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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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IN THE RANGE   50 < Z < 82, 50 < N < 82

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I.

The existence of deformed nuclei in the range $50 < Z < 82$, $50 < N < 82$ was first discovered experimentally in ref. [1] and then proved in ref. [2]. In ref. [3] the nuclear equilibrium deformations in this region were calculated for positive deformation only and with no account of pairing correlations, without explicit introduction of the Coulomb energy. In ref. [4] the equilibrium deformations of the ground states of nuclei in this region were investigated in the framework of the model taking into account superconducting pairing correlations and the quadrupole-quadrupole interactions. It was shown that a number of the Ba isotopes may have negative equilibrium deformation. The present paper aims to calculate the equilibrium deformations for the ground states of even-even nuclei in the range $50 < Z < 82$ and $50 < N < 82$ and to investigate the dependence of the results on the methods of calculation and the choice of parameters.

II.

The energies $\mathscr{E}_0(\beta, \gamma)$ of the ground states of even-even nuclei were calculated for the β-values from 0 to 0.5 and for the γ va-

lues from $0°$ to $60°$ in the same way as in ref./5/ by the Bes-Szymanski method /6/ improved in ref./7/ and by the Strutinsky method/8/.

In the calculations by the Strutinsky method /8/ the total energy of the ground state of an even-even nucleus is divided into the two parts

$$\mathcal{E}_0(\beta,\gamma) = \mathcal{E}_{drop}(\beta,\gamma) + \Delta\mathcal{E}(\beta,\gamma), \tag{1}$$

where $\mathcal{E}_{drop}(\beta,\gamma)$ is the energy in the liquide drop model, its parameters are determined from the experimental data on nuclear masses. The shell correction

$$\Delta\mathcal{E}(\beta,\gamma) = \Delta\mathcal{E}(Z) + \Delta\mathcal{E}(N) \tag{2}$$

consists of the proton and neutron parts and

$$\Delta\mathcal{E}(Z) = \mathcal{E}_p - \bar{\mathcal{E}}(Z), \tag{3}$$

where $\mathcal{E}_p$ is determined below in (6), the averaged energy

$$\bar{\mathcal{E}}(Z) = \int_{-\infty}^{\lambda} E \, g(E) \, dE \tag{4}$$

$$g(E) = \frac{1}{\sqrt{\pi}} \frac{1}{\tilde{\gamma}} \sum_\nu \exp\left\{-\left(\frac{E - E(\nu)}{\tilde{\gamma}}\right)^2\right\}$$

and λ is the chemical potential determined from the implicit equation $Z = \int_{-\infty}^{\lambda} g(E) \, dE$. The parameter $\tilde{\gamma}$ is close to the energy difference between shells.

In calculating by the Bes-Szymanski method the total energy may be represented in the form

$$\mathcal{E}_0(\beta,\gamma) = \mathcal{E}_p + \mathcal{E}_n + \mathcal{E}_0, \tag{5}$$

where $\mathcal{E}_p$, $\mathcal{E}_n$ are the energies of the proton and neutron systems, $\mathcal{E}_0$ is the Coulomb energy of a uniformly charged ellipsoid. The energy of the proton system is

$$\mathcal{E}_p = \sum_\nu E(\nu) \, 2 v_\nu^2 - \frac{C_p^2}{G_z}, \tag{6}$$

where the summation is performed over the average field one-particle levels, $E(\nu)$ is the one-particle energy, G_z is the constant of pairing interactions in the proton system, C_p is the correlation function, $2 v_\nu^2$ is the proton density in the ν state.

For an axial-symmetric ellipsoid the Coulomb energy can be written in the form/9/

$$\mathcal{E}_0(\epsilon) = \mathcal{E}_0(0) \, g(\epsilon), \tag{7}$$

where $\mathcal{E}_0(0) = 0.6 \frac{Z e^2}{R}$ is the Coulomb energy of an uniformly charged sphere, and

$$g(\epsilon) = \frac{(1 - 2/3 \, \epsilon)^{2/3} (1 + 1/3 \, \epsilon)^{1/3}}{\sqrt{2\epsilon - 1/3 \, \epsilon^2}} \ell n \frac{1 - 2/3 \, \epsilon}{1 + 1/3 \, \epsilon - \sqrt{2\epsilon - 1/3 \, \epsilon^2}} \tag{8}$$

for $\epsilon > 0$ and

$$g(\epsilon) = \frac{(1 - 2/3 \, \epsilon)^{2/3} (1 + 1/3 \, \epsilon)^{1/3}}{\sqrt{1/3 \, \epsilon^2 - 2\epsilon}} \, arctg \frac{\sqrt{1/3 \, \epsilon^2 - 2\epsilon}}{1 + 1/3 \, \epsilon} \tag{8'}$$

for $\epsilon < 0$. The deformation parameter ϵ is introduced in ref./10/, it is connected with the usually employed parameter β as follows

$$\epsilon = \frac{3/2 \sqrt{\frac{5}{4\pi}} \, \beta}{1 + 1/2 \sqrt{\frac{5}{4\pi}} \, \beta} \approx 0.95 \, \beta \, (1 - 0.32 \, \beta).$$

