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ON EQUILIBRIUM DEFORMATIONS OF THE GROUND AND EXCITED STATES OF STRONGLY DEFORMED NUCLEI

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О равновесных деформациях основных и возбужденных состояний сильно деформированных ядер

Проведен расчёт равновесных деформаций ядер и исследована зависимость энергии ядер от параметров деформации β и γ. Показано, что для сильно деформированных ядер полная энергия ядра в зависимости от β при $\gamma=0$ имеет два минимума, один при $\beta_0=0,2-0,3$ с энергией $-(4-8)$ Мэв относительно энергии ядра при $\beta=0$, другой при $\beta_0\approx(0,1-0,2)$ с энергией $-(1,5-5)$ Мэв. При изучении поведения полной энергии ядра $\mathcal{E}(\beta,\gamma)$ как функции параметров β и γ выяснено, что эта функция имеет только один минимум при $\gamma=0$ и $\beta\approx0,2-0,3$. Исследования равновесных деформаций возбужденных состояний показали, что не имеется возбужденных состояний с равновесной деформацией $\beta_e<0$, когда для основного состояния $\beta_0>0$. Приведенные выводы относятся к деформированным ядрам, далеким от переходной области. Для ядер переходной области, таких как, например, легкие изотопы тория, деформация возбужденных состояний может сильно отличаться как по β так и по γ от равновесной деформации основного состояния.

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On Equilibtium Deformations of the Ground and Excited States of Strongly Deformed Nuclei

The equilibrium deformations for nuclei have been calculated and the dependence of the energy on the deformation parameters β and γ have been studied. It is shown that for strongly deformed nuclei the total energy has two minima depending on β for $\gamma=0$; one at $\beta_0=0.2-0.3$ (with the energy $-(4-8)$ MeV with respect to the nuclear energy at $\beta=0$) the other at $\beta_0\approx(0.1-0.2)$ (with the energy $-(1.5-5)$ MeV). While studying the behaviour of the total energy $\mathcal{E}(\beta,\gamma)$ as a function of the parameters β and γ, it was found out that this function has only one minimum at $\gamma=0$ and $\beta\approx0.2-0.3$. Investigation of the equilibrium deformations of the excited states have shown that there exist no the excited states with the equilibrium deformation $\beta_e<0$ while for the ground state $\beta_0>0$. The conclusions mentioned refer to the deformed nuclei, far from the transition region. For the nuclei of the transition region, such as the light torium isotopes, the deformation of the excited states may differ considerably on β as well as on γ from that of the ground state.

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ON EQUILIBRIUM DEFORMATIONS OF THE GROUND AND EXCITED STATES OF STRONGLY DEFORMED NUCLEI

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I. The equilibrium deformations of the ground states of even-even nuclei have been studied in a number of papers. Bes ans Szymanski[1], Sobiczewski[2] et al. have calculated the equilibrium deformations for nuclei in the rare-earth region and in the actinide region. They used the one-particle energies of the Nilsson potential and took into account the pairing correlations of the superconducting type. In order to calculate the statical nuclear shape Baranger and Kumar[3] have used the model in which the pairing correlations and quadrupole-quadrupole interactions are taken into account. In the overwhelming majority of cases the calculated equilibrium deformations β_0 are in good agreement with experimental data. Das Gupta and Preston[4] have investigated the axial symmetry of the shape of nonspherical nuclei. They showed that the strongly deformed nuclei have axial symmetry and the static shape of these nuclei is a prolate ellipsoid of rotation. The word "strongly deformed nuclei" will be used for defining nonspherical nuclei which are outside the transition regions from deformed nuclei to spherical ones.

The investigation of the equilibrium deformations of the excited nuclei is in its initial stage. In ref.[5] it was considered in what cases the equilibrium deformations of the excited quasiparticle states may differ from those of the ground states.

In ref.[6] the equilibrium deformations of the excited states were investigated in connection with the problems of the structure of spontaneously fissioning isomers.

The main attention is focused, in this paper, on the calculation of the equilibrium deformations of the excited states. It is considered whether shape isomers may exist in heavy strongly deformed nuclei. Since the magnitude of the equilibrium deformations of the excited states are mainly defined by the behaviour of the total energy of the ground state of an even-even nucleus depending on the deformation parametres β and γ then the total energy $\mathcal{E}_0(\beta,\gamma)$ for strongly deformed nuclei is calculated.

II. The energies $\mathcal{E}_0(\beta,\gamma)$ of the ground states of even-even nuclei for the β values from 0 to 0.4 and for the γ values from 0° to 60° (provided that $\beta > 0$, γ changes from 0° to 60°, if β assumes the positive and negative values then γ changes from 0° to 30°) are calculated by the two methods: by the Bes-Szymanski method[1] and the Strutinsky method[7].

Following the Bes-Szymenski method the total nuclear energy can be represented in the form

$$\mathcal{E}_0(\beta,\gamma) = \mathcal{E}_p + \mathcal{E}_n + \mathcal{E}_c \quad , \tag{1}$$

where $\mathcal{E}_p$, $\mathcal{E}_n$ are the energies of the proton and neutron systems, $\mathcal{E}_n$ is the Coulomb energy of a uniformely charged ellipsoid. The energy of the proton system is of the form

$$\mathcal{E}_p = \sum_\nu E(\nu)\, 2\, v_\nu^2 - \frac{C_p^2}{G_Z} \quad , \tag{2}$$

where the summation is made over the average field one-particle levels,

$E(\nu)$ are the one-particle energies of the Newton potential/8/.

G_Z is the constant of the pairing interactions in the proton system and $2v_\nu^2$ is the proton density in the state ν. The constants G_N and G_Z and the parameters of the Newton potential (which in the case $\gamma=0$ turns into the Nilsson potential) are taken in the rare-earth region the same as in ref./9/ and in the actinide region the same as in ref./6/.

Following the Strutinsky method/7/ the total energy of the ground state of an even-even nucleus is divided into the two parts:

$$\mathcal{E}_0(\beta,\gamma) = \mathcal{E}_{drop}(\beta,\gamma) + \Delta\mathcal{E}(\beta,\gamma), \tag{3}$$

where $\mathcal{E}_{drop}(\beta,\gamma)$ is the energy in the liquid drop model, its parameters are determined from experimental data on the nuclear masses. The shell correction

$$\Delta\mathcal{E}(\beta,\gamma) = \Delta\mathcal{E}(Z) + \Delta\mathcal{E}(N) \tag{4}$$

consists of the proton and neutron parts, in this case

$$\Delta\mathcal{E}(Z) = \mathcal{E}_p - \bar{\mathcal{E}}(Z), \tag{5}$$

where $\mathcal{E}_p$ is determined in (2) and the averaged energy

$$\bar{\mathcal{E}}(Z) = \int_{-\infty}^{\lambda} E\, g(E)\, dE ,$$

$$g(E) = \frac{1}{\sqrt{\pi}}\frac{1}{\gamma}\sum_\nu \exp\left\{-\left(\frac{E-E(\nu)}{\gamma}\right)^2\right\} \tag{6}$$

and λ is the chemical potential, the parameter γ is close to the energy difference between the shells (that is 5-10 MeV). The constants G_N and G_Z were chosen so that obtain for $\beta = \beta_0$ and $\gamma = 0^{\circ}$ the correlation functions C_n and C_p close to the data in ref./10/. The schemes of the average field levels were chosen the same as in the Bes-Szymanski calculations.

III. Let us now discuss the results of calculation of the total energy of the ground states of even-even strongly deformed nuclei depending on the deformation parameters β and γ. Some of the obtained results a presented in Figs. 1-7. In Figs. 1 and 2 is given the function $\mathcal{E}_0(\beta, \gamma = 0^{\circ})$ for a number of the gadolinium and hafnium isotopes. The deepest minima of the functions $\mathcal{E}_0(\beta,\gamma)$ correspond to the equilibrium defomations which are denoted by β_0 and γ_0. It is seen from the figures that the minima of the functions $\mathcal{E}_0(\beta,\gamma)$ become deeper as far as the isotopes are moving away from the nuclei of the transition regions.

