ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E4 - 3687

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DEVELOPMENT OF STATISTICAL INTERPRETATION OF QUANTUM MECHANICS BY MEANS OF THE JOINT COORDINATE–MOMENTUM REPRESENTATION

ЛАБОРАТОРИЯ ЯДЕРНЫХ ПРОБЛЕМ

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DEVELOPMENT OF STATISTICAL INTERPRETATION OF QUANTUM MECHANICS BY MEANS OF THE JOINT COORDINATE-MOMENTUM REPRESENTATION

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К развитию статистической интерпретации квантовой механики на основе совместного координатно-импульсного представления

Рассмотрены задачи определения траектории или плотности вероятности в фазовом пространстве по известным статистическим распределениям для импульсов и координат. Впервые для типично квантового случая движения гармонического осциллятора в нормальном состоянии получена всюду положительная плотность вероятности в совместном координатно-импульсном представлении. Делается вывод, что основные особенности движения микрочастиц (нулевая энергия, соотношение неопределенности, туннельный эффект и принципиально неустранимая статистичность описания) обусловлены виртуальными процессами взаимодействия с нулевыми колебаниями физического вакуума.

**Препринт Объединенного института ядерных исследований.
Дубна, 1968.**

Tyapkin A.A. E4-3687

Development of Statistical Interpretation of Quantum Mechanics by Means of the Joint Coordinate-Momentum Representation

The problem of the determination of a trajectory or probability density in phase space according to the known statistical distributions for momenta and coordinates have been considered. For the first time for a typical quantum case of harmonic oscillator motion in the zero state the positive probability has been obtained everywhere in the joint coordinate-momentum representation. The conclusion has been drawn that the main peculiarities of microparticle motion (zero energy, uncertainty relation, the tunnel effect and the principally unremovable statistical character of description) are due to the virtual process of interaction with zero oscillations of physical vacuum.

**Preprint. Joint Institute for Nuclear Research.
Dubna, 1968**

I. Introduction

One of the most striking and dramatic pages in the history of the development of the world cognition is related to the originating of quantum mechanics which has given the strict quantitative description of all the variety of strange phenomena of the atomic world. The fundamental representations of new mechanics have not been, however, developed basing directly on clear understanding of the necessary transformation of the laws of conventional mechanics in passing over to microparticle motion.

The theory of atomic phenomena was build anew with the complete refusal from classical mechanics conceptions. The most unusual phenomenon in developing quantum mechanics was the fact that at first "the carcas" of the mathematical apparatus of the future theory was discovered, so to say, and only later the physical treatment of quantities contained in obtained mathematical equations was found. The modern quantum theory in full measure reflects this unusual phenomenological way of developing. The formulation itself of the qualitatively original laws of the microworld is directly related to the interpretation of all the formalisms of quantum mechanics and up to now it continues to raise disagreements in some insignificant, as it might seem, details which screen entirely various views on the essence of the laws established by quantum mechanics.

The most important problems for understanding quantum mechanics which have no conventional solution yet, need further more accurate determination of the formulations of the very bases of quantum theory, further clarification of some aspects of the relationship of quantum and classical laws.

The solution of problems related to the interpretation of quantum mechanics is at present of urgent significance in connection with the necessity to further radically change physical representations for the theoretical generalization of rich experimental information in elementary particle physics.

A phenomenological way for constructing quantum mechanics was inevitably to result in formal understanding of this theory with considerable lack in explaining the peculiarities of microparticle motion. Now it has become traditional not to pay attention both to the unusualness of quantum theory construction and to the limitedness of this theory explanation achieved. During the years of successful application of quantum mechanincs the number of physicists was reduced who share the opinion of A.Einstein on the incompleteness of quantum mechanics or of those solidary with Mandelstam's point of view[1] of the necessity to search for deeper and more complete interpretation of this theory. The great success of practical application of this theory to atomic phenomena allowed numerous Bohr's supporters to persuade the majority of physicists that further development of atomic phenomena treatment was impossible, that search for more complete description of these phenomena is groundless from the scientific point of view.

However, the progress of science will inevitably make us again consider the same problems, to re-estimate the consolidated opinions.

The period of rapid development of quantum theory application to various atomic world aspects was over long ago when it was possible to obtain valuable physical results without deducing any new fundamental equation or theorem and without caring for a deeper understanding of the formalism employed. The leading front of the theoretical physics moved from the atomic world to nuclear physics and elementary particle physics during recent decades. Here it faced immense difficulties in solving new problems basing on the use of the earlier formalism of quantum mechanics and the relativity theory. The present day theoretical physics needs extremely the reappraisal of quantities, the radical change of the "apparatus" used. This problem can hardly be solved basing on the formal interpretation of the present formalism of theoretical physics without essential deepening the quantum theory.

And that is why along with investigations carried out at the leading edge of theoretical physics where the majority of theoretical physicists are working, the investigation is needed in the deep rear of theoretical physics aimed at eliminating the lack of understanding of the earlier established laws and at searching for more complete description of these phenomena.