The quadrupole moment for the ground state is

$$Q_0 = \sum_\nu q_{\nu\nu} \, 2 v_\nu^2. \tag{9}$$

where $q_{\nu\nu}$ is the diagonal matrix element of the quadrupole moment operator.

III.

Let us consider the parameters used. This problem is of especial importance since the calculations of the properties of nuclei of the transition region are very sensitive to the choice of the parameters. The calculations are based on the one-particle levels of the Newton potential /11/ which at $\gamma = 0°$ transforms into the Nilsson potential with $<\ell^2>$ term. In all our calculations for protons in the range $50 < Z < 82$ we use the same scheme of levels which at $\gamma = 0°$ turns into the scheme suggested in ref./12/ for the rare-earth region and on which calculations made in ref./13/ are based.

For neutrons in the region $50 < N < 82$ we use three different schemes: Scheme I, at $\gamma = 0°$ transforms into the scheme suggested in ref. /12/ for the rare-earth region.

The scheme 2 at $\gamma = 0°$ transforms into the scheme used in ref./13/ for the calculation of the nuclear properties with the number of neutrons $58 < N < 136$.

The extrapolation scheme for which the parameters were chosen as follows: we took the potential parameters κ and μ given in ref. /12/ first for the region $126 < N < 184$ with the centre N=155, then the parameters for the region $82 < N < 126$ with the centre N=103 and extrapolated them linearly to the centre N=65 in the region $50 < N < 82$. As a result we obtained $\kappa = 0.0638$ and $\mu = 0.49$.

The pairing correlation constants G_N and G_z were taken from ref./13/, their values were renormalized since the calculation was made with 24 one-particle levels ($G_N A = 25$ MeV, $G_z A = 28.5$ MeV). The stability of the results against small changes in G_N and G_z is investigated. It is shown that the changes in G_N and G_z within (5-10)% do not lead to a noticeable change of the equilibrium deformation parameters.

IV.

For the ground states of even-even nuclei in the region $50 < Z < 82, 50 < N < 82$ the total energies $\mathcal{E}_0(\beta,\gamma)$ are calculated as functions of the deformation parameters β and γ and the equilibrium deformations, β_0, γ_0 are found. The calculations by the Bes-Szymanski method are made for schemes 1,2 and 3, the calculations by the Strutinsky method are made for scheme I. The results obtained with and without the account of the Coulomb energy are analysed. Figs. 1-6 give the results obtained by the Bes-Szymanski method according to scheme I.

The quadrupole moments calculated by eq.(9) coincide practically with those corresponding to uniformly charged ellipsoids what indicates to a self-consistent solution of the problem.

The results of calculations of the deformations corresponding to the energy minima $\mathcal{E}_0(\epsilon)$ at $\gamma = 0°$ denoted by ϵ_0^+ and at $\gamma = 60°$ denoted by ϵ_0^- are given in Figs. 1 and 2 in the form of contour maps. The values for ϵ_0^+ and ϵ_0^- correspond to the two minima of the function $\mathcal{E}_0(\beta, \gamma = 0°)$ depending on β, one at $\beta > 0$, the other at $\beta < 0$. The function of such a type is given in Fig. 5 for ^{126}Ba.

Fig. 3 gives the deformation energies $\mathcal{E}_{def}(\epsilon_0^+) = \mathcal{E}_0(\epsilon = 0) - \mathcal{E}_0(\epsilon_0^+)$ for the minima ϵ_0^+ at $\gamma = 0°$. It is seen that in the middle of the region the deformation energies are 8 MeV, what is close to those obtained in ref./5/ for nuclei of the middle of the regions $150 < A < 190$ and $A > 226$. Most nuclei of the region considered have the deformation energy higher than 2 MeV.

