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ON REACTIONS WITH TRANSFER OF TWO PARTICLES

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О реакциях с передачей двух нуклонов

В работе дается формула для вычисления спектроскопического фактора при возбуждении уровня конечного ядра в реакиях со сложными ядрами. Учитываются все промежуточные состояния, образованные из переданных нуклонов. Рассматривается и сравнивается спектроскопическая информация из реакций (Li^{6}, α) и (F^{18}, O^{16}) на легких ядрах.

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On Reactions with Transfer of Two Particles

In this paper, a formula is given for the spectroscopic factor determining the excitation of any level of the final nuclei in reactions between complex nuclei. All intermediate states which are formed by the transferred nucleons and which are compatible with the quantum numbers of the nuclei are taken into account. The reactions (Li^{6}, α) and (F^{18}, O^{16}) on light nuclei are considered in more details.

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Reactions with transfer of some nucleons are very suitable for the investigation of the nuclear structure. In the last time, reactions of the type (Li^6, α), (Li^6, d) were much investigated at energies of the lithium ions near the Coulomb barrier. At these energies, it is straightforward to make the assumption that the direct process consists of the transfer of an α - particle or a deuteron as a whole. Therefore, the cross section for the direct process is proportional to the reduced width for emission of an α - particle and a deuteron, respectively./ 1/ .

At higher energies of the incident particles one can no longer make the assumption that the transferred nucleons form a particle by filling up the lowest shells. For example, the nucleons coming from the 1s- shell[x)] of the initial nucleus are to be expected to go more easily into the 1s- shell of the final nucleus than into any other shell. As it will be shown in the following, such a result follows only if one takes into account all possible states which the transferred nucleons can form.

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x) Here, following Brody and Moshinsky/ 2/ the energy levels of the harmonic oscillator will be numbered as follows: 0s , 0p , 1s , 0d , 1p , 0f ...

1. *Formalisms*

The spectroscopic factor, usually defined for the separation of k nucleons from the nucleus A which form a particle by filling up the lowest shells, is given by the following overlap integral[3]

$$\binom{A}{k} \int \phi^*_A \, \phi_{A-k} \, \phi_k \, \Psi_{NLM} \, d\tau . \qquad (1)$$

Here, ϕ_A, ϕ_{A-k} are the wave functions of the initial and final nucleus and ϕ_k is the wave function of the particle formed by the k separated nucleons. For example, ϕ_k is the α - particle wave function $(0s)^4[4]^{11}S$ in the case of two neutrons and two protons separated from the nucleus A. Ψ_{NLM} describes the relative motion of the nuclei (A-k) and k

Under the assumption that the theorem of Elliott and Skyrme[4]

$$\chi = \Psi_{000}(\vec{R})\,\phi(\vec{\rho})$$

is true for the nuclei A and A-k, one can transform this overlap integral by a Talmi transformation into a simpler form[3]:

$$\binom{A}{k}\left(\frac{A}{A-k}\right)^{N+\frac{L}{2}} \langle \chi_A \,|\}\, \chi_{A-k}, \chi_k \rangle \, K_0 \, . \qquad (2)$$

Here, χ is the shell model wave function of the nucleus, R is the coordinate of the mass centre, and ρ are the inner coordinates. The energy of the harmonic oscillator is, as in the paper of Brody and Moshinsky[2], $E_{NL} = 2N + L + \frac{3}{2}$ in units of $\hbar\omega$. The first integral $\langle \chi_A \,|\}\, \chi_{A-k}, \chi_k \rangle$ is the overlap integral of the shell model wave function of the nucleus A with the shell model wave functions of the nucleus (A-k) and of the k separated nucleons. This overlap integral (fractional parentage coefficient) takes into account the structure of the nuclei in the initial and final states. The second integral

$$K_0 = \int \chi^*_k \, \phi_k \, \Psi_{NLM} \, d\tau \qquad (3)$$

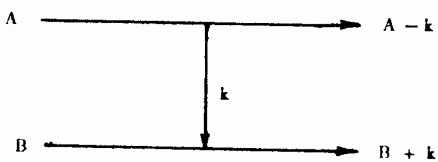
is the overlap integral of the wave function of the k separated nucleons with the wave function of the particle formed by these k nucleons by filling up the lowest shells.