2. On the Incompleteness of the Available Quantum - Mechanics Description of Micro-Object Motion

In the discussion with A.Einstein N.Bohr gained the victory not only over Einstein's erroneous confidence in existence of atomic phenomena the results of measurements in which are not described by means of quantum mechanics. Together with that, as if for one, Einstein's idea was rejected unjustifiably of the necessity of further theory development in order to establish the space-time description of the microworld phenomena. The value of the raising the question is not diminished also by the fact that A.Einstein erroneously hoped that it was possible to exclude in this way the probability from the description of quantum phenomena.

It cannot be said that this positive feature of Einstein's position has been lost among erroneous statements and remained unnoticed by its opponents. W.Heisenberg, for instance, in his book "PHYSIK UND PHILOSOPHIE" rather accurately formulates this part of A.Einstein's position as well: "... so erlaubt diese Deutung doch keine Beschreibung von dem was tatsachlich geschieht, unabhangig von oder zwischen unseren Beobachtungen. Aber irgend etwas muss doch geschehen, daran konnen wir nicht zweifeln. Dieses irgend etwas kann vielleicht nicht in den Begriffen Elektronen oder Wellen oder Lichtquanten beschrieben werden; aber sofern es nicht irgenwie beschrieben wird, ist die Aufgabe der Physik noch nicht erfullt."/2/ In his turn N.Born also has drawn a conclusion that " ... es nicht so sehr die Frage des Determinismus ist, die Einsteins Adleknung der heutigen Quantenphysik bedingt, sondern sein Glaube an die objective Realitat des physikalischen Geschehens, unabhangig vom Beobachter" /3/.

The incompleteness of quantum mechanics is obvious from the point of view of the space-time description of microparticle motion hidden from direct observation but uniquely connected with the results of irreversable processes of measurements. Even the most zealous supporters of the Kopenhagen interpretation hardly doubt about the existence within the given microconditions of microparticle motion before measurements. They just consider that the aim of physical theory is to describe only the results of irreversable measurements but not that of the space-time picture of micro-object motion hidden from observation. In this connection N.Bohr formulated the conception of physical reality whereas the physical principle of uncertainty was generalized into the philosophical principle of additivity. The limitedness of this point of view is especially clearly seen in rejecting by persistent supporters of the Kopenhagen school to give explanation to the interference of single photons after their passing through a half-transparent mirror. This experiment was suggested for consideration by A.Einstein. Quantum mechanics in its present state does not describe and explain the really existing process of photon motion from the half-transparent mirror to the photoplate detecting interference when the experiment is repeated several times /4/. But instead of seeing the limitedness of the quantum mechanics problem solved without any description of this undoubtedly existing in reality but directly unobservable motion, N.Bohr declares the statement of the problem describing such a motion to exceed the bounds of the problems of physical theory/5/. In this way N.Bohr managed both to avoid the answer to the question put by A.Einstein and to simultaneously conserve the myth on the incompleteness of quantum mechanics description. Such unnatural situation basing on the philosophical re-estimation of the notion of physical reality could be consolidated in physics for a long time only due to the fact that quantum mechanics gives by avoiding the description of directly unobservable motion of micro-objects the complete description of the results of measurements.

The principle of additivity suggested by N.Bohr not only reflects the fact that in various experiments we learn the aspects of reality supplementing each other, whose experimental investigation cannot be combined in experiments of a certain type. The principle of additivity, however, is not limited by this generalization of the content of the physical uncertainty principle. It unjustifiably prohibits the theoretical combination into one physical image of information obtained in incompatible experiments too. Just in this respect the principle of additivity exceeds the bounds of generalization of the physical law which had been formulated for the first time by W.Heisenberg as the principle of uncertainty.

Nobody proved yet the theorem on the impossibility of unique statistical description of quantum phenomena by basing on the combined use of physical quantities not measurable simultaneously. The Kopenhagen school declared such description to be outside physical reality by proceeding from positivistic principle of observability according to which only directly observable quantities should be introduced into physical theory. It is a surprise, of course, that the supporters of such points of view do not notice that the suggested principle of observability is denied by quantum mechanics itself which is based on the application of the wave function not measurable directly.

The fact that two separately measured quantities cannot be directly measured simultaneously does not mean that it is impossible to apprehend, to cognize the idea including both the quantities for no possibility has been proved yet to indirectly measure such quantities by basing on the results of direct measurements. On the contrary, the refusal from studying the motion of micro-objects in the given micro-conditions, for instance, of photon motion in the earlier discussed experiment, is a direct contribution to agnosticism. The development of such a statistical description though for a single but typical quantum case of motion could be the best proof of the failure to prohibit the theoretical application of the joint description of simultaneously not measurable quantities. The clarification of these basic disadvantages of the present-day formulation of existing theoretical description of quantum phenomena is of great importance. The conventional statement that quantum theory defines the state of micro-systems with respect to devices described by classical mechanics and which are original counting systems gives rise to a difficulty of principle to consistently describe quantum processes which occurred in the prehistoric period.

The above difficulty to consistently formulate the description of quantum processes in the prehistoric period is due, mainly, to the confuse in the determination of the notions "a device" and "measurement" in quantum mechanics. If in describing and studying sattelite motion the Earth was considered to be a device, this would give rise to a great surprise. A similar confusion still remains in quantum mechanics in the basis of this theory formulation.