It should be noted that the calculations for ^{152}Gd (and to less degree for ^{154}Gd) are more sensitive to the Nilsson potential parameters and to the G_N and G_Z values as compared with strongly deformed nuclei and therefore they are not quite unambiguous. The depth of the the well is close to the energy of zero oscillations. Therefore the calculated function $\mathcal{E}_0(\beta,\gamma)$ does not contradict a sharp disappearance of the deformation when the number of neutrons is 88 that is revealed in the properties of the first 2^+ states and in the anomaly of the behaviour of the coupling energy of the pair of last neutrons/11/.

The results in Figs. 1-4 are obtained by the Strutinsky method. However, for defomed nuclei the functions $\mathcal{E}_0(\beta,\gamma)$ calculated by the Bes-Szimanski and the Strutinsky methods are very close to each other. So, the calculations by the Bes-Szymanski method give for ^{156}Gd $\beta_0 = 0.33$ and $\mathcal{E}_0(\beta_0, \gamma = 0^\circ) = -4.8$ MeV (as compared with $\beta_0 = 0.30$, $\mathcal{E}_0(\beta_0, \gamma = 0^\circ) = -4.8$ MeV on Fig. 1) and for ^{154}Gd $\beta_0 = 0.3$ and $\mathcal{E}_0(\beta_0, \gamma = 0^\circ) = -3.1$ MeV (as compared ed with $\beta_0 = 0.27$, $\mathcal{E}_0(\beta_0, \gamma = 0^\circ) = -3.1$ MeV on Fig. 1) and so on.

From Fig. 1 and 2 it is seen that for $\gamma = 0^\circ$ the function $\mathcal{E}_0(\beta, \gamma = 0^\circ)$. has the two minima: one at $\beta > 0$, the other at $\beta < 0$ (or $\beta > 0$, $\gamma = 60^\circ$). In order to make oneself sure that a minimum of $\mathcal{E}_0(\beta, \gamma)$ exists for $\beta < 0$ it is necessary to calculate the function $\mathcal{E}_0(\beta, \gamma)$ for all the values of γ different from zero. Fig. 3 gives the behaviour of the function $\mathcal{E}_0(\beta_0, \gamma)$ depending on γ for the function values at minima with respect to β. From Fig. 3 it is seen that in all the cases the function $\mathcal{E}_0(\beta, \gamma)$ has only one minimum for $\beta_0 > 0$, $\gamma_0 = 0^\circ$. This fact is seen in Fig. 4 where for ^{174}Yb the behaviour of $\mathcal{E}_0(\beta, \gamma)$ is given for γ equal to 0°, 10°, 20°, 30°, 40°, 50° and 60°.

For the dysprosium, erbium and itterbium isotopes the behaviour of $\mathcal{E}_0(\beta, \gamma)$ is very similar to the behaviour of this same function for ^{174}Yb and ^{174}Hf, the functions $\mathcal{E}_0(\beta, \gamma)$ have less deep minima for the tungstem isotopes. The values obtained by us for strongly deformed nuclei $|\mathcal{E}_0(\beta = 0, \gamma = 0^\circ) - \mathcal{E}_0(\beta_0, \gamma_0)|$ are by 2-4 MeV smaller than those in ref./12/. The calculated values of the equilibrium deformation of β_0 are in good agreement with experimental data (see, e.g. ref./13/). The β_0 values found by us are far larger than those calculated in ref./2/ and somewhat smaller than those calculated in ref./12/.

The function $\mathcal{E}_0(\beta, \gamma)$ is also calculated for nuclei in the actinide region. Figs. 5-7 represent a part of the results of calculation of $\mathcal{E}_0(\beta, \gamma)$ performed by the Bes-Szymanski method.

It should be noted that the behaviour of $\mathcal{E}_0(\beta, \gamma)$ for ^{240}Pu is about the same as the curve I in Fig. I in ref./6/. In Fig. 7 one gives a contour diagram where the continuous lines correspond to the $\mathcal{E}_0(\beta, \gamma)$ values like -5, -4, -3, -2 and -1 MeV. The minimum of $\mathcal{E}_0(\beta,\gamma)$ is -5.7 MeV with respect to the $\mathcal{E}_0(\beta = 0, \gamma = 0^\circ)$ value. The curves such as in Figs. 6 and 7 are close to the functions $\mathcal{E}_0(\beta, \gamma)$ for ^{236}U, ^{240}Pu and for a number of curium and californium isotopes. For all the strongly deformed nuclei the function $\mathcal{E}_0(\beta, \gamma)$ has one minimum at $\gamma = 0^\circ$. The calculated values of β_0 are in rather good agreement with experimental data (see e.g. ref./14/) and are close to the values calculated in ref./2/.

As far as we are going to the transition region (i.e. for light uranium and thorium isotopes) the minina of the functions $\mathcal{E}_0(\beta, \gamma)$ become more and more shallow and the shape of the curve $\mathcal{E}(\beta, \gamma = \mathrm{const})$ more and more deflecting from the parabola. For example, for ^{228}Th

$$\mathcal{E}_0(\beta = 0, \gamma = 0^\circ) - \mathcal{E}_0(\beta_0 = 0{,}14, \gamma_0 = 0^\circ) = -0{,}6\,\mathrm{MeV},$$

$$\mathcal{E}_0(\beta = 0, \gamma = 0^\circ) - \mathcal{E}_0(\beta_0 = 0{,}10, \gamma = 60^\circ) = -0{,}2\,\mathrm{MeV}, -$$

the curve is nonsymmetrical with respect to the minimum.

Thus, for each of the strongly deformed nuclei the function $\mathcal{E}_0(\beta, \gamma)$ has one minimum at $\gamma = 0^\circ$ and the β_0 values are close to the experimental one. The results of calculations performed by the two methods practically coincide with one another. The results are stable against some changes of the G_N and G_Z

values and against such change of parameters in the Nilsson-Newton potential which does not lead to contradiction with experimental data on odd-mass strongly deformed nuclei.

For all the strongly deformed nuclei the shape of the curve $\mathcal{E}_0(\beta\ ,\ \gamma = \mathrm{const})$ is close to the parabola which testifies a small anharmonicity of the oscillations. This result is in agreement with the results obtained in ref./15/. In ref./15/ one gives the calculations of the admixture of two-phonon states to the one-photon states. The calculations have shown that for strongly deformed nuclei the admixtures to the lowest one-phonon states are small what shows up a small role of the anharmonic effects.

IV. We discuss now the behaviour of the one-particle levels of the average field depending on the deformation parameters β and γ. The behaviour of the one-particle levels at 0° or 60° depending on β is well studied, it is well shown in the Nilsson scheme (see, e.g. ref./10/).

In the case when $\neq 0^{\circ}$ or 60° the nucleus is nonaxial and the projection of the total momentum of the nucleus K on any axes is not a good quantum number. The case of nonaxial nuclei was investigated by A.S. Davydov et al./16/. For us it is important that for $\gamma = 0$ and 60° K is not a good quantum number.

Let us choose the constant value $\beta = \beta_1 > 0$ and vary the value γ from 0° to 60°. Let for $\gamma = 0^{\circ}$ the symmetry axis be the axis 0_z and $K = K_1$ be a good quantum number. Increasing γ from 0° to 30° the wave function of a given state except the component with $K = K_1$ (which is predominant) contains admixtures with other values of K. When γ is larger than 30° we have for $\gamma = 60^{\circ}$ an axially symmetrical nucleus with other symmetry axis which we denote by 0_y ($0_y \perp 0_z$) and for this state there is a good quantum number $K = K_2$. In the interval of γ from 30° to 60° the predominant component of the wave function is a component with $K = K_2$. This is seen from Table I. From the normalization condition of the wave function of the lowest state from subshell $i_{13/2}$ one has obtained the components of the wave function with different K for $\beta = 0.20$. For $\gamma = 0°$ the state considered is assigned by the quantum numbers $1/2 + [660]$ and the symmetry axis is the axis 0_z . At $\gamma \neq 60°$ the given state is assigned by $13/2 + [606]$ and the symmetry axis is the axis 0_y .

As an example we show the behaviour of the one-particle levels of the average field for nuclei in the actinide region depending on γ . Fig. 8 gives the scheme of the levels for the neutron system and in Fig. 9 for the proton system. The calculation is made for $\beta = 0.14$. This choice is explained by the fact that the function $\mathcal{E}_0(\beta, \gamma = 60°)$ has a minimum near $\beta = 0.14$ for the uranium and plutonium isotopes. The energies are given in units $\hbar \overset{o}{\omega}_0 = 41 A^{-1/3}$ MeV. On the left one gives the quantum numbers $K\pi [N n_z \Lambda]$ for $\gamma = 0°$ and on the right for $\gamma = 60°$.