The function $\mathcal{E}_0(\beta,\gamma)$ has one minimum, for some nuclei it lies at $\gamma = 0°$ for others at $\gamma = 60°$. Fig.4 gives the difference of the deformation energies

$$\Delta\mathcal{E}_{def} = \mathcal{E}_{def}(\epsilon_0^+) - \mathcal{E}_{def}(\epsilon_0^-) = \mathcal{E}_0(\epsilon_0^-) - \mathcal{E}_0(\epsilon_0^+)$$

corresponding to the minima at $\gamma = 0°$ and $\gamma = 60°$. The solid line

in Fig.4 corresponds to the case $\Delta\mathcal{E}_{def} = 0$. The positive value of $\Delta\mathcal{E}_{def}$ is denoted by $\oplus$ and points out that the equilibrium shape of the corresponding nuclei is a prolate ellipsoid of rotation. To the negative values of $\Delta\mathcal{E}_{def}$ denoted by $\ominus$ there correspond nuclei having the shape of an oblate ellipsoid of rotation. It is seen from Fig.4 that there is a large number of nuclei the equilibrium deformation of which is an oblate ellipsoid of rotation.

Basing on the data of Figs. 1 and 2, the equilibrium deformation parameters ϵ_0 or β_0 can be found as follows. For a given nucleus the difference $\Delta\mathcal{E}_{def}$ should be found from the curves of Fig.4. If $\Delta\mathcal{E}_{def} > 0$ then the equilibrium deformation is a prolate ellipsoid of rotation then the corresponding value of ϵ_0^+ from Fig.1 determines the equilibrium deformation. Otherwise the equilibrium shape is an oblate ellipsoid and the corresponding value of ϵ_0^- is found from Fig.2.

The contour maps of Figs 1-4 are calculated by the Bes-Szymanski method with schemes 2 and 3. The calculations according to all the three schemes give similar results. Calculating according to scheme 2 one obtained somewhat larger deformation parameters ϵ_0^+ and ϵ_0^- and higher deformation energies as compared to the scheme 2. The region with maximum deformation is shifted to smaller Z and N. The region of the negative values of $\Delta\mathcal{E}_{def}$ on a plot similar to Fig.4 in the range Z=56-60 and N=66-74 was increased.

In calculating with scheme 3 one obtained somewhat smaller parameters ϵ_0^+ and ϵ_0^- as compared to scheme I, the region of maximum equilibrium deformations being somewhat shifted to larger Z. The region of nuclei with positive equilibrium deformations is essentially extended. The values of $\Delta\mathcal{E}_{def}$ in the range Z=54-60, N=66-74 were decreased and does not exceed in its absolute value 0.4 MeV.

For the ground states of even-even nuclei the deformation energy of which is higher than 2 MeV, the parameter γ_0 is either 0° or 60° i.e. all the deformed nuclei are axially symmetric.

In ref./14/ corrections to the nuclear total energy connected with the projection of the nuclear wave functions on the state with a fixed momentum I and its projection M were introduced. The account

of such corrections leads to a small increase of the deformation energy, retaining β_0 practically unaffected.

V.

Of most interest in the region considered are strongly neutron-deficient isotopes of xenon, cesium, barium, lanthanum, cerium and some others. The parameters of the equilibrium deformations and the deformation energies for nuclei from the transition regions are far more sensitive to the calculation methods and the choice of the parameters as compared to strongly deformed nuclei. In this case it is important to have results independent of the calculation method and the choice of parameters and , in addition, to single out results corresponding to the best choice of the parameters.

The result which is independent of the calculation method and the choice of the parameters for the nuclei considered appears to be the following: energy difference between minima, corresponding to a prolate and oblate ellipsoid of rotation is very small. This is the main difference in the behaviour of the function $\mathcal{E}_0(\beta,\gamma)$ for nuclei of a given region as compared to the behaviour of $\mathcal{E}_0(\beta,\gamma)$ for nuclei in the region $150 < A < 190$ and $A > 226$.

Table I gives a small part of the results for a number of xenon, barium, cerium and neodymium isotopes. The calculations are performed by schemes 1,2,3 by the Bes-Szymanski method without and with the account of the Coulomb energy (denoted by NC and C respectively) and by the Strutinsky method (denoted by St.). The deformation energies $\mathcal{E}_{def}(\epsilon_0^F)$ and the deformation energy differences $\Delta\mathcal{E}_{def}$ are given in MeV.

From Table I it is seen that the calculations by scheme 2 give the most deep minima of the function $\mathcal{E}_0(\beta,\gamma)$ and the largest values of $|\Delta\mathcal{E}_{def}|$ as compared to the calculations by schemes I and 3. The account of the Coulomb energy leads to an increase in the deformation energy by 1-2 MeV and to an increase of the deformation parameters ϵ_0^+ and ϵ_0^- by 0.04-0.08. It is also seen that for some nuclei all the results indicate that their equilibrium deformation is an oblate ellipsoid of rotation.