Indeed, they call the physical conditions of micro-object motion given by the classical potential to be a preparatory part of the device forgetting that not the micro-object itself is the subject of quantum mechanics but the micro-object which is in certain physical conditions. The process of micro-object transition from some physical conditions of motion into other ones is incorrectly called measurement. In fact, the man's congnitive activity is principally connected only with the detection of this process with a device generating a macroscopic signal.

The test of quantum mechanics predictions along with the employment of macroscopic amplification in order to detect the change of quantum motion states requires, of course, the controlled conditions for the unique separation of the original statistic assembly of quantum systems under study, which is impossible without definite macroscopic physical conditions of micro-object motion. However, only the part which detects quantum transitions should be eliminated in describing quantum phenomena occurred in the prehistoric period, i.e. in describing them without relating to the experimental test of theoretical predictions. The experimental device itself generating the amplified macroscopic signal of micro-object quantum transition from some physical conditions into other ones, is not at all an integral part of the irreversable quantum process. Unfortunately, D.I.Blokhintsev[6] in his monograph has paid no attention to this circumstance in considering the macroscopic nature of a detector, of the equilibrium macroscopic instability of its initial state.

For instance, atom excitation, emission of light quanta by these atoms as well as photoionization of gas atoms by photons occur without any connection with the congnition activity of a subject from the macroworld.

The use of a generated photoelectron in the device as the initial impetus for producing a macroscopic avalanche under the effect of the electrical field in a photomultiplier, in a gas counter or a spark chamber cannot affect the earlier quantum processes of atom excitation, its photon emission and further atom photoionization in the counter. Despite the fact whether the electrical field is supplied to the counter or not, i.e., whether we have detected a photon or not, atom photoionization itself or the production of the Compton electron introduces the violation of wave coherency in quantum processes of the 1-st type eliminating a possibility to further separate with macroscopic means the assembly in which the interference effect might take place (Von Neumann's terminology[7]).

Thus, if in the experiment with a half-transparent mirror the photons before hitting a photoplate will pass through two symmetrically placed counters of Compton electrons, independently of voltage over the counter the interference of stipes on the photoplate will be eliminated proportionally to the effectiveness of Compton scattering process.

3. Substantitation of a Necessity to Establish the Joint Coordinate-Momentum Description of Micro-Object Motion

The behaviour of quantum systems consisting of an assembly of separate micro-objects not interacting with each other which are in identical macroscopic conditions of motion is a subject for description in quantum mechanics.

Both in quantum and in classical mechanics physical conditions for object motion are given by potential energy functions. But in classical physics these potential energy functions are used after solving motion equations in order to obtain the dynamic or statistical description of motion in space and time or in the phase space of coordinates and momenta. In quantum mechanics potential energy functions determine unambiguously only some auxuliary quantity, the so-called wave function of the state under study. The physical meaning of the latter is expressed in the predictions of the probabilities of micro-object transition from the state under study into any other motion state occurring in nature.

Note that the probability character of the present description corresponds to the the statistical nature of these transitions but it is not an artificial reason of the accepted description. Indeed, with one and the same type of sudden changes of external physical conditions in various specimens of similar quantum system transitions into various states take place which differ by the eigen-values of a certain dynamical variable. Therefore, the objective law is displayed in the probability distribution of such transitions.

Since a macroscopic signal informing of the transition of each micro-object into another state of motion can be obtained in specially prepared conditions, consequently, one can say that the wave function gives the statistical description of the results of any possible measurements of quantum systems. In accordance with the type of an effect upon quantum systems these processes are classified as the measurements of various dynamic variables. Among the so-called observables in quantum mechanics we have also dynamic variables (coordinates and momenta) with which motion in classical physics is described. But in quantum mechanics these quantities are not combined in the description of micro-object motion in phase space.

Among various possible states of quantum systems the so-called eigen-states of physical quantities are of special importance. Without describing micro-object motion itself within the given physical conditions defining a quantum-mechanical state the quantum mechanics separates such states and corresponding to them functions of potential energy from possible ones with which certain physical quantities conserve constant values. It is obvious that the corresponding measurements carried out for each separate specimen from the statistical body of quantum systems being in the given eigen-state will provide the same result. Such a body has a definite value of the physical quantity due to its conservation in each specimen in the process of motion.

In order to create quantum systems being in the eigen-state it is necessary to bring about certain physical conditions represented by a certain potential function.

The peculiarity of quantum system properties is that in nature there are no such states of micro-object motion in which the space coordinate in any direction and the momentum component in the same direction remained constant. (By the way, in classical mechanics these conditions are valid only in the state of object rest). Hence, the eigen-states of these dynamic variables do no coincide. In order to bring about the eigen-state by the momentum the absence of any forces in space is necessary, whereas for the eigen-state by the coordinate an infinitely narrow potential box with infinitely high potential walls is required.

A direct consequence of the fact that in nature there are no states having constant values of the coordinate and momentum is the impossibility to simultaneously measure the coordinate and momentum, since the process of measurement employs transition occurring in each case of the micro-object from the state under study into one of the existing eigen-states with the constant values of measurable quantities.