In reactions with complex nuclei, the transferred nucleons do not come from the Os-shell of the initial nucleus, as a rule. Therefore, they do not form with necessity a particle by filling up the lowest shells, for example the Os-shell in the case of $k \leq 4$. This means that not only the overlap integral (3), but all the overlap integrals

$$K_i = \int \chi_k^* \left(\phi_k \Psi_{NLM} \right)_i d\tau \qquad (4)$$

compatible with the quantum numbers of the initial and final nuclei determine the cross section of the reaction.

Therefore, the spectroscopic factor determining the excitation of any level of the final nucleus $B+k$ in a reaction between complex nuclei of the type



is given by the following expression:

$$(2J_1+1)(2J_4+1)S^{1/2} = \binom{A}{k}^{1/2} \binom{B+k}{k}^{1/2} \sum_k C^{T_1 T_{1z}}_{T_2 T_{2z},\, T_k T_{kz}} C^{T_4 T_{4z}}_{T_3 T_{3z},\, T_k T_{kz}}$$

$$\times \sum_M C^{J_1 M_1}_{J\mu,\, L_k M_k} C^{J\mu}_{J_2 M_2,\, S_k \mu_k} C^{L_k M_k}_{\ell m,\, LM} C^{J_4 M_4}_{J'\mu',\, L'_k M'_k} C^{J'\mu'}_{J_3 M_3,\, S_k \mu'_k} C^{L'_k M'_k}_{\ell m,\, L'M'} \qquad (5)$$

$$\times \int \phi_1^*(\vec{\rho}_1)\, \phi_2(\vec{\rho}_2)\, \phi_{n\ell m}(\vec{\rho}_k)\, \Psi_{NLM}(\vec{R}_2 - \vec{R}_k)\, d(\vec{R}_2 - \vec{R}_k)\, d\vec{\rho}_2\, d\vec{\rho}_k$$

$$\times \int \phi_4(\vec{\rho}_4)\phi^*_3(\vec{\rho}_3)\phi^*_{n\ell m}(\vec{\rho}_k)\Psi^*_{N'L'M'}(\vec{R}_3-\vec{R}_k)d(\vec{R}_3-\vec{R}_k)d\vec{\rho}_3 d\vec{\rho}_k \quad (5)$$

$$\times\ R \times R'.$$

Here $C^{J_1M_1}_{J\mu,L_kM_k},\ldots$ are Clebsch-Gordan coefficients; $\vec{J}=\vec{J}_2+\vec{S}_k$, $\vec{J}'=\vec{J}_3+\vec{S}_k$ are the channel spins; L_k, L'_k are the orbital momenta of the k nucleons in the nuclei A, and $B+k$, respectively. The indices 1,2,3,4, correspond to the nuclei A, $A-k$, B, $B+k$. The sum $\sum_k$ goes over all possible states of the k nucleons which are compatible with the quantum numbers of the nuclei. The sum $\sum_M$ goes over

$$\mu,\ \mu',\ M_k,\ M'_k,\ \mu_k,\ \mu'_k,\ M_2,\ M_3,\ m,\ m',\ M,\ M',\ M_1,\ M_4 .$$

The values R, R' contain all uncertainties coming from the radius dependence/3/. In the following, it will be assumed that the R, R' are independent of the quantum numbers NL, and $N'L'$, respectively.