Let us compare the results obtained by the Bes-Szymanski method and the Strutinsky method. It is seen from Table I that the deformation energies $\mathcal{E}_{def}(\epsilon_0^-)$, $\mathcal{E}_{def}(\epsilon_0^+)$ calculated by the Strutinsky method are lower by 0.5-1.5 MeV as compared to the Bes-Szymanski method. When going to the middle of the region $50 < N < 82$, $50 < Z < 82$ this divergence increases, the deformation parameters calculated by the Strutinsky method are smaller by 0.02-0.05 than the $\epsilon_0^{\mp}$ values obtained by the Bes-Szymanski method with the account of the Coulomb energy. The $\epsilon_0^{\mp}$ and $\mathcal{E}_{def}(\epsilon_0^{\mp})$ values calculated by the Strutinsky method lie between the corresponding $\epsilon_0^{\mp}$ and $\mathcal{E}_{def}(\epsilon_0^{\mp})$ values calculated by the Bes- Szymanski method with and without the account of the Coulomb energy.

In ref./8/ it is indicated that the ϵ_0 and $\mathcal{E}_{def}$ values obtained by the Bes-Szymanski method depend on the conservation of the volume of the nucleus. The analysis showed that in calculations by the Nilsson scheme with the term $<\ell^2>$ /12/ the volume conservation condition is fulfilled better than in schemes without $<\ell^2>$/10/. However in the calculations by the Bes-Szymanski method it is difficult to take into account with sufficient accuracy the effect of the Coulomb energy and ensure volume conservation for the potential, containing the spin-orbital interaction.

VI.

For the ground states of even-even nuclei the total energy $\mathcal{E}_0(\beta,\gamma)$ is of the form similar to that represented in Fig.5 (the lower solid curve) and in Fig.6 for ^{126}Ba . The function $\mathcal{E}_0(\beta,\gamma)$ is calculated by the Bes-Szymanski method taking into account the Coulomb energy. It is seen from Figs.5 and 6 that the function $\mathcal{E}_0(\beta,\gamma)$ has one minimum at $\beta = -0.25$ and $\gamma = 0^\circ$ and the equilibrium deformation of ^{126}Ba is an oblate ellipsoid. The dependence $\mathcal{E}_0(\beta,\gamma)$ given in Fig.6 shows that there is a valley for $\beta = 0.25$ when γ changing from 0° to 60°. ($\beta < 0$ corresponds to $\beta > 0$ and $\gamma = 60^\circ$).

Therefore there is a smooth transition from the minima at $\gamma = 0^\circ$ to the minimum at $\gamma = 60^\circ$.

Nuclei of the region considered are characterized by a large softness with respect to γ -vibrations, the functions $\mathcal{E}_0(\beta,\gamma)$ change weakly and monotonously with γ changing from 0° to 60°. Therefore in studying rotational nuclear states in the region considered one should take into account the connection of rotations with gamma vibrations.

The small value of the difference $\Delta\mathcal{E}_{def}$ for the ground states of even-even nuclei implies the possible existence of excited quasiparticle states the equilibrium deformations of which differ strongly from those in the ground states. Fif.5 gives the energies $\mathcal{E}(\rho_1,\rho_2)$ of the two-quasiparticle excited states as a function of β . It is seen that ^{126}Ba in the ground and two-quasiparticle nn($523\downarrow$, $532\downarrow$) states is an oblate ellipsoid. The two-quasiparticle states with

nn($523\downarrow$, $402\downarrow$ $-$ $532\downarrow$, $411\downarrow$) and pp($420\uparrow$, $541\uparrow$ $-$ $402\uparrow$, $514\uparrow$) have positive equilibrium deformation corresponding to prolate ellipsoid.

Thus, due to weak dependence of the function $\mathcal{E}_0(\beta,\gamma)$ on γ in nuclei of the region considered two types of states may exist: some of them correspond to the prolate nuclear shape, the others to the oblate one.

The electromagnetic transitions with the change of the nuclear shape are hindered (though in this case the hindrance is not large) therefore the states of this kind may be called shape isomers.

An analogous situation takes place in other even-even nuclei as well as in odd and odd-odd nuclei. According to our calculations the shape isomers may exist for a number of isotopes of Xe , Cs, Ba, La, Ce and others. Their experimental observation is of great interest.