Figs. 8 and 9 show that the energy of most levels depends weakly on γ . However, there is a number of levels the energy of which increases by 0.5 MeV when γ changes from 0° to 60°. This can lead in principle, to the existence of quasiparticle excited states with $\gamma_e = 60°$.

Using the formulas of the superfluid nuclei model/17/ it is not difficult to calculate the behaviour of the total energies depending on β and γ for odd-mass nuclei with a quasiparticle on the level ρ and for two-quasiparticle excited state of even-even nuclei with quasiparticles on the levels ρ_1 and ρ_2 . The energy of the one-quasiparticle state of the system which consists of an odd number of protons and the quasiparticle of which is on the level ρ is of the form

$$\mathcal{E}_p(\rho\ ;\beta,\gamma) = E(\rho) + 2\sum_{\nu\neq\rho} E(\nu)v_\nu^2 - \frac{C_p^2(\rho)}{G_z} \qquad (7)$$

The energy of the two-quasiparticle state of the system which consists of an even number of protons with the quasiparticles on the levels ρ_1 and ρ_2 reads:

$$\mathcal{E}_p(\rho_1,\rho_2;\beta,\gamma) = E(\rho_1) + E(\rho_2) + 2\sum_{\nu\neq\rho_1\rho_2} E(\nu)v_\nu^2 - \frac{C_p^2(\rho_1\rho_2)}{G_z} \qquad (8)$$

In order to calculate $\mathcal{E}_p(\rho\ ;\beta,\gamma)$ and $\mathcal{E}_p(\rho_1,\rho_2;\beta,\gamma)$ for each state and for each value of β and γ one solves the equations (see ref./17/) for determining the correlation functions $C_p(\rho)$, $C_p(\rho_1,\rho_2)$ and the chemical potentials $\lambda(\rho)$ and $\lambda(\rho_1,\rho_2)$. Similar formulas are taken from the calculations for the neutron system states.

The total energies of the one-quasiparticle states of $\mathcal{E}(\rho;\beta,\gamma)$ and of the two-quasiparticle states of $\mathcal{E}(\rho_1,\rho_2;\beta,\gamma)$ are calculated by the formulas (1) or (3), (5) where instead of (2) one inserts (7) or (8). The total energies of the two-quasiparticle states of odd-odd nuclei $\mathcal{E}(\nu_0,s_0;\beta,\gamma)$ are calculated by the formulas (1) or (3), (5), where instead of $\mathcal{E}_p$ and $\mathcal{E}_n$ one inserts the expressions (7).

From the condition of the absolute minimum of the total energies $\mathcal{E}(\rho,\beta,\gamma)$, $\mathcal{E}(\rho_1,\rho_2,\beta,\gamma)$ and $\mathcal{E}(\nu_0,s_0;\beta,\gamma)$ it is not difficult to determine the parameters β_e and γ_e for the equilibrium deformation of the one-quasiparticle states of strongly deformed nuclei.

V. Let us consider the behaviour of the total energies of one- and two-quasiparticle states and find the appropriate equilibrium values of the deformation parameter β_e and γ_e for strongly deformed nuclei.

First, we consider the one-quasiparticle states. As was mentioned in ref.[5] an one-quasiparticle excited state with equilibrium deformation β_e which differs from the deformation β_o for $\gamma_e = \gamma_o = 0$ may exist if the decrease of the quasiparticle state energy with changing β is larger than the increase of the energy of an even-even core. For one-quasiparticle states one may formulate the following rules when deflections of β_e from β_o are possible: $\beta_e > \beta_o$ if a quasiparticle is either on the hole level of the average field the energy of which rapidly increases with increasing β or on the particle level the energy of which strongly decreases with increasing β; $\beta_e < \beta_o$ if a quasiparticle is either on the hole level the energy of which rapidly decreases with increasing β or on the particle level the energy of which strongly increases with increasing β. In ref.[5] one gives some example of one-quasiparticle states in which $\beta_e \neq \beta_o$. In calculating in ref.[5] the functions $\mathcal{E}(\rho ; \beta, \gamma = 0^o)$ for the ground one-quasiparticle states were taken from ref.[18]. The functions $\mathcal{E}(\rho ; \beta, \gamma = 0^o)$ calculated by us for the ground states of odd-mass strongly deformed nuclei have (as compared with ref.[18]) more plane minimum for $\beta > 0$. Therefore in these calculations we obtain a somewhat larger deflection of β_e from β_o as compared with ref.[5]. The largest difference is for the following states: according to our calculations, for $\gamma = 0^o$ the equilibrium deformation of the excited state 1/2 – [541] ^{175}Lu is by 0.04 larger than β_e (in ref.[5] $\Delta\beta = 0.01 - 0.02$); in ^{175}Gd for the state 1/2 + [400] $\Delta\beta = 0.05$ (as compared with $\Delta\beta = 0$ in ref.[5]) for the state 1/2 –[505] $\Delta\beta = -0.05$ (as compared with $\Delta\beta = -0.01$ in ref.[5]); in ^{177}Hf the equilibrium deformations β_e of the states $7/2-[503]$ and $9/2-[505]$ by 0.02 smaller than β_0, while in ref.[5] $\beta_e = \beta_0$ and so on.

In the actinide region, e.g. in ^{241}Pu for the one-quasiparticle states $13/2+[606]$, $1/2-[761]$, $11/2+[615]$ $\Delta\beta = 0.04 - 0.05$, while for the states $1/2-[770]$, $3/2-[761]$, $7/2+[613]$ $\Delta\beta = -(0.03-0.04)$. In ^{241}Am the one-quasiparticle states $11/2-[505]$, $1/2+[651]$ have $\beta_e = \beta_0 + 0.04$ and the states $11/2+[615]$, $9/2-[505]$ have $\beta_e = \beta_0 - 0.03$ and so on.

Thus the calculation performed prove the conclusion drawn in ref.[5] that some low-lying one-quasiparticle excited states in odd-mass strongly deformed nuclei may have equilibrium deformations which differ from the equilibrium deformations of the corresponding even-even nuclei. These deflections do not, apparently, exceed 0.05.

Let us investigate the question as to whether there exist one-quasparticle states with $\gamma_e \neq 0$ for $\beta_e > 0$. As is shown below the energies $\mathcal{E}_0(\beta,\gamma)$ of the ground states of even-even nuclei sharply increase with increasing γ from 0° to 10° and higher. From Figs. 8 and 9 it is seen that the change in the energy of the one-particle levels with increasing γ is essentially smaller as compared with the change of the energies of the one-particle levels with increasing β. These two facts allow one to understand the results of the calculations performed according to which in strongly deformed nuclei there are no one-particle states with $\gamma_e \neq 0$.

As is shown above, the function $\mathcal{E}(\beta > 0, \gamma = 60^\circ)$ has a minimum. The problem is investigated whether this minimum can lead to the existence in odd-mass strongly deformed nuclei of one-quasi-particle states with $\gamma_e = 60^\circ$. Table 2 shows the behaviour of the energies depending on γ for one-quasiparticle states in ^{235}U and ^{235}Np for $\beta = 0.14$. From the table it is seen that for a number of states, e.g. for $1/2+[640]$ in ^{235}U, $1/2+[660]$, $1/2-[530]$ in ^{235}Np (the values of $K\pi[Nn_z\Lambda]$ correspond to $\gamma = 60°$) the function $\mathcal{E}(\rho;\beta,\gamma)$ has, in addition to the deep minimum for $\gamma = 0°$ and $\beta = \beta_e$, a very plane minimum for $\gamma = 60°$ and $\beta = 0.14$. However the depth of this minimum is very small, it is smaller than the energy of zero oscillations and therefore it is impossible to speak about the existence of the isomers of such a type.

Thus, in strongly deformed odd-mass nuclei the one-quasiparticle states must have $\gamma = 0°$.

