') \times$$

$$\times (p''\ell'', P''L'', \lambda'' | t_{red} | p_3^0 \ell_3^0, p_4^0 \ell_4^0 \ell_{34}^0).$$

A simple parity consideration shows, that there is no mixing between the even and odd parity of ℓ.

When the potential v contains a hard core, the system (2.5) has to be replaced by a similar one, in which the matrix elements of the potential are redefined. This procedure is performed exactly in /5/.

The algebraic system (2.5) is infinite. The infinity is originated by the sums over ν and $N = 2P + L$. The infinity which is caused by the coupling in the center-of-mass motion, can be treated sufficiently accurately with the help of a suitable cut-off as was shown in /4/. In our case the Hamada-Johnston potential is taken as acting in the S, P and D states. By this assumption L is automatically limited to a finite number of values and the infinity in N arises only from the infinity in P. The ν-infinity which occurs in the kernel of the system (2.5), can be considered in the same way,

because the expansion of the kernel is convergent [11]. In this way the system (2.5) can be converted to a finite algebraic system.[x)]

In the present calculation it is shown that the cut-off $P \leq \bar{P} = 6$ is sufficiently accurate, but the cut-off $\nu \leq \bar{\nu} = 26$ is too low.

The solution of the system (2.5) with the cuts $P \leq \bar{P}$, $\nu \leq \bar{\nu}$ has been obtained by an iterative perturbation method, in which the zero-th approximation is taken as decoupled in the center-of-mass motion, i.e. the term containing $f^{(\nu, \subseteq^*)}_{p\ell P L, p'\ell' P'L', \lambda}$, is omitted.

III. Results

The computed nuclear characteristics of ^4He are dependent on the frequency ω. We have considered two values corresponding to the one-particle shell model:

$$\hbar\omega = 21.32 \text{ MeV} \qquad \hbar\omega = 25.10 \text{ MeV}.$$

The Hamada-Johnston potential is used and considered as acting in the S, P and D states only.

In Table I we find the unperturbed energy $E_o - \frac{3}{2}\hbar\omega$, the correcrions $\Delta E^{(1)}$, $\Delta E^{(2)}$, the "Coulomb energy" E_o, the

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x) *It should be noted that the treatment of the hard-core part of v differs slightly from that of ref. [5]. The cut-off in the kernel applies here only to the regular part of v, while the hard-core part is treated exactly, in the same way as in ref. [4]. Since the regular part of v gives the binding, this approach may be expected to lower the absolute value of the binding energy.*

sum of these four denoted by E_{int} (in MeV), and the mean square radius in the CM-system (in f). These quantities refer to the value $\hbar\omega = 25.10$ MeV. The first three rows correspond to the cut-off value $\bar{P} = 6$ and different values of $\bar{\nu}$. In the last row the results for $\bar{P} = 0$ are shown.

The "Coulomb energy" has been obtained as the difference between the energy of the system calculated with the Coulomb interaction and the energy calculated without it. It does not represent only the pure Coulomb energy but also the rearrangement energy due to the Coulomb interaction.

Table I

$\bar{\nu}$	$\bar{P}$	$E_0 - \frac{3}{2}\hbar\omega$	$\Delta E^{(1)}$	$\Delta E^{(2)}$	E_c	E_{int}	$(r.m.s.)_{CM}$
26	6	112.96	-139.32	-1.2112	0.8233	-26.748	1.448
24	6	112.96	-137.25	-1.5051	0.8231	-24.972	1.462
22	6	112.96	-135.00	-1.8783	0.8239	-23.094	1.476
26	0	112.96	-148.27	-0.9706	0.8608	-35.420	1.419

In Table II the dependence of the results on the frequency is shown for the cuts $\bar{\nu} = 26$, $\bar{P} = 6$ and for two values of ω .

Table II

$\hbar\omega$	$E_0 - \frac{3}{2}\hbar\omega$	$\Delta E^{(1)}$	$\Delta E^{(2)}$	E_c	E_{int}	$(r.m.s)_{CM}$
25.10	112.96	-139.32	-1.2112	0.8233	-26.748	1.448
21.32	95.94	-118.60	-0.3409	0.7762	-22.225	1.518

The computed values should be compared with the experimental data for ^4He :

$$E_{int} = -28.295 \text{ MeV}, \qquad r.m.s. = 1.65 \text{ f.}$$

The ^4He charge distribution $\rho_{CM}(r)$ calculated for the cut-off values $\bar{\nu} = 26$, $\bar{P} = 6$ and for the above values of ω are plotted in fig.1 together with the experimental curve /13/.

Similarly, fig.2 shows the ^4He charge from factor $F(q^2)$ which is given in the first Born approximation for the spherical charge distribution $\rho(r)$ by

$$F(q^2) = 4\pi \int_0^{+\infty} \rho(r) \frac{\sin q r}{q r} r^2 dr \ .$$

The "experimental" curve from ref. /13/ is given by

$$F_{exp}(q^2) = [1 - (a^2 q^2)^6] e^{-b^2 q^2} \qquad a = 0.316 \text{ f}$$

$$b = 0.681 \text{ f.}$$

IV. Discussion

The performed calculations were primarily intended to give some information about the plausibility of the approximations involved (the cut-offs for P and ν) and about the dependence of the results on the oscillator frequency ω .