References

1. R.K.Sheline, T.Sikkeland, R.N.Chande. Phys. Rev. Lett., _7_, 446 (1961).

2. J.E.Clarkson, R.M.Diamond, I.S.Stephens and I.Perlman. Nucl. Phys., _A93_, 272 (19767).

3. E.Marshalek, L.W.Person, R.K.Sheline. Rev. Mod. Phys., _35_, 108 (1963),.

4. K.Kumar, M.Baranger. Phys. Rev. Lett., _12_, 73 (1964).

5. D.A.Arseniev, L.A.Malov, V.V.Pashkevich, V.G.Soloviev, Izv. AN SSSR (ser. phys.), _32_, 864(1968).

6. D.R.Bes, Z.Szymanski. Nucl. Phys., _28_, 42 (1961); Z.Szymanski. Nucl. Phys., _28_, 63 (1961).

7. A. Sobiczewski. Nucl. Phys., A93, 501; Nucl. Phys., _A96_, 258(1967).

8. W.M.Strutinsky. Yad. Phys., 3, 614 (1966); Nucl.Phys., _A95_, 420(1967).

9. S.G.Nilsson. Lectures at International School of Physics "Enrico Fermi" 26-June-15 July. Varenna (Como) Italy (1967).

10. S.G.Nilsson. Mat.Fys.Medd.Dan.Vid.Selsk., _29_, No 16 (1955).

11. T.D.Newton. Can. Journ. Phys., _38_, 700 (1960).

12. G.Gustafson, I.L.Lamn, B.Nilsson, S.G.Nilsson. Arkov.für Fysik, _36_, 613 (1967).

13. L.A.Malov, V.G.Soloviev, I.D.Khristov. Yad. Phys., _6_, 1186 (1967).

14. E.B.Balbutcev, I.N.Mikhailov, Izv. AN SSSR, _30_, 1118 (1966).

Table I

Deformation Parameters $\varepsilon_0^{\mp}$, Deformation Energies $\mathcal{E}_{def}(\varepsilon_0^{\pm})$ (in MeV) and Deformation Energy Deference $\Delta\mathcal{E}_{def}$ (in MeV)

Nuclei		Scheme I			Scheme 2		Scheme 3
		NC	C	St	NC	C	C
1	2	3	4	5	6	7	8
$^{120}_{54}Xe_{66}$	ε_0^-	0,17	0,24	0,19	0.18	0.25	0.23
	$\mathcal{E}_{def}(\varepsilon_0^-)$	0.6	1.9	1.0	1.3	2.8	1.9
	ε_0^+	0.11	0.25	0.15	0.15	0.25	0.24
	$\mathcal{E}_{def}(\varepsilon_0^+)$	0.2	1.6	0.6	0.6	2.1	2.0
	$\Delta\mathcal{E}_{def}$	-0.4	-0.4	-0.4	-0.7	-0.7	+0.1
$^{122}_{54}Xe_{68}$	ε_0^-	0.16	0.22	0.20	0.18	0.24	0.22
	$\mathcal{E}_{def}(\varepsilon_0^-)$	0.6	1.9	1.1	1.4	2.9	1.9
	ε_0^+	0.10	0.21	0.14	0.15	0.22	0.22
	$\mathcal{E}_{def}(\varepsilon_0^+)$	0.2	1.3	0.7	0.8	2.0	1.7
	$\Delta\mathcal{E}_{def}$	-0.4	-0.6	-0.4	-0.6	-0.9	-0.2
$^{124}_{54}Xe_{70}$	ε_0^-	0.14	0.21	0.17	0.17	0.23	0.20
	$\mathcal{E}_{def}(\varepsilon_0^-)$	0.5	1.5	0.9	1.2	2.5	1.6
	ε_0^+	0.10	1.18	0.13	0.14	0.19	0.18
	$\mathcal{E}_{def}(\varepsilon_0^+)$	0.2	1.0	0.7	0.8	1.8	1.2
	$\Delta\mathcal{E}_{def}$	-0.3	-0.5	-0.2	-0.4	-0.7	-0.4
$^{126}_{54}Xe_{72}$	ε_0^-	0.11	0.17	0.13	0.14	0.20	0.17
	$\mathcal{E}_{def}(\varepsilon_0^-)$	0.2	0.9	0.6	0.8	1.8	1.0
	ε_0^+	0.10	0.15	0.15	0.12	0.16	0.15
	$\mathcal{E}_{def}(\varepsilon_0^+)$	0.1	0.7	0.5	0.6	1.5	0.9
	$\Delta\mathcal{E}_{def}$	-0.1	-0.2	-0.1	-0.2	-0.3	-0.1