A number of two-quasiparticle states in strongly deformed even-even nuclei have the equilibrium deformations β_e differing from β_o when $\gamma = 0$. According to our calculations in ^{240}Pu the two-quasiparticle neutron states

$$K\pi = 7-, 6- \quad 13/2+[606],\ 1/2-[761],$$

$$K\pi = 7-, 6- \quad 13/2+[606],\ 1/2-[501],$$

$$K\pi = 6+, 7+ \quad 13/2+[606],\ 1/2+[880]$$

have the equilibrium deformation $\beta_e = \beta_o + 0.04$ and the state

$$K\pi = 2-, 5- \quad 7/2+[613],\ 3/2-[761]$$

has $\beta_e = \beta_o - 0.04$ and so on.

As in odd-mass nuclei the two-quasiparticle excited states of even-even strongly deformed nuclei have $\gamma_e = 0$.

Table 3 gives the behaviour of the two-quasiparticle states for ^{234}U with changing γ from 60° to 30° for $\beta = 0{,}14$. From this table it is seen that for some states, e.g. neutron two-quasiparticle

$$K\pi = 3+,\ 2+\quad 5/2+[633],\quad 1/2+[640],$$

$$K\pi = 5-,\ 6-\quad 11/2-[725],\quad 1/2+[640]$$

and proton two-quasiparticle

$$K\pi = 3-,\ 4-\quad 7/2-[503],\ 1/2+[660],$$

$$K\pi = 1-,\ 0-,\ 1/2-[541],\ 1/2+[660]$$

the function $\mathcal{E}(\rho_1, \rho_2; \beta, \gamma)$ has, in addition to the deep minimum for $\gamma = 0^{\circ}$ and $\beta = \beta_e$ a minimum for $\beta = 0.14$ and $\gamma = 60^{\circ}$. However, as in the case of odd-mass nuclei, the depth of this minimum is smaller than the energy of zero oscillations and an isomer for $\gamma = 60^{\circ}$ in the case of even-even nuclei will not exist.

The largest deviations of β_e from β_o at $\gamma = 0$ must be observed in odd-odd nuclei where it is more easy to find two levels of the average field the energy of which rapidly changes with β. So, the calculations of ref./6/ showed that in many odd-odd nuclei in the transuranium region the excited states

$$K\pi = 12-\ p\,11/2-[505],\quad n\,13/2+[606],$$

$$K\pi = 11-\ p\,11/2-[505],\quad n\,11/2+[615],$$

$$K\pi = 6+\ p\,11/2-[505],\quad n\,1/2-[761]$$

have equilibrium deformations which are by 0.08- 0.10 larger than β_o. Basing on these calculations we have tried to explain the structure of spontaneously fissioning isomers.

The calculations showed that a number of excited states of odd-odd nuclei must have equilibrium deformations β_e essentially smaller than β_o for $\gamma_e = 0$. For example, the equilibrium deformations of the excited states of nuclei with $Z = 105$ and $A = 258$ and 262

$$K\pi = 0-, 9- \quad p\,9/2 - [505], \; n\,9/2 + [604]$$

$$K\pi = 10+, 1+ \quad p\,11/2 + [615], \; n\,9/2 + [604]$$

are by 0.03 - 0.05 smaller than β_o. For the isomers with $\beta_e < \beta_o$ in odd-odd nuclei with $Z > 103$ the spontaneous fission and alpha decay half-lives can noticeably increase as compared with the ground states. This fact may turn out to be very essential in obtaining heavier transuranium elements.

Table 4 shows the behaviour of some two-quasiparticle levels in the odd-odd nucleus ^{286}Np depending on γ for $\beta = 0.14$. From Table 4 it is seen that for their states the deepest minimum is at $\gamma = 0^{o}$. In those states where there is an additional minimum at $\gamma = 60^{o}$ and $\beta = 0.14$, e.g.

$$K\pi = 0-, 1- \quad n\,1/2 + [640], \; p\,1/2 - [541],$$

$$K\pi = 1-, 0- \quad n\,1/2 + [640], \; p\,1/2 - [530],$$

$$K\pi = 1+, 0+ \quad n\,1/2 + [640], \; p\,1/2 + [660]$$

its depth is far larger than in the case of odd and even-even nuclei however even in this, the most favorable case, we may not speak about the existence of isomers with $\gamma_e = 60^{\circ}$, since the depth of this minimum is apparently, lower than the energy of zero oscillations.

VI. The following conclusions concerning the strongly deformed nuclei can be drawn on the basis of the calculations performed:

1. The total energies of the ground states of even-even nuclei have minima at $\gamma_0 = 0^{\circ}$ and at the values of β_0 which are in agreement with experimental data on equilibrium deformations. The shape of the curves $\mathcal{E}_0(\beta, \gamma)$ near the minimum is a parabola which leads to a small anharmonicity of oscillations.

2. Some one-quasiparticle and two-quasiparticle excited states may have equilibrium deformations β_e' which differ from β_0 .

3. There are no excited states with $\beta_e' < 0$, when $\beta_0 > 0$.

4. In one- and two-quasiparticle excited states nuclei retain the axial-symmetric shape, i.e. $\gamma_e' = 0$.

These conclusions are related to strongly deformed nuclei which are outside of the transition region from spheroidal nuclei to deformed nuclei. Some of these conclusions appear to be wrong for the nuclei of the transition regions. It is just in the nuclei of the transition region the appearance of shape isomers should be expected in particular, isomers for which $\beta_e < 0$ while for the ground state $\beta_0 > 0$. In conclusion we express our gratitute to A. Sobiczewski and V.M. Strutinsky for the routines of calculation on electronic computers.

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T a b l e   I

The Contribution of the Components with Different Projection K of the Total Momentum on the Axes O_z and O of the Wave Function of the Lowest level from the Subshell $i_{13/2}$ for $\beta = 0.20$. This level is assigned for $\gamma = 0^o$ by 1/2+[660] and for $\gamma = 60^o$ by 13/2 + [606]

γ in degrees	K Projection Values $1/2$	$3/2$	$5/2$	$7/2$	$9/2$	$11/2$	$13/2$	With Respect to the Axis
0^o	1,0	0	0	0	0	0	0	
10^o	0,733	0,223	0,040	0,003	2.10^{-4}	4.10^{-6}	4.10^{-8}	
20^o	0,600	0,300	0,085	0,014	0,001	6.10^{-5}	1.10^{-6}	O_z
30^o	0,532	0,320	0,118	0,027	0,003	3.10^{-4}	8.10^{-6}	
30^o	1.10^{-4}	8.10^{-6}	2.10^{-3}	7.10^{-7}	0.056	2.10^{-7}	0.942	
40^o	1.10^{-5}	2.10^{-7}	5.10^{-4}	8.10^{-9}	0.025	8.10^{-10}	0.975	O_y
50^o	1.10^{-7}	6.10^{-10}	3.10^{-5}	4.10^{-12}	0.006	4.10^{-12}	0.994	
60^o	0	0	0	0	0	0	I	

T a b l e   2

The Behaviour of the Energies (in MeV) of the One-Quasi-particle Excited States with Changing γ from 60° to 30° for β = 0.14.

Nucleus	$K\pi[Nn_z\Lambda]$ for γ = 60°	γ 60°	50°	40°	30°
^{235}U	7/2+ [613]	2.52	2.44	2.18	1.78
	5/2+ [633]	2.53	2.45	2.21	1.85
	3/2+ [642]	2.58	2.48	2.21	1.81
	1/2+ [651]	2.65	2.57	2.30	1.95
	5/2+ [622]	2.67	2.74	2.46	2.17
	1/2+ [640]	2.92	2.99	2.96	2.60
^{235}Np	5/2− [523]	2.52	2.44	2.18	1.78
	11/2+ [615]	2.56	2.47	2.21	1.79
	3/2− [532]	2.63	2.53	2.26	1.87
	1/2− [541]	2.73	2.73	2.57	2.17
	1/2+ [660]	3.57	3.64	3.57	3.34
	1/2− [530]	3.57	3.60	3.53	3.31

T a b l e 3

The Behaviour of the Energies (MeV) of the Two-Quasiparticle States of ^{234}U with Changing γ from 60° to 30° for β = 0.14

(Quantum Numbers $K\pi[Nn_z\Lambda]$ are Related to γ = 60°)