By analogy with (2), the spectroscopic factor can be written in the form

$$S^{1/2} = R\,R'\binom{A}{k}^{1/2}\binom{B+k}{k}^{1/2}$$

$$\times \sum_{\substack{L_kL'_kS_kT_k\\ JJ'}} C^{T_1T_{1z}}_{T_2T_{2z},T_kT_{kz}}\, C^{T_4T_{4z}}_{T_3T_{3z},T_kT_{kz}} \langle\chi_1|\}\chi_2\chi_k\rangle\langle\chi_4|\}\chi_3\chi_k\rangle \quad (6)$$

$$\times \sum_{n\ell NLN'L'} \left(\frac{A}{A-k}\right)^{N+\frac{L}{2}} \left(\frac{B+k}{B}\right)^{N'+\frac{L'}{2}} K(n\ell, NL, L_k)\,K'(n\ell, N'L', L'_k),$$

where

$$\langle \chi_a || \chi_b, \chi_k \rangle = \sum_{\substack{\mu, M_k \\ M_b, \mu_k}} C^{J_a M_a}_{J\mu, L_k M_k} C^{J\mu}_{J_b M_b, S_k \mu_k} \int \chi^*_a \chi_b \chi_k \, d\tau \qquad (7)$$

is the f.p.c. The integral

$$K(n\ell, NL, L_k) = \sum_{m,M} C^{L_k M_k}_{\ell m, LM} \int \chi^*_k(\vec{r}_1) \phi_{n\ell m}(\vec{\rho}_k) \Psi_{NLM}(\vec{R}_k) \, d\vec{R}_k \, d\vec{\rho}_k \qquad (8)$$

does not depend on the quantum numbers m, M, M_k. For transfer of two nucleons, the K, K' are given by transformation brackets (Moshinsky coefficients):

$$K(n\ell, NL, L_k) = \langle n\ell, NL : L_k | n_1\ell_1, n_2\ell_2 : L_k \rangle$$

$$K'(n\ell, N'L', L_k) = \langle n\ell, N'L' : L'_k | n_3\ell_3, n_4\ell_4 : L'_k \rangle . \qquad (9)$$

If both transferred nucleons originate from the Os-shell $(A \leq 4)$ then

$$K(n\ell, NL, L_k) = 1$$

$$K'(n\ell, N'L', L'_k) = \langle 00, N'L'_k : L'_k | n_3\ell_3, n_4\ell_4 : L'_k \rangle \equiv K'_0 .$$

In this case, the relative probability for excitation of the levels of the final nucleus $(B+k)$ is given by the usual spectroscopic factor for particles in their lowest states[x].

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x) *This result is in contrast to the theory of Gledenning, who assumes $n \neq 0$ for two-particle transfer with particles originating from the Os-shell.*

2. *Spectroscopic Information from Transfer Reactions between Complex Nuclei*

In Table 1, the sums $\sum KK'$ (without the correction factors $\left(\frac{A}{A-k}\right)^{N+\frac{L}{2}}\left(\frac{B+k}{B}\right)^{N'+\frac{L'}{2}}$) are given for some configurations. As is seen from the table. $\sum KK'$ is maximal. if the initial and the final states of the transferred nucleons are the same ($n_3\ell_3 = n_1\ell_1$; $n_4\ell_4 = n_2\ell_2$). The more strongly the states in the initial and final nucleus differ from one another, the smaller is the sum $\sum KK'$.

For comparison, the overlap integrals $K_oK'_o$ are also given in Table 1. These overlap integrals show another dependence on the states in the initial and final nucleus. The higher the shells in the initial and final nucleus from which the transferred nucleons are coming and into which they are going, the smaller the $K_oK'_o$. This is independent of whether these shells are equal or not because $K_oK'_o$ projects out of $\sum KK'$ the case in which the transferred nucleons form an intermediate particle in its lowest state (both nucleons in the Os-shell, $n = 0$, $\ell = 0$).