Thus, quantum mechanics leads to the impossibility of bringing about such macroscopic conditions of micro-object motion which allowed to separate the statistical body of quantum systems with the constant values of the momentum and coordinate. If the condition of conserving one of the variables to be constant holds, then according to the principle of uncertainty we obtain for the statistical body of such systems an indefinite value of another quantity. It should be noted that the term "an indefinite value" having a specific, concrete meaning with respect to the statistical body of similar quantum systems is often used rather lamely for a separate quantum system when one forgets the fact that a separate measurement always gives a certain value of the measured quantity. Quantum mechanics as P.Dirac has emphasized in his book "PRINCIPLES OF QUANTUM MECHANICS" (ref./8/, p. 145) proceeds from the abstract possibility of an as precise as desired single measurement of the coordinate and momentum of a micro-object.

Suppose we have a statistical assembly of electrons not interacting with eachother

~~infinite~~?

localized in a certain area of space due to the use of identical potential boxes. If we are able to instantly remove the potential walls of the box and let the electron into the space free of force fields affecting the electron, this will allow us to carry out as precise as desired measurement of the momentum of the single electron. In each specific measurement we shall obtain a certain value of momentum. But the measurement of various specimens of the available body of identical systems will provide different results. Dispersion characterizing the spread of obtained results in the statistical body is just contained in the uncertainty relation.

Naturally we shall consider that measurement is a specail type of effect which despite the destruction of the not studied quantum system allows to obtain the values of the measured quantity which are inherent to the object under study. Then the difference of experimental results of measurements carried out using separate specimens of the body of similar systems should describe the change of the given measured quantity in the process of micro-object motion in the state under study. Then the goal of further development of quantum mechanics should be the description of the micro-object motion in the phase space p and q hidden from the direct observation but uniquely related to the statistical bodies of the results of measurements.

Since quantum mechanics gives the correct prediction of statistical distribution of the results of all possible measurements, it is natural to suggest that the formalism of the existing theory hiddenly contains all the information on micro-object motion in the phase space p and q in any given state.

Just in this respect, against the Kopenhagen school statements, some scientists made attempts to develop the conventional formalism of quantum theory. As P.Dirac remarks in his book[8] (p.187), von Neumann was the first to introduce for the quantum system the density F (p , q) in the phase space which is analogous to the Gibbs density in classical statistical physics. But as far as quantum systems are concerned, this statistical function cannot be measured directly since the statistical body of systems corresponding to it is hidden from the macroworld as it cannot be separated by creating macroscopic conditions for micro-object motion. In this respect the function of probability density in phase space is entirely similar to the wave function also not measured directly in experiment but related theoretically with all the statistical distributions of the results of measurements. However, the development of the quantum theory on the basis of the distribution function in mixed coordinate-momentum space would deepen the theory since it would allow together with the predictions of statistical distributions of the results of all possible measurements to obtain information on the micro-object motion hidden from direct observation.

Note that the question of acquiring information on such motion does not contradict the conclusion drawn earlier by Von Neumann on the "hidden parameters". As far back as 1932 in his excellent monograph "MATHEMATISCHE GRUNDLAGEN DER QUANTENMECHANIK" /7/ which still remains the most strict and accurate description of quantum mechanics, Von Neumann drew a conclusion on the impossibility of introducing into quantum mechanics hidden parameters not taken into account which would allow to separate sub-assemblies without momentum and coordinate dispersion and to establish on this basis "true" causal motion of micro-particles (p. 240-241). Thus, the conclusion of Von Neumann refers to the parameters not taken into account in theory but directly observed in experiment, since he speaks of the separation with their help of assemblies not described by conventional quantum theory.

Acknowledging that modern quantum mechanics gives the complete description of all really directly observed quantities we pay attention to the inevitable existence of the parameters of micro-object motion which are hidden from the direct observation. These parameters cannot be used for separating with macroscopic means the assemblies contradicting the present day quantum theory and to available experimental data. But these parameters principally hidden from the direct observation should be cognizable by establishing their relation to all the assembly of observed quantities.

It should be stressed that the solution of the problem of establishing the unique description of micro-object motion principally hidden from observation will not allow to predict any new effect or result of measurements in the sphere of quantum phenomena. Then, what is this description needed for? First of all, it is needed to fill in the gap in the explanation of quantum effects for which the existing theory provides only statistical prediction of the results of observation. In other words, in order to give concrete explanation, for instance, to micro-particle motion with the tunnel effect, to the reason of statistical spread of the results of repeated measurements and at last to the unity of corpuscular and wave properties of matter a new experiment would be necessary to run, if the question arose of substituting the old explanation by a new one. But just the paradox of our time is that for forty years of quantum mechanics existence there has been no explanation whatever of the above effects. Thus, quantum mechanics in its present-day form giving correct and complete description of statistical distribution of the results of all possible measurements does not provide, nevertheless, in an obvious form all what could be established concerning micro-object motion in the microworld basing on these results of measurements.

Besides, it should be noted that the elimination of the gap in the explanation of quantum effects basing on the establishment of the nature of quantum effects hidden from the direct observation will permit in due course to predict new experimental results not in the field of atomic physics but in the deeper spheres of physics where even a formal theoretical formalism has not been developed yet on the basis of available representations. Strictly speaking, molecular theory as well which had opened wide horizons for physicists gave no practical results just to phenomenological thermodynamics.