$K\pi$	$K_1\pi_1[Nn_z\Lambda]$ for γ = 60°	$K_2\pi_2[Nn_z\Lambda]$ for γ = 60°	γ = 60°	50°	40°	30°
			N e u t r o n		**S t a t e s**	
6+, 1+	5/2+[633]	7/2+[613]	4.36	4.28	3.99	3.58
2-, 9-	7/2+[613]	11/2-[725]	4.37	4.28	3.98	3.54
5+, 2+	7/2+[613]	3/2+[642]	4.47	4.34	4.04	3.60
4+, 3+	7/2+[613]	1/2+[651]	4.56	4.49	4.16	3.79
5+, 0+	5/2+[633]	5/2+[622]	4.57	4.53	4.35	4.05
3-, 8-	11/2-[725]	5/2+[622]	4.59	4.55	4.36	4.04
1+, 6+	7/2+[613]	5/2+[622]	4.60	4.55	4.36	4.04
3+, 2-	5/2+[633]	1/2+[640]	4.84	4.92	4.88	4.51
5-, 6 -	11/2-[725]	1/2+[640]	4.87	4.94	4.91	4.52
			P r o t o n		**S t a t e s**	
6+, 1+	7/2-[503]	5/2-[523]	4.29	4.21	3.95	3.56
2-, 9-	7/2-[503]	11/2+[615]	4.29	4.21	3.95	3.56
4+, 3+	7/2-[503]	1/2-[541]	4.61	4.63	4.48	4.03
3-, 4-	7/2-[503]	1/2+[660]	5.54	5.61	5.56	5.36
1-, 0-	1/2-[541]	1/2+[660]	5.89	6.05	6.10	5.82

T a b l e IV

The Behaviour of the Energies (in MeV) of the Two-Quasi-Particle States with Changing γ from 60^o to 30^o for β = 0.14

(Quantum Numbers $K\pi\,[Nn_z\Lambda]$ are Related to $\gamma = 60^o$)

$K\pi$ For γ =60°	Neutr. States $K_1\pi_1[Nn_z\Lambda]$ For γ =60°	Prot. States $K_2\pi_2[Nn_z\Lambda]$ For γ =60°	γ 60°	50°	40°	30°
1-, 6-	7/2+ [613]	5/2- [523]	2.52	2.44	2.18	1.78
3+, 8+	5/2+ [633]	11/2 [615]	2.57	2.49	2.24	1.86
11-, 0-	11/2-[725]	11/2+ [615]	2.56	2.47	2.21	1.79
4+, 7+	3/2+ [642]	11/2+ [615]	2.62	2.51	2.23	1.82
5+, 6+	1/2+ [651]	11/2+ [615]	2.69	2.60	2.33	1.97
8+, 3+	5/2+ [622]	11/2+ [615]	2.72	2.67	2.49	2.18
1-, 4-	5/2+ [622]	3/2- [532]	2.78	2.73	2.54	2.26
0-, 1-	1/2+ [640]	1/2- [541]	3.13	3.28	3.34	3.00
1-, 0-	1/2+ [640]	1/2- [530]	3.97	4.15	3.31	4.14
1+, 0+	1/2+ [640]	1/2+ [660]	3.97	4.18	4.35	4.17

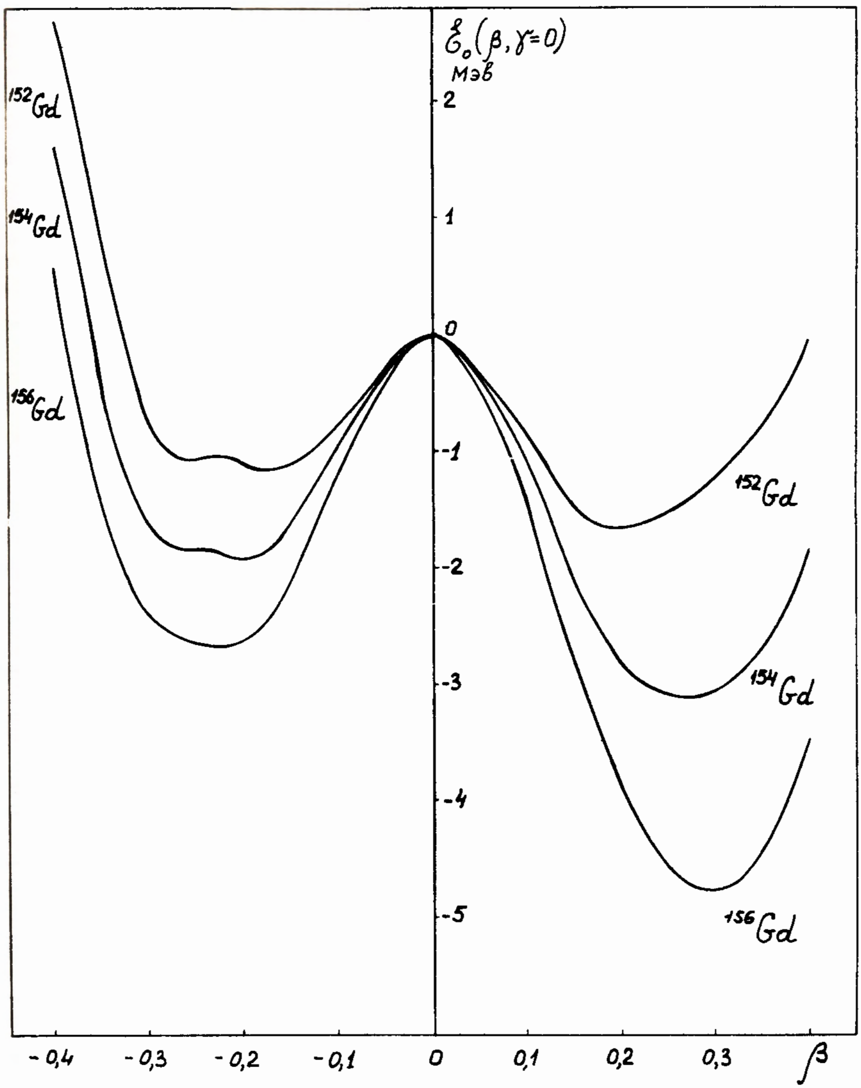

Fig. 1. The behaviour of the total energy of the gadolinium isotopes (in MeV) depending on the deformation parameter β from $\beta = -0.4$ to $\beta = 0.4$ for the nonaxiality parameter $\gamma = 0^{\circ}$.