The results may be summarized as follows:

1. The accuracy of our approximations does not depend on ω (therefore only the results for $\hbar\omega = 25.1$ are shown in detail).

2. The difference of the order 00.1 between the computed values for $\bar{P} = 5$ and $\bar{P} = 6$ indicates that the use of the cut-off in P is legitimate. On the other hand, when comparing the data for $\bar{P} = 6$ and $\bar{P} = 0$ in Table I, we see considerable differences. They are caused by the influence of the center-off-mass motion of the nucleon pair on the corresponding t-matrix.

3. The absolute value of the binding energy ($|E_{int}|$) increases with the increasing cut-off $\bar{\nu}$ and the value of the r.m.s slowly decreases. This dependence is qualitatively correct (cf. the footnote on p.12) but is too strong for E_{int} . It is necessary to take a larger value of ν .)Unfortunately, this exceeds the limited storage capacity of the computer we have used). In addition., the use of the same cut-off both for the regular and hard-core part of the kernel /5/ may improve convergence.

4. The calculated charge distribution and charge form factor display all the qualitative features of the experimental curves.

The charge dsitribution and charge form factor depend on the cut-off $\bar{\nu}$ rather weakly, approximately in the same way as the r.m.s. We can expect that higher values of $\bar{\nu}$ will not change the agreement with experiment.

5. We have considered only two values of ω since, in view of the bad convergence with respect to $\bar{\nu}$, it was not worth-while to repeat the calculation. The quantity $|E_{int}|$ tends to increase with increasing ω and the r.m.s. to decrease. We assume therefore that lower values of ω ($\hbar\omega \approx 20$ MeV) may give better results. This assumption is supported by the behaviour of the computed charge distribution. The strange maximum on the curve for $\hbar\omega = 25.1$ MeV is probably due to the worse convergence of the Goldstone expansion for this ω .

6. The small value of the second-order correstion to E_{int} indicates that the Goldstone expansion is applicable. This may also serve as an argument for different self-consistent approaches based on the requirement $\Delta E^{(2)} = 0$. On the other hand, the first-order correction to the charge distribution which in the perturbation theory correponds to the second-order correction to energy and is usually neglected in the self-consistent approaches is important. This correction fully accounts for the correct shape of the form-factor curve.

We hope that more extended calculations on a bigger computer will give a correct description of the ^{4}He ground state.

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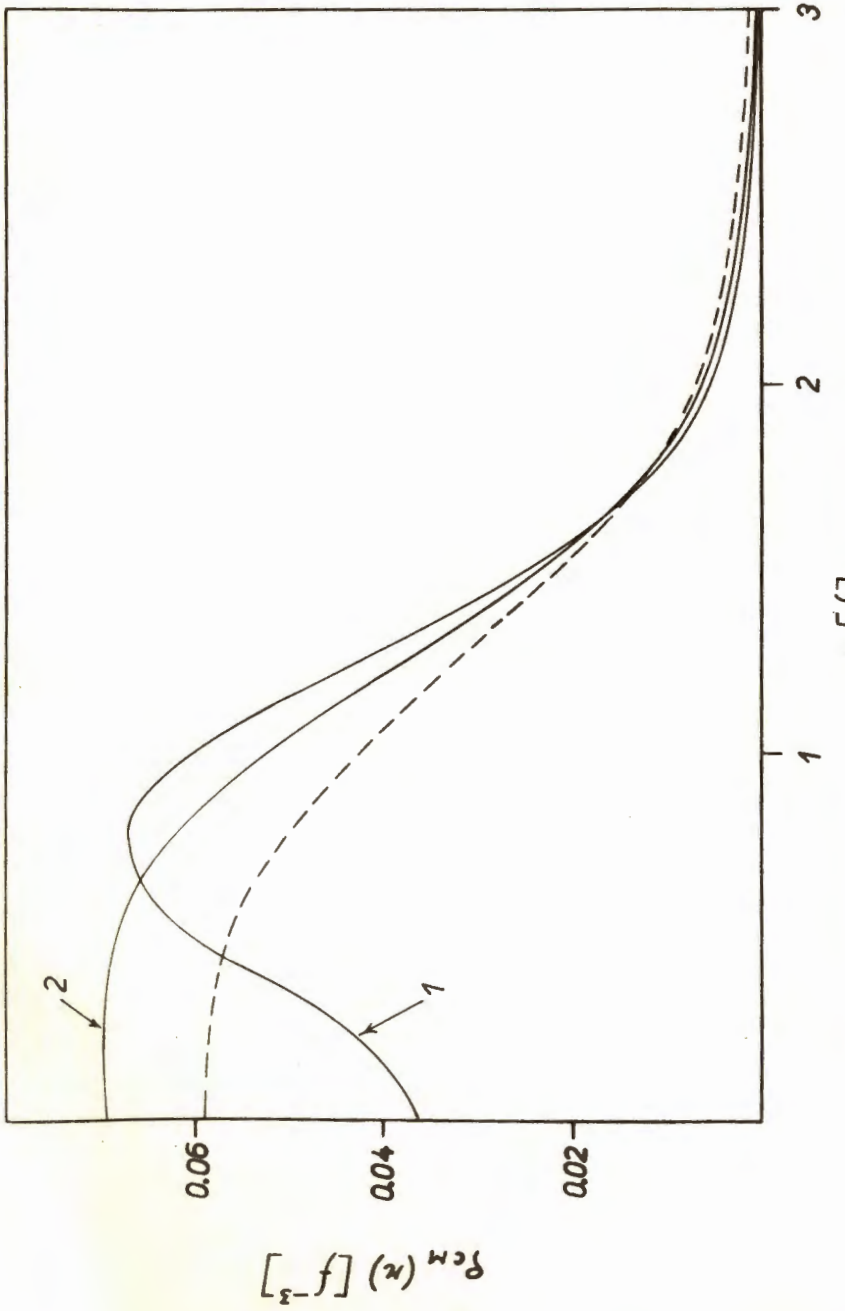