1	2	3	4	5	6	7	8
	ε_0^-	0.20	0.26	0.24	0.21	0.27	0.25
$^{124}_{56}Ba_{68}$	$E_{def}(\varepsilon_0^-)$	1.4	3.3	2.0	2.3	4.4	3.3
	ε_0^+	0.17	0.26	0.22	0.18	0.26	0.25
	$E_{def}(\varepsilon_0^+)$	0.8	2.8	1.6	1.5	3.6	3.2
	ΔE_{def}	-0.6	-0.5	0.4	-0.8	-0.8	-0.1
	ε_0^-	0.18	0.25	0.22	0.20	0.26	0.25
$^{126}_{56}Ba_{70}$	$E_{def}(\varepsilon_0^-)$	1.1	2.9	1.7	2.0	4.0	2.9
	ε_0^+	0.15	0.22	0.17	0.16	0.23	0.22
	$E_{def}(\varepsilon_0^+)$	0.7	2.2	1.4	1.5	3.1	2.5
	ΔE_{def}	-0.4	-0.7	-0.3	-0.5	-0.9	-0.4
	ε_0^-	0.14	0.23	0.18	0.17	0.25	0.23
$^{128}_{56}Ba_{72}$	$E_{def}(\varepsilon_0^-)$	0.7	2.0	1.3	1.4	3.1	2.1
	ε_0^+	0.14	0.19	0,17	0.15	0.20	0.18
	$E_{def}(\varepsilon_0^+)$	0.5	1.6	1,2	1.2	2.5	1.8
	ΔE_{def}	-0.2	-0.4	-0,1	-0.2	-0.6	-0.3
	ε_0^-	0.11	0.18	0.13	0.14	0.20	0.18
$^{130}_{56}Ba_{74}$	$E_{def}(\varepsilon_0^-)$	0.3	1.0	0.7	0.9	2.0	1.1
	ε_0^+	0.10	0.15	0.15	0.13	0.16	0.15
	$E_{def}(\varepsilon_0^+)$	0.2	1.0	0.7	0.8	1.7	1.2
	ΔE_{def}	-0.1	0	0	-0.1	-0.3	+0.1

| 1	2	3	4	5	6	7	8
	ε_0^-	0.21	0.28	0.25	0.22	0.28	0.27
$^{128}_{58}Ce_{70}$	$E_{def}(\varepsilon_0^-)$	1.8	4.1	2.5	2.8	5.2	4.0
	ε_0^+	0.18	0.26	0.22	0.20	0.26	0.25
	$E_{def}(\varepsilon_0^+)$	1.4	3.6	2.3	2.2	4.6	3.8
	ΔE_{def}	-0.4	-0.5	-0.2	-0.6	-0.6	-0.2
	ε_0^-	0.18	0.26	0.22	0.20	0.28	0.25
$^{130}_{58}Ce_{72}$	$E_{def}(\varepsilon_0^-)$	1.2	3.0	1.8	2.1	4.2	3.1
	ε_0^+	0.15	0.22	0.17	0.17	0.23	0.21
	$E_{def}(\varepsilon_0^+)$	1.0	2.6	1.8	1.8	3.6	2.8
	ΔE_{def}	-0.2	-0.4	0	-0.3	-0.6	-0.3
	ε_0^-	0.13	0.22	0.18	0.16	0.25	0.22
$^{132}_{58}Ce_{74}$	$E_{def}(\varepsilon_0^-)$	0.6	1.8	1.1	1.3	2.9	1.9
	ε_0^+	0.14	0.18	0.15	0.15	0.20	0.18
	$E_{def}(\varepsilon_0^+)$	0.5	1.7	1.2	1.3	2.6	1.9
	ΔE_{def}	-0.1	-0.1	0. 1	0	-0.3	0
	ε_0^-	0.19	0.28	0.25	0.21	0.29	0.27
$^{132}_{60}Nd_{72}$	$E_{def}(\varepsilon_0^-)$	1.6	3.9	2.4	2.6	5.2	4.0
	ε_0^+	0.18	0.26	0.24	0.19	0.26	0.24
	$E_{def}(\varepsilon_0^+)$	1.5	3.7	2.6	2.4	4.8	3.8
	ΔE_{def}	-0.1	-0.2	0.2	-0.2	-0.4	-0.2
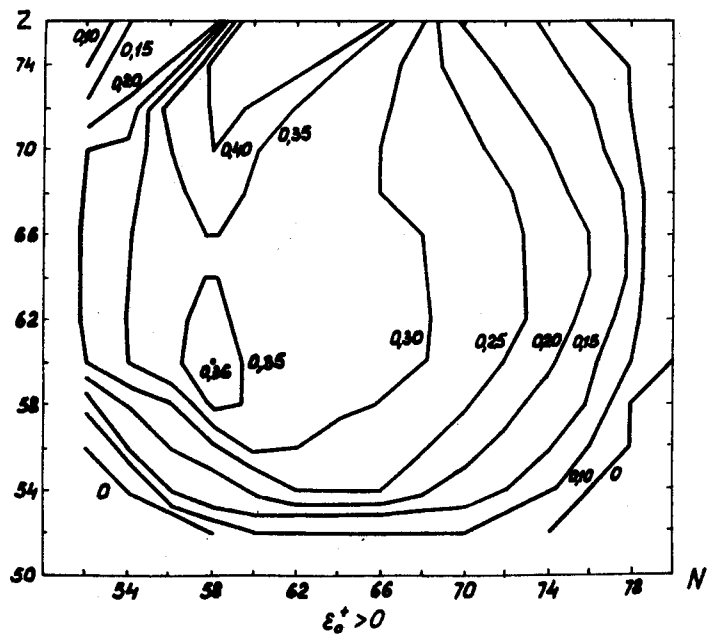
Fig.1. Contour map for the equilibrium parameter ϵ_0^+ as a function
of the number of neutrons N and protons Z. The numbers along
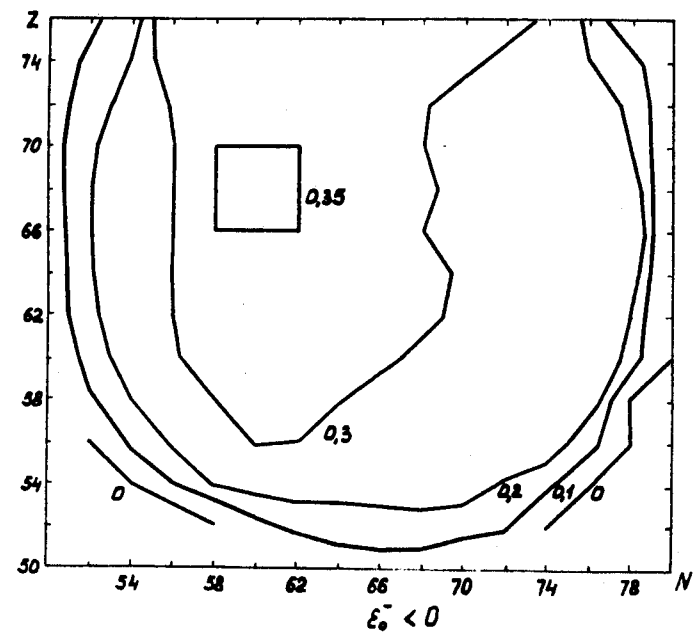the cúrves give the ϵ_0^+ values.