Prof. D.I.Blokhintsev in his monograph "PRINCIPAL PROBLEMS OF QUANTUM MECHANICS" (ref.[6]) quite correctly remarks that the assumption on the hidden causal motion of quantum particles to which principally hidden, not measurable parameters correspond cannot be rejected basing on the considerations called the Neumann theory[7](pp. 133-147), since Von Neumann proceeds from the observability of parameters hidden in the formalism of modern quantum theory. Nevertheless, the methods of mathematical statistics, in fact, allow to strictly prove the same conclusion for the unobservable parameters as well. Further it will be shown that for the principally hidden micro-particle motion the representation of motion along certain trajectories in phase space is banned since such motion is incompatible in terms of mathematical statistics with probability distributions given in quantum theory for the observable quantities.

This means that the attempts of some physicists[9-12] to develop quantum theory for establishing dynamical motion are condemned to fail. The impossibility of creating the causal, not statistical description of the results of measurements in quantum processes has been proved by Von Neumann. Further we shall prove the necessity of involving statistics not only to predict results but also to describe micro-particle motion principally hidden from direct observation.

Searching for a unique description of hidden-particle motion, using the methods of mathematical statistics a non-classical way of motion as complex as desired should be admitted, if necessary. Their unique compatibility with all the amount of experimental results predicted with the present day theory can be the only criterion of the correctness of the discovered properties of hidden micro-particle motion. Therefore, the inverse method for finding the unobserved function of the distribution of the $F(p,q)$ probability density in phase space proceeding from the analysis of the mathematical formalism of the available quantum mechanics is quite natural.

Just this way of obtaining the mixed density matrix or the function of the joint momentum-coordinate distribution has been chosen by E.Wigner[13], G.Beil[14], Ya.P.Terletsky[15], D.I.Blokhintsev[16], P.Dirac[17] and J.E.Moyal[18], However, the above authors have not succeeded in obtaining the description of the hidden motion of micro-particles since obtained probability densities in phase space were expressed either by the complex function or had negative values. The use of such quasi-probability distribution functions means the development of new versions of the formalism for describing the results of observations in quantum mechanics only in their external form close to that of classical statistical physics.

The problem of uniqueness of the solutions obtained by the above authors has been discussed by R.L.Stratanovich/19/. G.V.Ryazanov/20/ could not avoid negative "probabilities" in generalizing the approach developed by R.Feynman/21/ as well.

In describing in terms of classical statistics specific quantum properties relating to the interference effects of quantum mechanics, negative values were always obtained for the $F(p,q)$ hidden function found theoretically. However, nobody has proved yet the theorem of the impossibility to solve the inverse problem of determining the real and everywhere positive function $F(p,q)$ of the hidden probability distribution in the space of simultaneously not measurable dynamic variables p and q with the known distributions of $f(p)$ and $\rho(q)$. Since we discuss here really existing hidden and as complicated as desired micro-particle motion for whose cognition no principal obstacles should exist, failures in solving this problem should be considered only as a proof of its complicacy.

The consideration and solution of an analogous inverse problem in classical physics can promote the overcoming of difficulties in solving the problems set in quantum mechanics. As far as is known, in classical statistical physics no problem was solved on the unique determination of probability density in phase space using known probability distributions for coordinates and momenta. At the same time this problem can be set both for object performing dynamical motion along certain trajectories in phase space and for objects moving along stochastic trajectories.

4. Statistical Description of the Motion of Individual Objects In Classical Physics

Prior to solving the inverse problem in classical mechanics let us refer to the statistical description of the dynamical motion of individual objects. Unfortunately, the description has not been widely used in practice despite the fact that just in its statistical form classical mechanics allows a direct comparison with experimental results.

The initial notions of the so-called statistical mechanics (phase space, microcanonical distributions, phase space invariance with respect to canonical transformations of variables, etc.) are considered in formulating the fundamental problem of statistical physics. M.Born/32/ has investigated a general case of statistical dynamics describing the combinations of separate particle motions along trajectories strictly following dynamical laws with the given distribution of the initial values of velocities and coordinates. To be frank, the author erroneously identified his proof of the limitedness of predictions of classical object motion states with the undeterminacy of the laws of classical mechanics. Both in its conventional and statistical forms classical michanics proceeds consistently from the Laplass predetermination of future motion states. Statistical calculations given in ref./32/ are based entirely on the acknowledgement of determinacy of classical object motion since for each object of the original statistical body a strictly determined trajectory is taken but not a stochastic one in phase space.

