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ЛАБОРАТОРИЯ НЕЙТРОННОЙ ФИЗИКИ

THE SCATTERING OF LIGHT BY AN ELECTRON MOVING IN A REFRACTIVE MEDIUM

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In a number of cases the optical properties of condensed matter are important for high energy physics. It became apparent for the first time after the discovery by Vavilov and Čerenkov of the new phenomenon which is often called the Čerenkov effect. As is known. in the Vavilov-Čerenkov effect light is emitted in the directions which form the angle θ_0 at the velocity v according to the formula

$$\cos\theta_0 = \frac{c}{v\,n_0}, \tag{1}$$

where c is the light velocity and n_0 is the refractive index of light for the frequency ω_0. In addition, if we take into account that the radiation intensity is proportional to the square of the charge, we shall see that this phenomenon depends on three quantities of equal importance. Two of these, namely, the charge and the velocity, characterize the particle, while the third, the refractive index, characterizes the medium.

It is noteworthy that originally physicists were inclined to consider the Vavilov-Čerenkov effect to be a purely optical phenomenon. It was regarded as an example of super-light velocity optics. At present this radiation has found essential applications in high energy physics, and it is regarded now as related to nuclear physics. As a matter of fact, it would be more correct to say that in this phenomenon the optical properties of the medium and particle properties are in close connection.

A number of other phenomena exists in which the optical properties of the medium prove to be very important. The so-called transition radiation can serve as an example of such a case. The electromagnetic field of the particle depends not only on the particle charge and velocity but on the properties of the medium in which the particle is travelling, as well. If the particle crosses the boundary of two media, say it travels from a vacuum into a solid, the field varies. In this case there should appear radiation which is called the transition radiation. As in the Vavilov-Čerenkov effect, in this case radiation also occurs when the particle is moving uniformly.

Over the ten last years this kind of radiation has been studied in rather good detail by physicists of different countries. At an initial stage, for experimental reasons, it was observed in the bombardment of metallic surface with electrons. It was even supposed that it might be specific for metals and be associated with vibrations of electron plasma in a metal. However, Tanaka and Katayama /1/ have shown that in agreement with the theory of transition radiation, this radiation occurs also on the boundary of a vacuum and a dielectric.

The study of transition radiation is at such a stage now that one can speak of this as one of the methods of investigating the optical properties of a medium.

At the same time some recent papers indicate that at relativistic particle energies the transition radiation acquires some unexpected features, which will permit its application in high energy physics, as well. Thus here again optics turns out to be closely connected with nuclear physics.

In my lecture I would like to dwell upon a comparatively new problem which connects optics with nuclear physics, namely, the scattering of light by a rapidly moving electron. This phenomenon drew attention by the fact that visible light can be transformed into gamma-rays as a result of scattering at the expense of the electron energy. The appearance of the powerful light sources of lazers has made it possible to study this scattering experimentally. The first successful experiments of this kind have been already carried out /2/.

In order to simplify the formulae, I shall restrict myself to the consideration of only such a case when a small fraction of the electron energy is transformed into light. This requirement is always satisfied for the scattering of ordinary visible light if the electron energy does not exceed a few GeV. In this case one can make use of the classical Doppler formulae to vary the frequency. Apparently, in the scattering of light a double transformation of the frequency takes place. If the incident light beam to be scattered forms an angle θ_0 with respect to the electron velocity and its frequency is ω_0, the light frequency affecting the electron will be equal to

$$\Omega = \omega_0 \, (1 - \beta \cos \theta_0). \tag{2}$$

According to the Doppler formula the light scattered at an angle θ with respect to the electron velocity will have the following frequency:

$$\omega = \frac{\Omega}{1 - \beta \cos\theta} = \frac{\omega_0 (1 - \beta \cos \theta_0)}{1 - \beta \cos \theta}. \tag{3}$$

The frequency of the light resulting from scattering seems to be highest if the initial beam of light is directed towards the electrons and the light is scattered in the direction of the electron motion. In this case $\cos\theta_0 = -1$, $\cos \theta = 1$ and the frequency of the light is expressed as follows

$$\omega = \frac{\omega_0 (1+\beta)}{1-\beta} \approx \frac{4\omega_0}{1-\beta^2} = 4\omega_0 \left(\frac{E}{mc^2} \right)^2 . \tag{4}$$

At the total electron energy of the order of GeV such transformation of visible light results in light with photons of the energy of already hundreds of MeV. As a result of the scattering, the direction of light propagation is reversed and the change of the frequency is maximal. If the incident light was also directed along the beam, that is the scattering angle was equal to zero, the light frequency would remain unchanged. This is so evident that at first sight it is not worth mentioning. In fact, however, such a case of light scattering is possible in which this is invalid. This can occur if light is scattered by a particle moving in a refractive medium rather than in a vacuum. I would like to draw your attention to some expected peculiarities of this phenomenon, which have not yet been studied experimentally. The theory of the scattering of light by a particle moving in a refractive medium, was considered for the first time in the recent papers by Gailitis and Tsytovich /3,4/.