The reduced widths for emission of particles in their lowest states (for emission of deuterons, tritons, α-particles and so on) from an excited level of a nucleus (B+k) are given by (2). They are proportional to

$$K'_0 \equiv K(00, N'L', L'_k). \tag{10}$$

In reactions between complex nuclei, however, the probability for excitation of any level of the nucleus (B+k) is given by (6). It is proportional to

$$z = \sum_{\substack{n\ell N'L' \\ NL}} \left(\frac{A}{A-k}\right)^{N+\frac{L}{2}} \left(\frac{B+k}{B}\right)^{N'+\frac{L'}{2}} K(n\ell, NL, L_k)\, K'(n\ell, N'L', L'_k). \tag{11}$$

Therefore, in reactions between complex nuclei, one does not directly measure the reduced widths for emission of particles in their lowest states. The components with different L'_k and with different

$n_3\ell_3$, $n_4\ell_4$ are to be multiplied by a weight factor, which depends on the properties of the nuclei A and A-k.

In particular, the contributions from the admixtures of higher configurations to the wave function of the nucleus (B+k) are also to be multiplied by a weight factor. For reactions with heavy ions, according to Table 1, this factor is not smaller than that by which the contribution of the main configuration must be multiplied. This gives, in principle, the possibility to investigate experimentally the admixtures of higher configurations to the wave function of the nucleus (B+k) by comparing several reactions with heavy ions.

3. *Examples*

In Table 2, the values

$$z = \sum_{\substack{n\ell NL \\ N'L'}} \left(\frac{A}{A-k}\right)^{N+\frac{L}{2}} \left(\frac{B+k}{B}\right)^{N'+\frac{L'}{2}} K(n\ell, NL, L_k)\, K'(n\ell, N'L', L'_k) \quad (12)$$

are given for the reactions (Li^6, α) and (F^{18}, O^{16}) on the nuclei Li^6 and C^{12} with formation of the final nuclei Be^8 and N^{14}. In both cases, the transferred nucleons are in a state with zero orbital momentum in the initial nucleus. The wave functions are /6,7/

$Li^6_{g.s.}$: $(0p)^2\ {}^{13}S$

$F^{18}_{g.s.}$: $0{,}82\,(0d)^2\ {}^{13}S + 0{,}55\,(1s)^2\ {}^{13}S + \ldots$

As it is seen from the table, the values z are almost the same for the two incident nuclei Li^6 and F^{18}. Therefore, if there are differences, for example, in the angular distributions of the α-particles from the (Li^6, α) reaction and of the O^{16}-nuclei from the (F^{18}, O^{16}) reaction on the same target nucleus with formation of the final nucleus in the same state, they are due to the dynamics (different Q values and Coulomb parameters), rather than to the structure.

In the reaction $Li^6(Li^6, \alpha)Be^8$ at low energies (1.5 MeV) of the incident particles a maximum in the cross section corresponding to the formation of Be^8 at an excitation energy of about 20 MeV was observed [8]. This corresponds either to the existence of a level with a very great reduced deuteron width lying in the neighbourhood of the threshold for the decay $Be^8 \to Li^6 + d$ at 22.3 MeV, or it follows from the fact that many levels are lying close to one another in this energy region which together lead to the observed maximum in the cross section, as it follows from shell model calculations [1]. Such a maximum in the cross section of the reactions $Li^6(Li^6, \alpha)Be^8$ and $Li^6(F^{18}, O^{16})Be^8$ is to be expected also at higher energies of the incident particles corresponding to calculations made on the basis of the shell model.

In Table 3 the spectroscopic factors

$$\theta = \frac{1}{RR'} S^{1/2} \; ; \; \theta^2 = \binom{A}{k}\binom{B+k}{k} \sum_{L'_k} \{ \langle \chi_1 | \} \chi_2, \chi_k \rangle \langle \chi_4 | \} \chi_3, \chi_k \rangle \times z \}^2$$

for $C^{12}(Li^6, \alpha)N^{14}$ and $C^{12}(F^{18}, O^{16}_{g.s.})N^{14}$ with formation of N^{14} in its ground and second excited state (3.9 MeV) are given. The wave functions for the two levels are [6]

$$\Psi_{g.s.} = 0{,}950\,[442]\,^{13}D - 0{,}247\,[442]\,^{13}S - 0{,}259\,[433]\,^{11}P$$