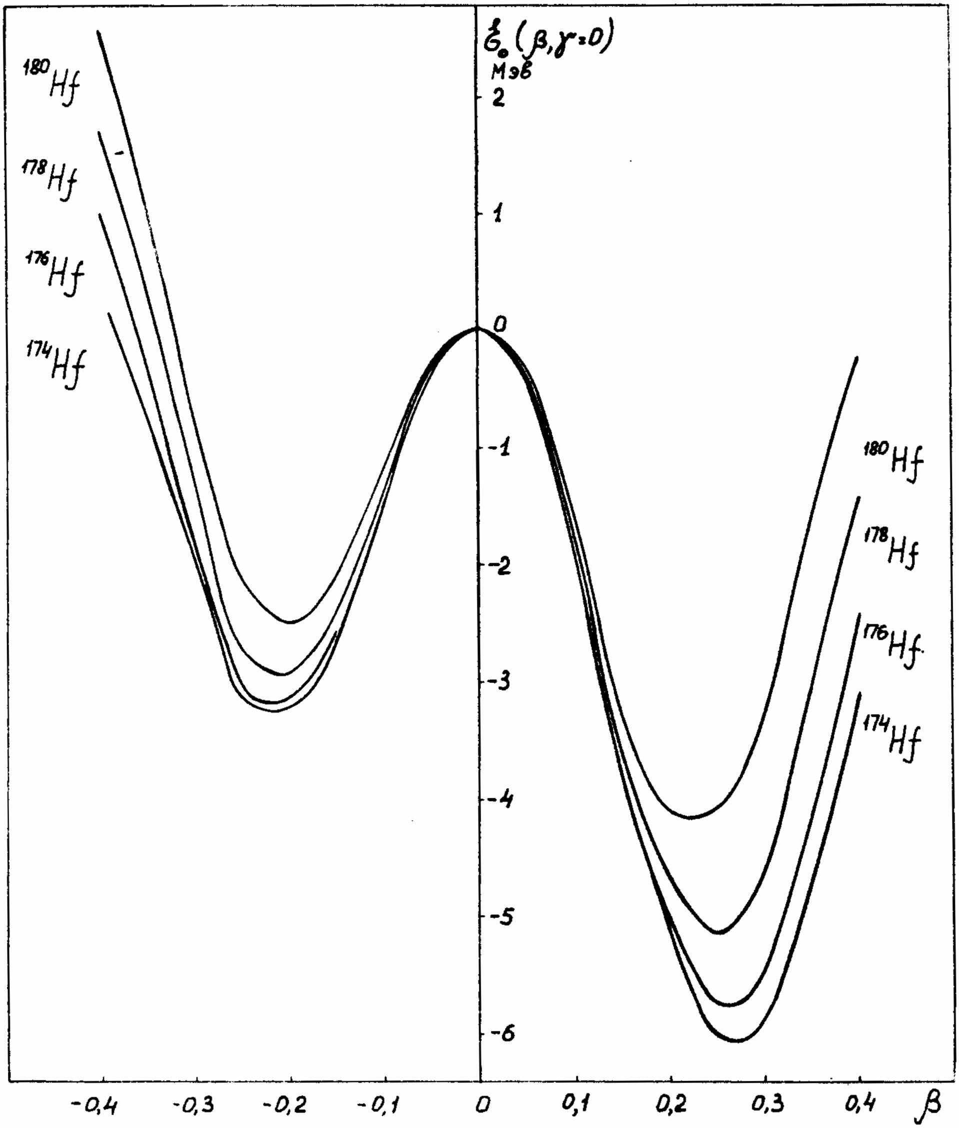

Fig. 2. The behaviour of the total energy of the hafnium isotopes (in MeV) depending on β (from -0.4 to + 0.4) for $\gamma = 0^{\circ}$.

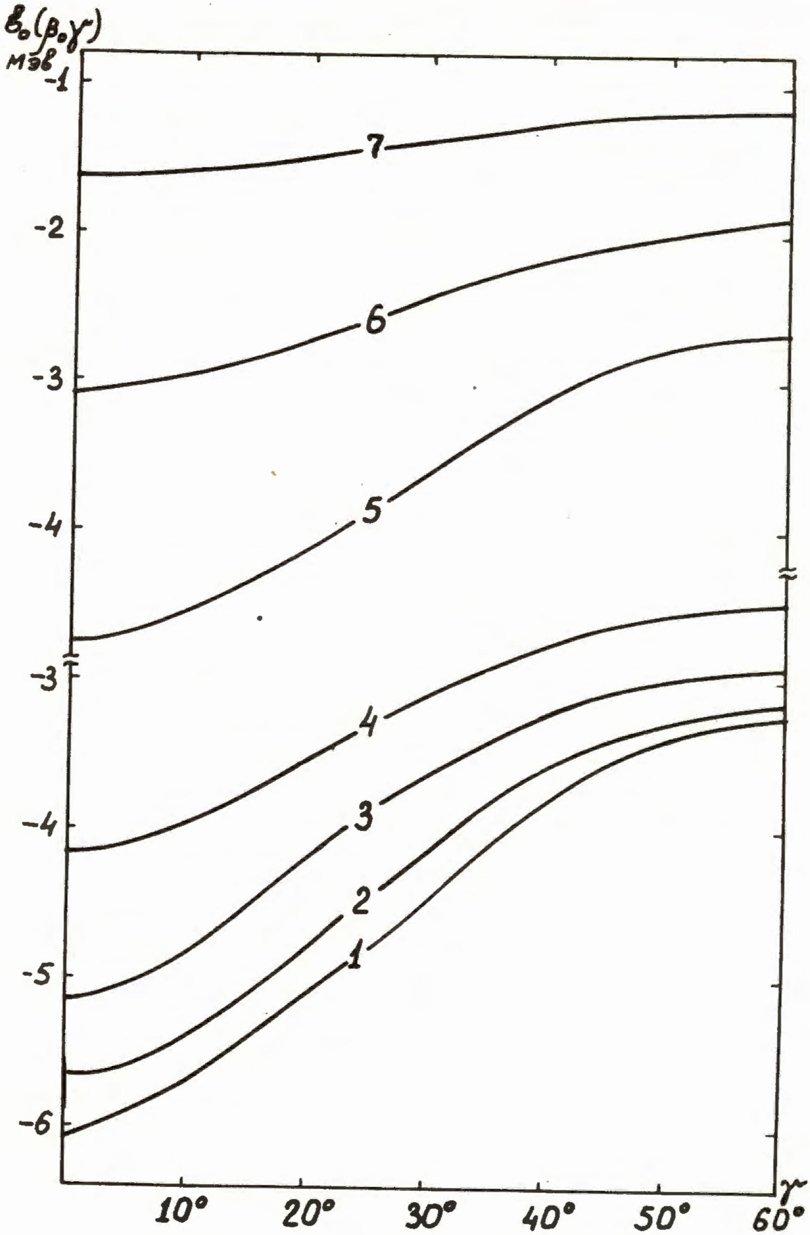

Fig. 3. The behaviour of the total energy of the gadolinium and hafnium isotopes (in MeV) depending on γ (from 0° to 60°) for the β value equal to the equilibrium value of β_0 for each γ. The curves correspond to the following isotopes:

1 – ^{174}Hf ; 2 – ^{176}Hf ; 3 – ^{178}Hf ; 4 – ^{180}Hf

5 – ^{156}Gd ; 6 – ^{154}Gd ; 7 – ^{152}Gd .

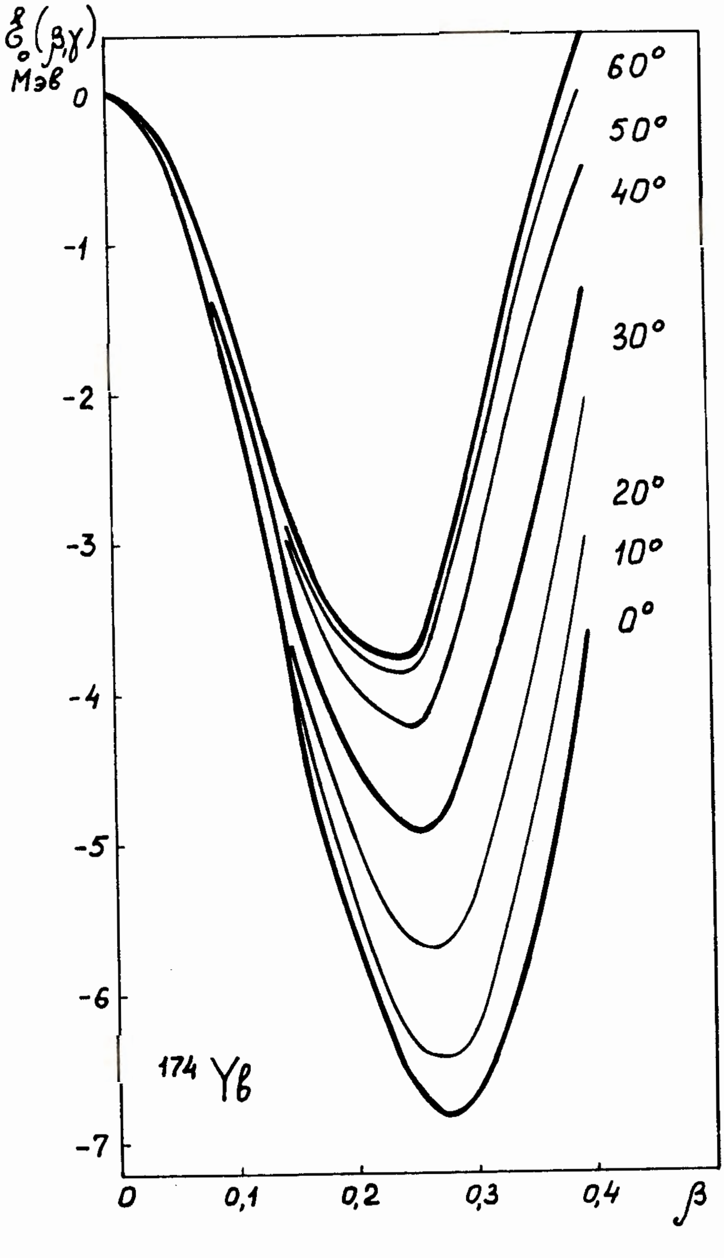

Fig. 4. The behaviour of the total energy (in MeV) of ^{174}Yb depending on β different values of γ from 0^{o} to 60^{o}.