Fig.1. ^{4}He charge distribution $\rho(r)[f^{-3}]$. The solid lines denoted by 1,2 correspond to $\hbar\omega$ = 25,1 and 21,32 MeV respectively. The experimental curve is represented by the dashed line.

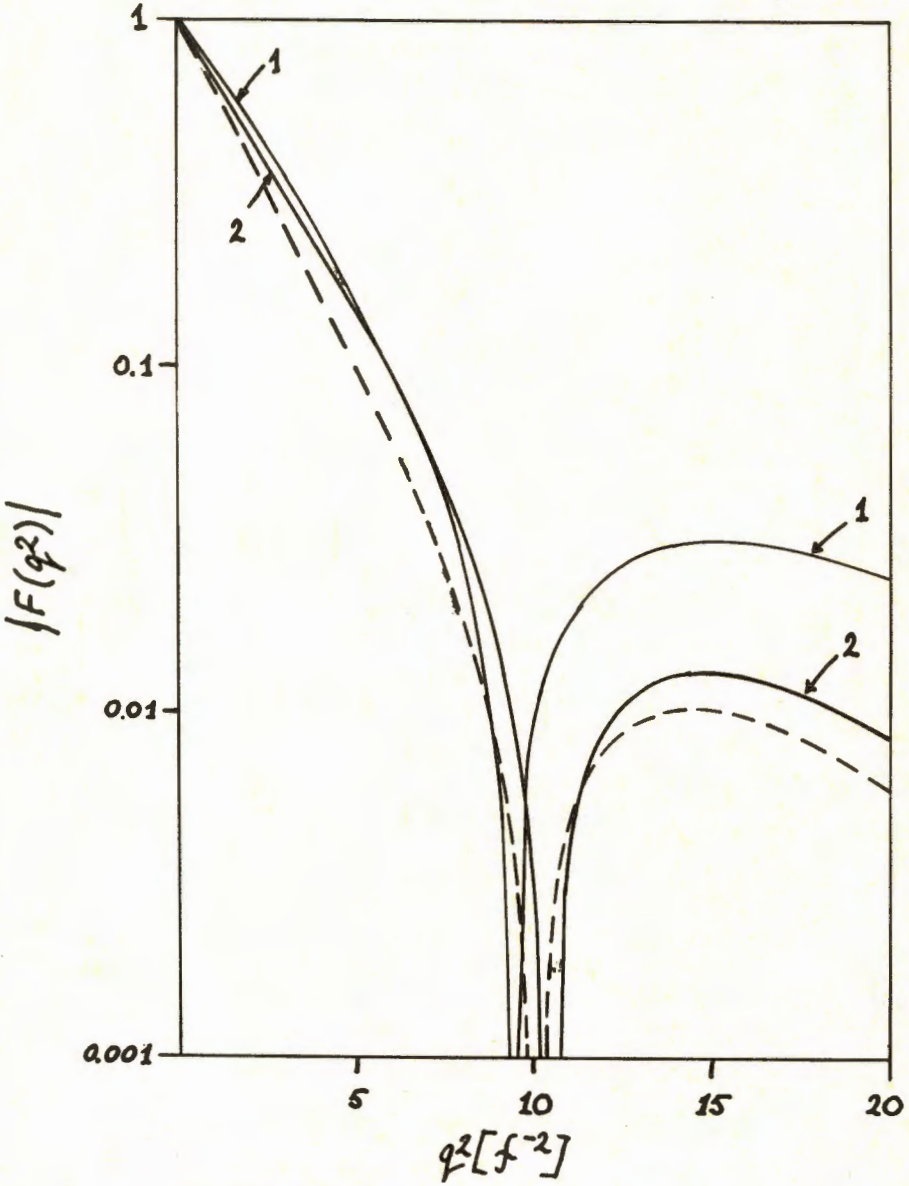

Fig. 2. The absolute value of the form-factor of charge ^{4}He. The same designations, as in fig. 1.