The statistical dynamics of the motion of separate classical particles is a limiting case in quantum mechanics since in vanishing of Planck's constant to zero quantum mechanics turns out directly into stochastic classical mechanics. This limiting transfer has been studied by Ya.P.Terletsky/15/. Thus, some features of the statistical representations of quantum mechanics are common to the stochastic form of classical mechanics and, therefore, they cannot be ascribed to the specific properties of quantum mechanics laws. Owing to this, the analysis of the stochastic formulation of classical mechanics turns out to be rather useful for clarifying a number of simple problems intricated by some clumsy, imperfect formulations of the conventional description of quantum mechanics. The exhaustive analysis of stochastic mechanics can also promote further clarification of such initial problems of statistical mechanics as the proof of ergodicity and the substantiation of the irreversibility of processes. For instance, ergodicity in the case of statistical dynamics results directly from the Liouville theorem on phase volume conservation in the canonical transformation of variables corresponding to the variation of coordinates and velocities according to the dynamical laws of classical mechanics. Indeed, the continuum of particles having various total energies which are plotted by a dotted line in phase space due to the Liouville theorem will move for a short period Δt so that the dots of phase space describing the continuum will pass the volume proportional to the time interval Δt and which is independent of the chosen time t. This invariance of the phase volume traversed for the time Δt, just means ergodicity. (See, for instance, ref./7/, p.331).

The time variation of the probability density F (p, q, t) in phase space for the continuum of particles not interacting with each other, moving according to the laws of classical dynamics is described by the well-known Liouville equation. If uniform probability density distribution along one of the particle trajectories(and the zero value of the probability density) for the remaining dots of phase space corresponds to the initial statistical assembly of the system under consideration, such microcanonical distribution remains constant in time. This kind of the time-independent state is of special interest since the solution of the inverse problem put by ourselves leads in this case to the determination of a certain trajectory in phase space from the analysis of the statistical distributions ρ (q) and f(p). It is also worth noting that the description of the time-independent assembly of classical particles located on a single phase trajectory is quite identical to the description of the motion of a single particle at various stochastic periods of time taken according to the law of uniform distribution. Ergodicity in this case is a direct consequence of the dependence of statistical distributions of quantities describing the object motion state, upon the variation of these quantities in time which is considered to be a stochastic quantity. Now it is evident also that the relative durations of the system residing in a certain region of phase space should be considered not as formal probabilities deprived of proper statistical bodies as it has been stated, for instance, by M.A.Leontovich/33/, but as the most conventional probabilities referring to the body of states at stochastic moments of time*).

In order to study the motion of an individual classical particle we can employ a device measuring only the particle coordinate and separately a device measuring only the particle momentum. Applying these devices in two series of independent measurements we obtain some statistical distributions of results of measurements performed.

In this case two versions are possible of holding the conditions of the independency of separate measurements. If one neglects the effect of measurement on the object under study, all the measurements can be subsequently carried out with one and the same object at independent stochastic moments of time. If each separate measurement results in considerable violation of motion of the object under study, the statistical investigation

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*) This approach in which time is considered to be a stochastic quantity can be successfully applied also to the stochastic description of the motion of a particle belonging to the equilibrium assembly of particles not interacting with each other, for instance, to ideal gas. In this case stochastic quantitites describing the state of individual object motion will be functions of both the stochastic period of time and the stochastic quantity of particle total energy. The proof of ergodicity, naturally, remains the same since averaging over quantities in time means statistical averaging over the continuum of states at various stochastic periods of time.

The consideration of time as a stochastic quantity is tolerable also for a quasistationary assembly of systems performing conditionally periodical motion if the period of the motion is much shorter than relaxation time. So, ergodicity or more exactly quasiergodicity can be strictly proved for those mechanical system whose statistical description of motion can be based on accepting the observation time of the motion state to be a stochastic quantity suffering uniform distribution.

is possible since measurements can be carried out with various specimens of the statistical bodies of similar systems.

Such an approach to the investigation of the motion of an individual classical object is applicable not only for describing motion along stochastic trajectories of the type of Brownian particle motion but for a conventional causal motion along a definite trajectory as well. The methods of mathematical statistics should evidently allow to obtain the description corresponding to this specially chosen method of experimental investigation for any type of motion. In other words, the statistical distributions of the results of independent measurements of coordinates and momenta should be predicted theoretically for any type of motion. In its turn, there should be a theoretical possibility of the unique reconstruction of motion description in the phase space of coordinates and momenta by the statistical distributions of coordinates and momenta separately found in experiment.

This method of investigations is peculiar of the applicability of its results also to the case when each measurement leads to the destruction of the system under study. Only due to the possibility available in classical mechanics to carry out measurement without violating the effect studied this principally important method of investigation turned out to be undeveloped in classical statistical physics.

Consider first the general case of periodic motion of a classical object along a certain trajectory in phase space. For the sake of simplicity consider the case of one-dimensional motion.

Considering the results of measurements of coordinates and momenta as a function of a stochastic quantity of observation time $q = q(t)$ and $p = p(t)$ and taking for t the uniform distribution $\varphi(t)dt = \frac{1}{T}\,dt$, where the normalizing factor T is a period, we find the following expressions for the probability density of acquiring the coordinate and momenta values in independent stochastic measurements:

$$\rho(q) = \frac{2}{T}\frac{1}{|\dot{q}|} = \frac{2}{T}\frac{1}{\left|\frac{\partial H}{\partial p}(q)\right|} \tag{1a}$$

$$f(p) = \frac{2}{T}\frac{1}{|\dot{p}|} = \frac{2}{T}\frac{1}{\left|\frac{\partial H}{\partial q}(p)\right|}\,, \tag{1b}$$

where H (p,q) is a Hamiltonian for the motion under study.