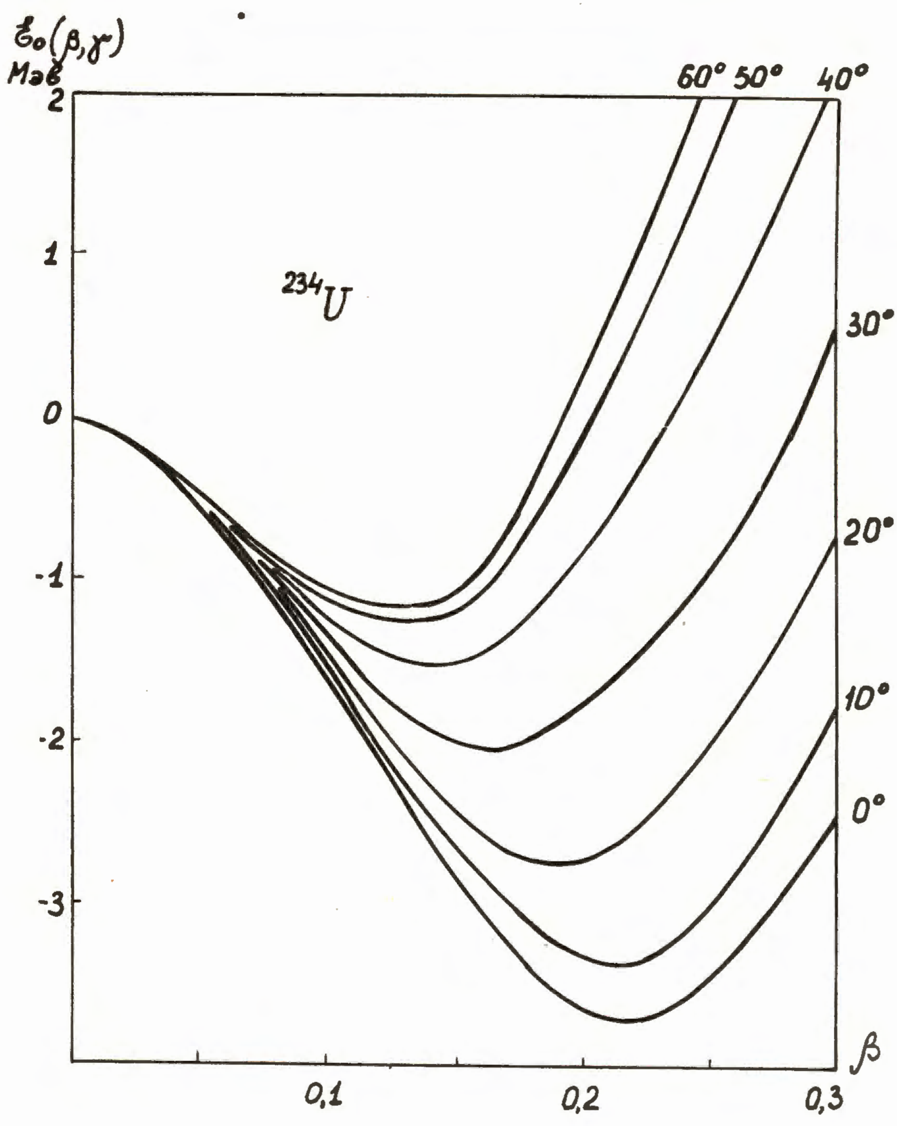

Fig. 5. The behaviour of the total energy $\mathcal{E}_0(\beta, \gamma)$ (MeV) for ^{234}U depending on β for the γ values from 0° to 60°.

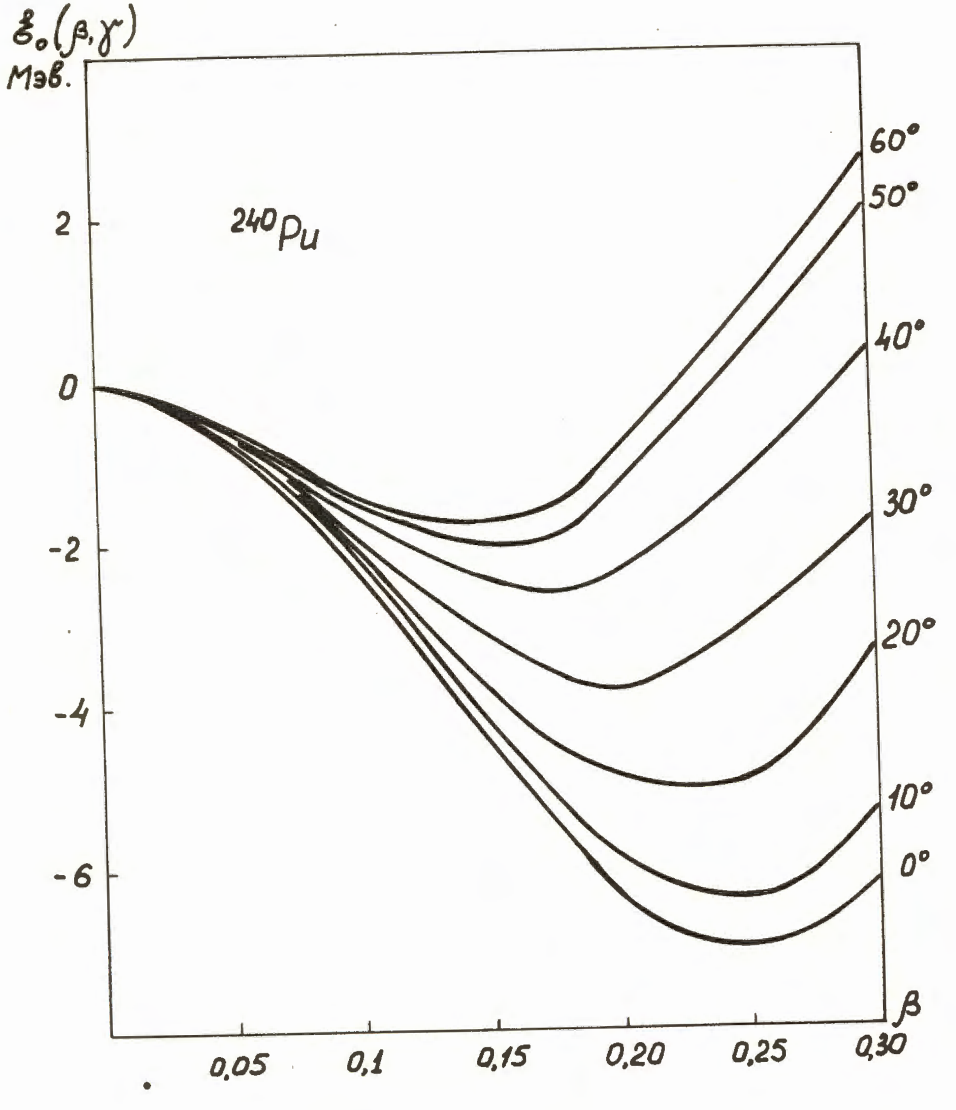

Fig. 6. The behaviour of the total energy $\mathcal{E}_0(\beta, \gamma)$ (MeV) for ^{240}Pu depending on β for the γ values from 0° to 60°.