Fig.2. Contour map for the deformation parameter ϵ_0^- as a function
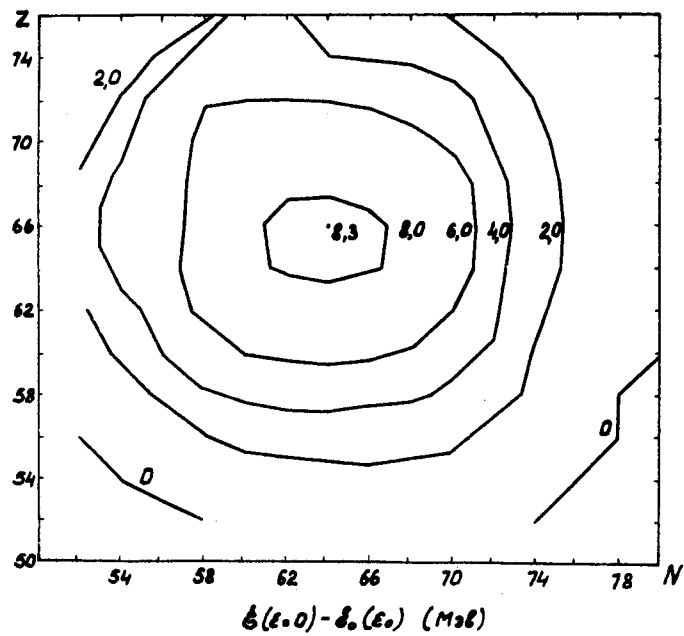of the number of neutrons N and protons Z. The numbers along
the curves give the ϵ_0^- values.

Fig.3. Equipotential curves for the deformation energy

$$\mathcal{E}_{def} \, (\epsilon_0^+) = \mathcal{E}_0(\epsilon = 0) - \mathcal{E}_0(\epsilon_0^+)$$