In discussing this problem one can make use of the data on the Doppler effect in a refractive medium. Let us assume that a particle or an atom has as its natural frequency Ω (more exactly $\Omega = \Omega'\sqrt{1-\beta^2}$, where Ω' is the proper frequency at $\beta = 0$). It is easy to write the Doppler formula by analogy with the Doppler formula for vacuum. Apparently, it is necessary to replace the velocity of light in vacuum c by the light phase velocity in the medium $c/n(\omega)$. Hence we obtain

$$\omega' = \frac{\Omega}{1 - \beta n(\omega')\cos\theta} \qquad \beta n(\omega')\cos\theta < 1 \,. \tag{5}$$

This relation will determine the spectrum of the Doppler frequencies, which we shall further call normal. At the same time, in some cases another spectrum is possible which is usually called anomalous. In fact, for small values of θ it is possible that $\beta n(\omega)\cos\theta > 1$. In this case the frequency is expressed by the formula for the anomalous Doppler effect as follows

$$\omega'' = \frac{\Omega}{\beta n(\omega'')\cos\theta - 1} \qquad \beta n(\omega'')\cos\theta > 1 \,. \tag{6}$$

In both cases the refractive index $n(\omega)$ corresponds to the Doopler frequency ω' or ω''. A number of peculiarities, which have been repeatedly discussed are associated with this fact.

For further discussion it is essential that there exist two physically different processes, which lead to the occurence of the normal and the anomalous Doppler frequencies, respectively.

One can make certain of this by means of a simple quantum consideration based on the employment of the laws of conservation of energy and momentum. This consideration results in the following. In the case of the normal Doopler effect the emitter is a system in excited state. The emission occurs in the usual way, i.e. the photon $\hbar\omega'$ is emitted in a spontaneous transition from the excited state to the normal one (Fig. 1a).

Anomalous frequencies arise in a different way. Their source is a system in the unexcited state. Spontaneous excitation of the system is accompanied by the emission of the photon $\hbar\omega''$ (Fig. 1b). This process is possible since the system is in motion and the laws of conservation of energy and momentum imply the change of kinetic energy during radiation. The source of energy for the emission of the photon and for the excitation of the atom is the resultant decrease of the kinetic energy. Now we shall apply the above considerations to the scattering of light by an electron moving in a refractive medium. The formula for the frequency of the scattered light can be easily written by analogy with that for vacuum as follows

$$\omega = \frac{\omega_0(1-\beta n_0\cos\theta_0)}{1-\beta n(\omega)\cos\theta} \qquad \begin{array}{l} \beta n_0 \cos\theta_0 < 1 \\ \beta n(\omega)\cos\theta < 1 \end{array} \quad . \tag{7}$$

In the numerator β is replaced by βn_0, where n_0 is the refractive index for the initial frequency of the light ω_0, and in the denominator β is replaced by $\beta n(\omega)$, where $n(\omega)$ is the refractive index for the frequency ω emitted. Let us assume that $n_0 > 1$. Then for relativistic velocities we obtain two possible cases depending on the value of θ_0.

If the angle θ_0 is sufficiently large, $\beta n_0 \cos\theta_0 < 1$. Then the numerator of the equation is positive. Since according to the definition ω is positive, the denominator (7) must also be positive and, therefore, $\beta n(\omega)\cos\theta < 1$. Consequently, the frequencies of the scattered light belong to the spectrum which we call the normal Doppler effect. For $n \to 1$ it develops into the spectrum occuring in the scattering by an electron moving in a vacuum. At small values of θ_0, $\beta n_0 \cos\theta_0 > 1$ and the numerator in negative. This implies that the denominator should be also negative, so $\beta n(\omega)\cos\theta > 1$. Therefore the frequencies of the scattered light belong to the spectrum, which we call anomalous. As a result, we obtain

$$\omega = \frac{\omega_0(\beta n_0 \cos\theta_0 - 1)}{\beta n(\omega)\cos\theta - 1} \qquad \begin{array}{l} \beta n_0 \cos\theta_0 > 1 \\ \\ \beta n(\omega)\cos\theta > 1 . \end{array} \tag{8}$$

As is known, the spectrum of anomalous frequencies is complex. Consequently this spectrum is complex, too, i.e., at the given ω_0, β, $n_0 \theta_0$ and θ there are at least two frequencies ω satisfying this equation. These frequencies have no analogs in the scattering of light by an electron moving in a vacuum.