$$\Psi_{3.9\,MeV} = 0{,}954\,[442]\,^{13}S + 0{,}243\,[442]\,^{13}D + 0{,}173\,[433]\,^{11}P$$

in the model with intermediate coupling. For C^{12} the wave function [6]

$$\Psi = 0{,}896\,[44]\,^{11}S + 0{,}413\,[431]\,^{13}P + \ldots$$

was used. As is seen from the table, the components with $L'_k = 0$ and with $L'_k = 2$ are to be multiplied by a different weight factor in the reaction with formation of the ground state and in the reaction with formation of the second excited one, although both levels have $J^\pi = 1^+$, $T = 1$. In the case of the second excited level, the transition is almost wholly determined by the $L'_k = 0$ component, while in the case of the ground state the $L'_k = 2$ component is the more important one. One expects that the different L'_k values of the two transitions lead to different effects on the diffraction structure of the angular distributions, may be to effects of such a type as they were observed by Bock et al. /9/ in the reaction $B^{11}(O^{16}, N^{15})C^{12}$ with proton transfer. Besides, the excitation of the two levels in dependence of the energy of the incident nuclei will be different from one another.

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Table 1 Overlap integrals $\sum_{n\ell NLN'L'} KK'$ and K_0K_0' for transfer of two nucleons with $T_k = 0, S_k = 1$

$(n_1\ell_1, n_2\ell_2)^2\ 1^3L_k$ / $(n_3\ell_3, n_4\ell_4)^2\ 1^3L_k'$	$(0s)^2\ 1^3S$		$(0p)^2\ 1^3S$		$(1s)^2\ 1^3S$		$(0d)^2\ 1^3S$	
	$\Sigma KK'$	K_0K_0'	$\Sigma KK'$	K_0K_0'	$\Sigma KK'$	K_0K_0'	$\Sigma KK'$	K_0K_0'
$(0s)^2\ 1^3S$	1,000	1,000	0,707	0,707	0,456	0,456	0,408	0,408
$(0p)^2\ 1^3S$	0,707	0,707	1,000	0,500	0,204	0,322	0,815	0,288
$(0p)^2\ 1^3D$	0,707	0,707	0,500	0,500	0,850	0,322	0,053	0,288
$(Is)^2\ 1^3S$	0,456	0,456	0,204	0,322	1,000	0,208	0,000	0,186
$(0d)^2\ 1^3S$	0,408	0,408	0,815	0,288	0,000	0,186	1,000	0,167
$(0d)^2\ 1^3D$	0,289	0,289	0,516	0,204	0,110	0,132	0,522	0,118
$(0d)^2\ 1^3G$	0,612	0,612	0,433	0,432	0,652	0,279	0,083	0,250

Table 2 Overlap integrals z for (Li^6, α) and (F^{18}, O^{16}).

$$z = \sum \left(\frac{A}{A-k}\right)^{N+\frac{L}{2}} \left(\frac{B+k}{B}\right)^{N'+\frac{L'}{2}} K K'$$

$A \rightarrow A+k$	L'_k	(Li^6, α)	(F^{18}, O^{16})
$Li^6 \longrightarrow Be^8$	0	1,500	1,111
	2	1,000	0,808
$C^{12} \longrightarrow N^{14}$			
1p shell	0	1,375	1,024
	2	0,875	0,721
2s-2s admixture	0	0,520	0,749
0d-0d admixture	0	I,204	I,098
	2	0,781	0,551
	4	0,883	0,708

Table 3 Spectroscopic factors Θ for $C^{12}_{g.s.}(Li^6, \alpha)N^{14}$ and $C^{12}_{g.s.}(F^{18}, O^{16}_{g.s.})N^{14}$

		(Li^6, α)	(F^{18}, O^{16})
$N^{14}_{g.s.}$	$L'_k = 0$	0,171	0,128
	$L'_k = 2$	−0,344	−0,284
$N^{14}_{3.9\ MeV}$	$L'_k = 0$	−0,840	−0,625
	$L'_k = 2$	−0,073	−0,060