Emphasize that the appearence of statistics in the distribution of the results of coordinate and momentum measurement of a classical object moving along a definite trajectory is related not to experimental errors but to stochastic selection of the moment of measurement. Various values obtained in these experiments characterize the fact that the object under study resides at various points of space in the states having various momenta. In this connection the meaning of the obtained distributions is very simple. For instance, the probability $\rho(q)dq$ of acquiring in measurements the value q in the interval dq is proportional to the time of real object residing in this region of space. The distribution of the probability density $f(p)$ has a similar meaning.

The obtained distributions $\rho(q)$ and $f(p)$ are characterized by the dispersions $\overline{(\Delta q)^2}$ and $\overline{(\Delta p)^2}$ differing from zero, if only the object under study is not at rest. With the definite (fixed) value of energy these dispersions turn out to be correlated. Moreover, if one considers a classical harmonic oscillator of the total energy $\frac{\hbar\omega_0}{2}$, one finds for it the same relation for dispersions as for a quantum oscillator in the zeroth state, i.e., $\overline{(\Delta q)^2}\,\overline{(\Delta p)^2} = \frac{\hbar^2}{4}$.

Since our description of the results of measurements of dynamical variables of the classical object is, entirely analogous in form to the statistical description of time-independent states in quantum mechanics, the obtained results clearly show the groundlessness of the interpretation of the term "uncertainty" applied in quantum mechanics to a separate micro-object. Only K.V.Nikolsky[22] and D.I.Blokhintsev[23] in their courses of studies on quantum mechanics followed the mathematical statistical language which had been used for the first time by Von Neumann[6] to describe the essence of quantum mechanics.

On the other hand, these authors obviously underestimated, however, the main peculiarity of statistical assemblies consisting of not interacting particles as a means for clarifying the properties of motion of individual objects.

Strictly speaking, the dispersions $\overline{(\Delta q)^2}$ and $\overline{(\Delta p)^2}$ both in classical and in quantum mechanics refer to the statistical assemblies of independent repeated measurements with similar physical systems. The objective character of measurements implies the true realization of these distributions in the assembly of similar systems in the process of motion of objects studied. Since the assembly consists of particles not interacting with each other, the statistical distributions referring to the assembly are at the same time the statistical form for describing individual particle motion. With the given energy differing from zero of the classical oscillator we are not able using any variations of the potential functions to create motion corresponding to a statistical assembly having zero values of both the dispersions.

The motion itself of the classical object under study in the phase space but not the fact that the object has no definite momentum at each point of space leads to this circumstance. The impossibility of creating an assembly with zero dispersions with the given oscillator energy implies also the impossibility of simultaneous measuring the coordinate and the momentum when a device is used which distroys the motion under study. Obviously, this limitation of experimental possibilities cannot deprive a classical object of its property to move at each point of space at a definite velocity. The most important fact is that the application of the more perfect device measuring simultaneously the particle momentum and coordinate is not the only method for proving this property.

Rather a simple theoretical analysis of the distributions $\rho(q)$ and $f(p)$ obtained only in the independent separate measurements of coordinates and momenta allow to uniquely establish the presence of motion along a definite trajectory of the phase space.

Relation (1a) by itself establishes the relation between the absolute value of velocity or the momentum and the coordinate.

Indeed,

$$|p| = 2\frac{m}{T}\frac{1}{\rho(q)} \qquad (2)$$

We are only to exclude from this relation the unknown parameters m and T.

Time differentiation over time of relation (2) gives

$$|\dot{p}| = \frac{2}{T}\frac{|\rho'(q)||p|}{\rho^2(2)} \qquad (3)$$

Substituting this value of $\dot{P}_\delta$ to (1b) we obtain the relation correlating the dynamical variables p and q only by the functions $\rho(q)$ and $f(p)$ known from measurements.

$$|p|f(p) = \frac{\rho^2(q)}{|\rho'(q)|} \qquad (4)$$

Relation (4) determines, consequently, the equation of the trajectory in the phase space of variables p and q.

Using it, for example, for a classical oscillator, having the distributions

$$\rho(q) = \frac{1}{\pi}\frac{1}{\sqrt{q_0^2 - q^2}}$$

and

$$f(p) = \frac{1}{\pi}\frac{1}{\sqrt{p_0^2 - p^2}},$$

we find known equation of the ellipse for the phase trajectory of the classical oscillator $\frac{p^2}{p_0^2} = 1 - \frac{q^2}{q_0^2}$.

The most important fact is that in deducing relations (1a), (1b) and in solving the inverse problem of determining the phase trajectory by using statistical distributions measured we did not employ the dynamical laws of classical mechanics. This circumstance has a principal meaning since it makes relation (4) applicable for finding out the hidden motion along a definite trajectory independently of effective dynamical laws. Consequently, the fact of motion along a definite trajectory due to the kinematic factors causes certain dependence of separate statistical distributions on q and p which are the projections of the combined micro-canonical distribution in phase space.

Thus, basing only on the measured distributions $\rho(q)$ and $f(p)$ without making the simultaneous measurement of the momentum and the coordinate and without making any suggestions on dynamical laws governing the motion of an object under study we can by using relation (4) establish the motion itself along a trajectory and determine the trajectory itself of object motion in phase space.