Both cases considered differ in values of θ_0. The boundary one will be the angle $\theta_0 = \theta_c$, satisfying the formula (1).

At this particular angle the inverse Vavilov-Čerenkov effect occurs.

The two cases of light scattering in a medium which have been considered here can be called normal scattering events. The scattering of light occurs in the ordinary way here, i.e. the absorption of the photon $\hbar\omega_0$ is followed by the generation of the photon $\hbar\omega$ of the scattered light (Fig. 2a).

For normal frequencies the absorption of the photon is followed by an increase in the energy of the moving electron vibrations, and the emission of a scattered photon decreases it.

In the case of the anomalous spectrum of frequencies the absorption of the photon corresponds to the damping of vibrations and emission corresponds to their increase.

These two kinds of light scattering are not the only possible ones as is sometimes supposed /5/.

It is easy to ascertain that another mechanism of scattering is also possible. The formula for the frequency variations for light scattering can also have the following form

$$\omega = \frac{\omega_0(\beta n_0 \cos\theta_0 - 1)}{1 - \beta n(\omega)\cos\theta} \qquad \begin{matrix} \beta n_0 \cos\theta_0 < 1 & \beta n(\omega)\cos\theta > 1 \\ \beta n_0 \cos\theta_0 > 1 & \beta n(\omega)\cos\theta < 1 \end{matrix} \tag{9}$$

We shall call this case the anomalous scattering of light.

Depending on the value of θ_0 we have two possibilities here too, but the anomalous and normal frequencies change places compared with normal scattering. Indeed, if θ_0 is sufficiently large, then $\beta n_0 \cos\theta_0 < 1$ and the numerator is negative, and it is necessary that $\beta n(\omega)\cos\theta > 1$, that is anomalous frequencies appear. And vice versa, at small values of θ_0, when $\beta n_0 \cos\theta_0 > 1$ normal frequencies are emitted. Let us consider, as an example, the case when $\theta_0 = 0$ and $\theta = 0$. Thus the initial beam of light is directed along the velocity, and the scattering photon is emitted in the same direction. If the electron energy is sufficiently high, the energy of the scattered photon can be so high that it will practically have $n = 1$. In this case the frequency equation will have the following form

$$\omega \approx \frac{\omega_0(\beta n_0 - 1)}{1 - \beta} . \tag{10}$$

This is the case when a considerable transformation of frequency takes place. The visible light can be transformed into gamma-

-rays though scatteing occurs at zero angle. This result completely contradicts the considerations expressed at the beginning of the lecture and whose apparent obviousness I have noted. The result will not seem so paradoxical if one pays attention to the fact that the electron is moving at a velocity exceeding the phase velocity of light with the frequency ω_0. In this sense it may be said that the electron outruns the light. In the coordinate system, connected with the electron, the wave moves toward the electron. Thus, this case is, to some extent, equivalent to the scattering of light at the angle δ_1.

The mechanism of the anomalous scattering is considerably different from that of normal scattering. One can deduce the following from the quantum consideration. The initial beam of light does not weaken in the process of scattering but becomes even stronger. Instead of the absorption of the initial photon $\hbar\omega_0$ the stimulated emission of the photon $\hbar\omega_0$ occurs. Thus the photon $\hbar\omega_0$ is converted into two photons $\hbar\omega_0$, emitted at the angle θ_0 and, in addition, the photon of the scattered light $\hbar\omega$ is generated (Fig.2). It is impossible to state in advance that the anomalous scattering of light will result in the increase of the intensity of the initial beam of light. In fact, simultaneously with the anomalous scattering of light normal scattering will take place, in which the intensity of light decreases.

If the electron energy is not very high (it has been assumed at the beginning of the lecture that the electron energy is limited) classical treatment similar to that for the Thompson scattering of light /3/ applies. The moving electron, vibrating in the field of the electromagnetic wave, radiates the whole spectrum of frequencies which covers both normal and anomalous scattering. In the case of the classical treatment the normal and the anomalous scattering seem to arise simultaneously, but in the quantum case the difference between their mechanisms becomes apparent. The possibility of the stimulated radiation was mentioned by V.N.Tsytovich /4/ in his quantum theory of light scattering. However the conditions under which this radiation occurs have, apparently, not been considered.

The formula for the anomalous scattering can be easily obtained in the quantum form which explains the mechanism of the process immediately. If the above consideration of the mechanism of anomalous scattering is right, then only the photon of stimulated radiation and that of the scattered light should be taken into account in the laws of conservation of energy and momentum. In fact, the initial photon remains unchanged, therefore it need not be taken into account.