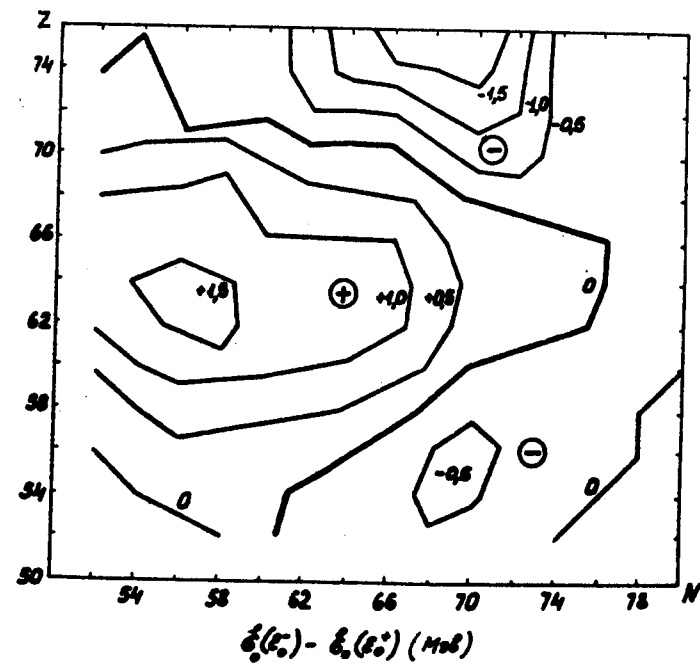at $\epsilon_0^+ > 0$ as a function of the number of neutrons N and protons Z.

Fig.4. Equipotential curves for the energy difference

$$\Delta \mathcal{E}_{def} = \mathcal{E}_{def}(\epsilon_0^+) - \mathcal{E}_{def}(\epsilon_0^-) = \mathcal{E}_0(\epsilon_0^-) - \mathcal{E}_0(\epsilon_0^+)$$

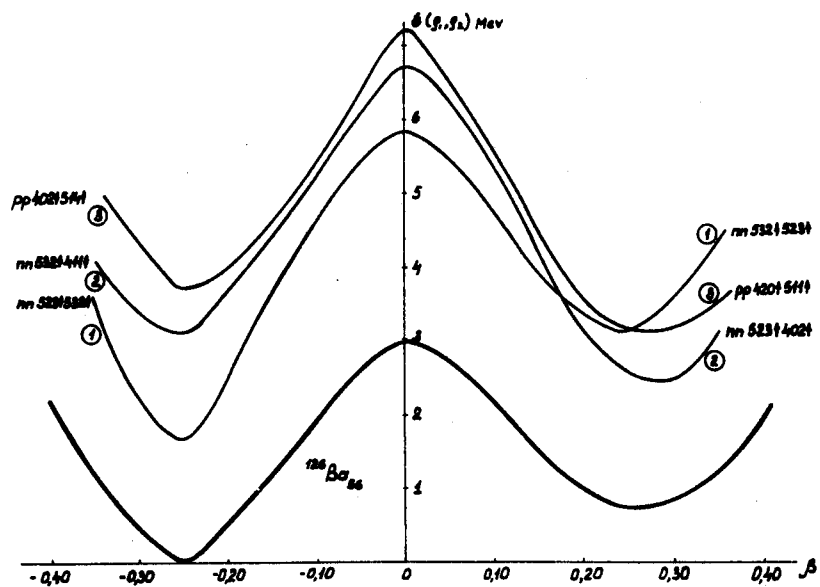as a function of the number of neutrons N and protons Z.

Fig. 5. Behaviour of the total energy of the ground and three two-
quasiparticle states of ^{126}Ba (in MeV) as a function of the de-
formation parameter β from $\beta =0.4$ to $\beta =-0.4$ at $\gamma=0°$.
For the two-quasiparticle states quantum numbers $N n_z \Lambda \Sigma$
are given on the right for $\gamma = 0°$, on the left at $\beta < 0$, which
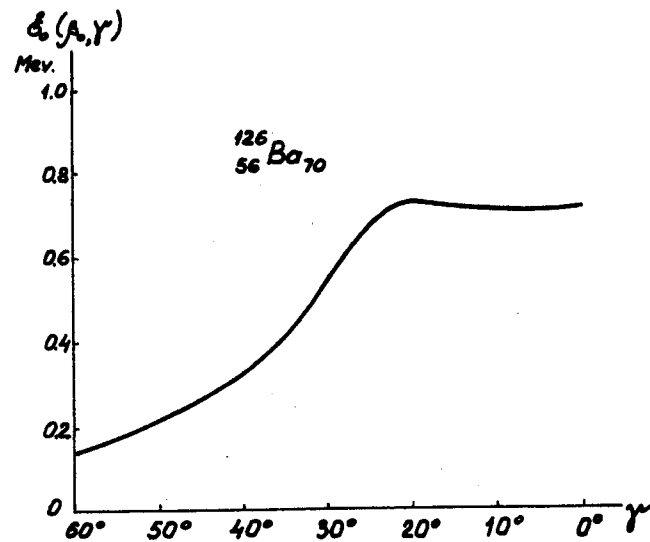corresponds to $\beta > 0$, $\gamma = 60°$.

Fig. 6. Behaviour of the total energy $\mathcal{E}(\beta,\gamma)$ (in MeV) for ^{126}Ba as
a function of γ at $\beta_0 = 0.25$.