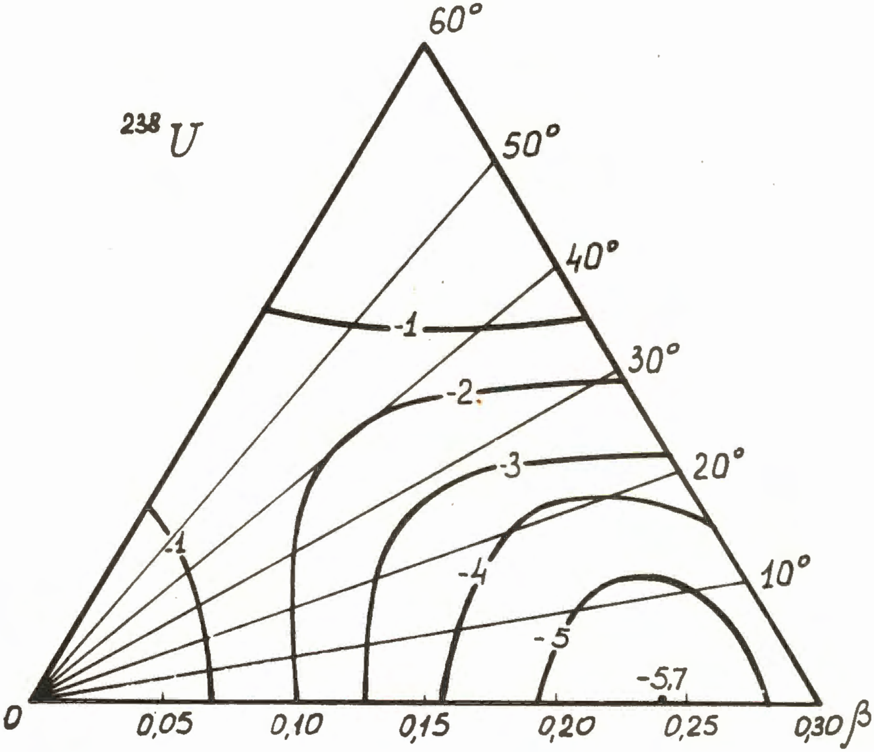

Fig. 7. The total energy $\mathcal{E}_0(\beta, \gamma)$ for ^{238}U. The minimum energy is −5.7 MeV. The continuous lines are the behaviour of $\mathcal{E}_0(\beta, \gamma)$ = Const for −5, −4, −3, −2 and −1 MeV. The function $\mathcal{E}_0(\beta, \gamma)$ has a minimum at $\beta_0 = 0.24$ and $\gamma_0 = 0$.

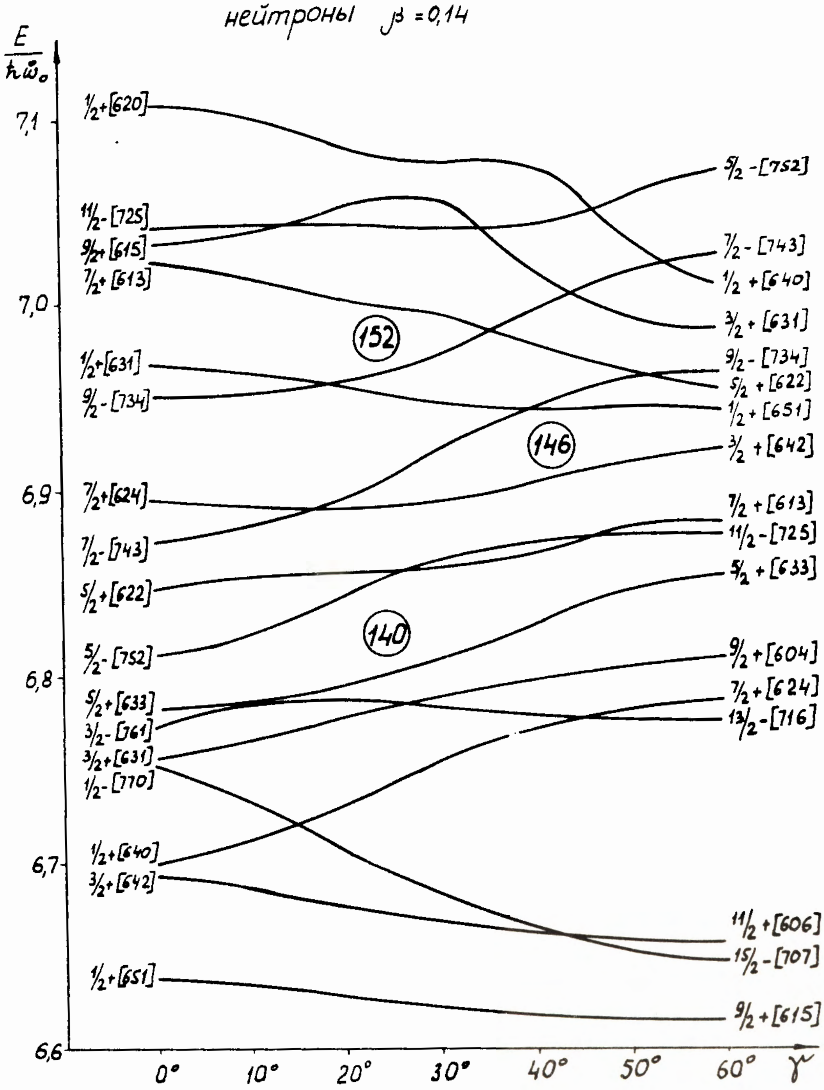

Fig. 8. The behaviour of the average field one-particle levels for the neutron system depending on γ (in degrees) for $\beta = 0.14$. On the left are the quantum numbers $K\pi[N n_z \Lambda]$ for $\gamma = 0^\circ$, on the right $\gamma = 60^\circ$.

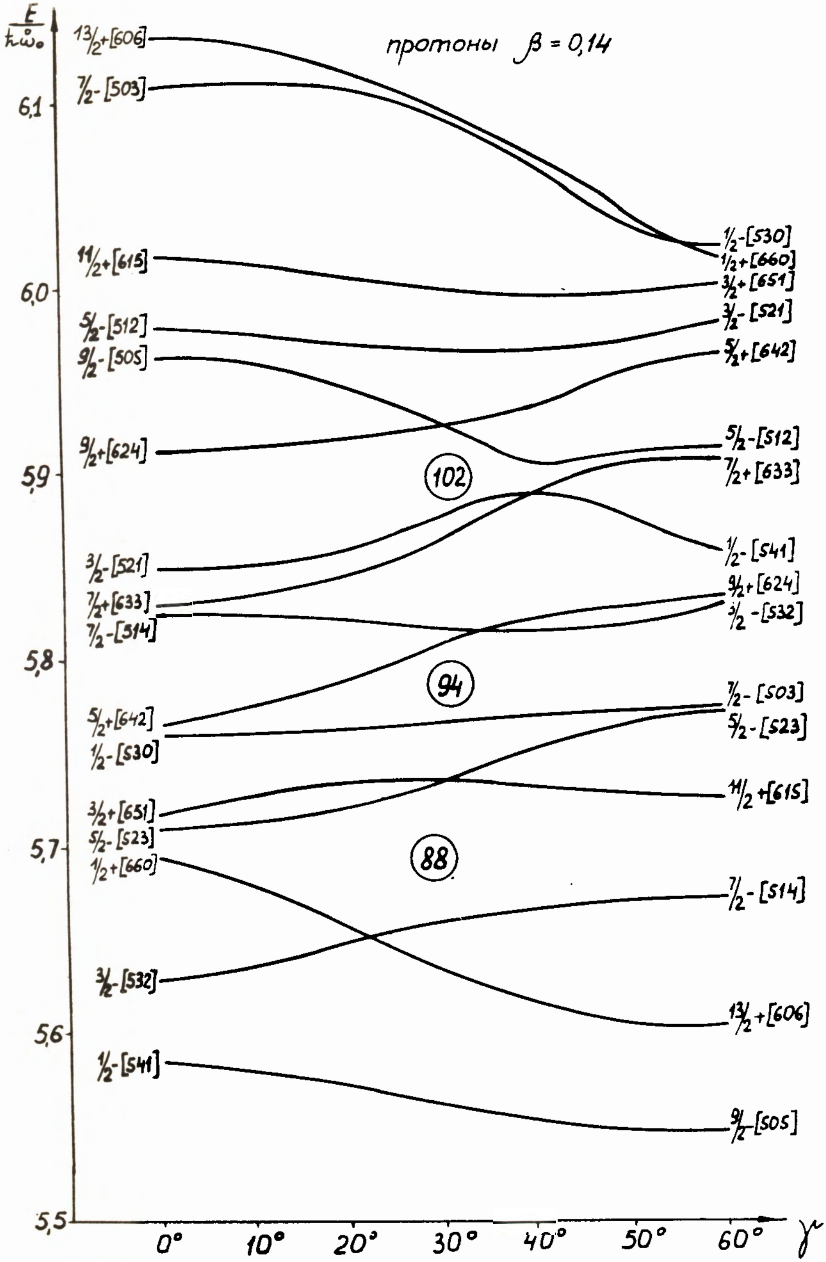

Fig. 9. The behaviour of the one-particle levels of the average field for the proton system depending on γ (in degrees) for $\beta = 0.14$. On the left are given the quantum numbers $K\pi[N n_z \Lambda]$ for $\gamma = 0^{\circ}$, on the right for 60°.