The projections of the momenta of the stimulated radiation photon and of the scattered photon of the direction of the velocity are equal to

$$\frac{n_0}{c} h \omega_0 \cos \theta_0 \quad ; \quad \frac{n(\omega)}{c} h \omega \cos \theta , \tag{11}$$

respectively. If we take their sum and multiply it by v , we obtain the work of the recoil force of the emitted photons. Apparently this work must be equal to the change of the electron energy and, consequently, to the energy of the emitted photons. As a result, we get

$$\beta n_0 h \omega_0 \cos \theta_0 + \beta n(\omega) h \omega \cos \theta = h \omega_0 + h \omega ; \tag{12}$$

If all the terms of this equation are devided by h , it is easy to ascertain that this equation completely coincides with the equation (9) for ω in the case of the anomalous scattering.

The same equation (12) may have also another meaning and it may be regarded as a condition for the spontaneous emission of two photons simultaneously. In other words, this is the condition of the two-photon Vavilov-Čerenkov effect which has so far not been studied experimentally. For the case of the emission of two protons this condition plays the same part as the known formula (1) for the emission of one photon (it turns into this common formula (1) if ω is assumed to be zero).

Of course, the same equation can be employed to deduce the equation for normal scattering. For this purpose it is sufficient to assume that the photon $\hbar\omega_0$ is not emitted but absorbed. This means that the plus sign before the terms containing ω_0, should be replaced by a minus in both sides of the equation (12).

We know that the discovery of the Vavilov-Čerenkov effect gave rise to a large number of theoretical papers in which numerous related effects have been considered. The attention paid to this field is explained by uncommon results from the viewpoint of electrodynamics, though they come only from it. Only a few of these effects have been studied experimentally. Unlike the transition radiation, many of these are, in fact, very hard to investigate experimentally. In my opinion, the light scattering by an electron is of special interest since its experimental study seems to be feasible.

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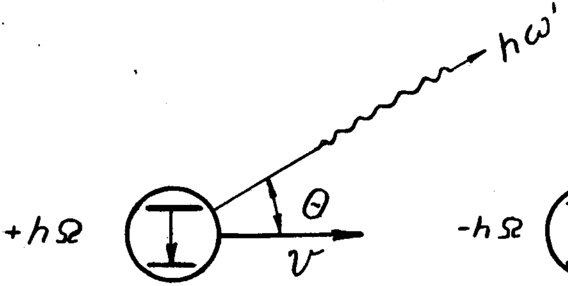

Fig. 1a

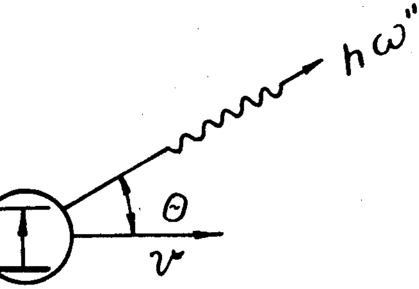

Fig. 1b

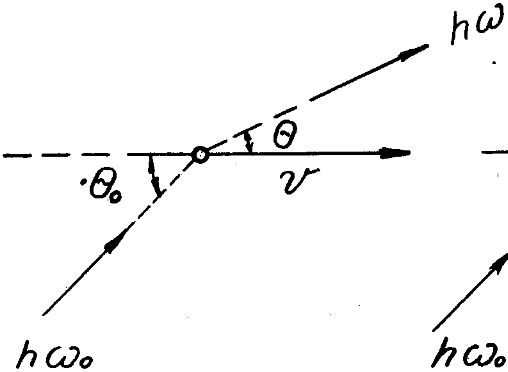

Fig. 2a

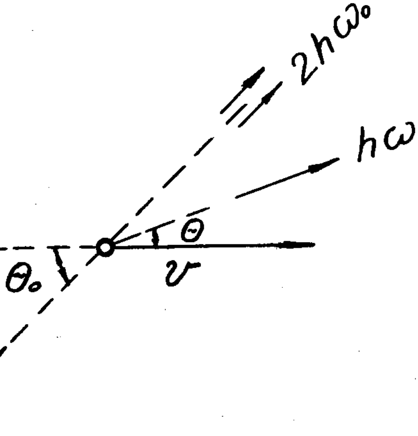

Fig. 2b

In addition to the paper by I.M.Frank entitled "The Scatt ring of Light by an Electron Moving in a Refractive Medium" (JINR Preprint E4-3760).

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Erratum

On p. 10, line 10 should read π instead of δ.