This means that we can apply relation also to the analysis of hidden microparticle motion in various quantum-mechanical states. Since in this aspect of physics it is principally impossible to simultaneously measure the momentum and the coordinate of the microparticle, the above method of analysing is the only possibility to clear out the hidden motion along a certain trajectory. Introducing into relation (4) distributions given by quantum mechanics for time-independent states we make sure that the obtained equation has no solutions. Equality (4) does not hold with any real values of p for definite q if, for instance, a distribution function for a quantum oscillator is taken. This violation of equality (4) takes place for all quantum-mechanical states with the wave function presenting the superposition of flat waves.

The absence of solutions for the trajectory equation means that the quantum-mechanical distributions $\rho(q)$ and $f(p)$ are not compatible with the representation of the hidden motion along a single trajectory. Only the free motion of micro-particles with the given momentum is an exception, i.e., the case when wave interference is absent.

Consequently, the conclusion of Von Neumann is generalized also for the case of entirely hidden, principally not observed parameters. This means that the statistical character of quantum-mechanical description is due not only to the stochastic selection of the measurement time of the separate specimen from the statistical assembly of similar objects being in the time-independent state of motion, but by a more complex character of the hidden, directly unobservable motion.

It should be noted that D.Bohm /11/ has made a completely wrong conclusion on the existence of hidden quantum-mechanical motion of particles along dynamical trajectories.

A system of two equations obtained from the Shroedinger equation was interpreted by the author as equations of the dynamical description for microparticle motion basing on the fact that one of these equations is of the Jacobi-Hamiltonian type with an additional quantum-mechanical potential. As has been shown earlier by B.T.Gelikman[24] with an example of the Brownian motion, in classical physics such a system of equations is description of statistical but not dynamical motion of the particle. The hidden parameter treated by D.Bohm as the velocity of a separate particle at a definite point of space is, in fact, only the mean velocity of the particle continuum at the given point.

The search for the theoretical description of the hidden motion of quantum mechanics should be carried out taking into account the stochastic character of this motion on the basis of statistical analysis allowing to determine everywhere positive probability density F (p, q) in phase space. The above-developed approach is easily generalized for the case of more complicated motion in phase space. For the sake of conserving complete clearness, it is reasonable to make this generalization using the examples of object motion from classical mechanics.

Let the independent and separate measurements of q and p be carried out in a statistical assembly of the system whose total energy is distributed according to the law W($\mathcal{E}$). The results of measurements of the distribution functions $\rho(q)$ and $f(p)$ for a time-independent case will be identical both for an assembly consisting of a mechanical mixture of systems having various but constant energy values for each specimen and for the assembly consisting of similar non-conservative systems whose total energy varies due to independent stochastic variations of perturbations. The interpretation of statistical results will be, of course, different for such assemblies.

The observed distributions in our case will be as follows

$$\rho(q) = \int_{\varepsilon_1(q)}^{\infty} \frac{2m}{T(\varepsilon)} \frac{W(\varepsilon)\partial\varepsilon}{|p(\varepsilon,q)|} \tag{5a}$$

$$f(p) = \int_{\varepsilon_2(p)}^{\infty} \frac{2}{T(\varepsilon)} \frac{W(\varepsilon)\partial\varepsilon}{|\dot{p}(\varepsilon,p)|} . \tag{5b}$$

On the other hand, these functions are related with the integral relations to probability density in phase space

$$\rho(q) = \int_{-\infty}^{\infty} F(p,q)\,dp \tag{6a}$$

$$f(p) = \int_{-\infty}^{\infty} F(p,q)\,dq . \tag{6b}$$

Comparing these two relations we have

$$F(p,q) = \frac{W(\mathcal{E})}{T(\mathcal{E})} \quad , \qquad (7)$$

where the functions $W(\mathcal{E})$ and $T(\mathcal{E})$ are also unknown. It follows from this relation that probability density necessary for us should be searched for as some function F ($\mathcal{E}$) of only one variable $\mathcal{E} = \frac{p^2}{2m} + U(q)$.

Expressing the absolute velocity $\left(\frac{p}{m}\right)$ in relation (5a) through energy, we obtain for the unknown function F ($\mathcal{E}$) the following Wölter integral equation of the first type:

$$\rho(q) = \sqrt{2m} \int_{\mathcal{E}_1}^{\infty} \frac{F(\mathcal{E})\,d\mathcal{E}}{\sqrt{\mathcal{E} - \mathcal{E}_1}} \quad , \qquad (8)$$

where q in the function ρ (q) should be expressed by the lower integral limit $\mathcal{E}_1 = U(q)$.

Equation (8) is identical to that found in the known Abel problem. It has the only solution[25] of the following form:

$$F(\mathcal{E}) = \frac{1}{\pi\sqrt{2m}} \frac{d}{d\mathcal{E}} \int_{\mathcal{E}}^{\infty} \frac{\rho(q = q(\mathcal{E}_1))\,d\mathcal{E}_1}{\sqrt{\mathcal{E}_1 - \mathcal{E}}} \quad . \qquad (9)$$

Thus, the solution of the inverse problem has been obtained by ourselves using the dynamical laws of the classical mechanics. In order to determine the probability density F($\mathcal{E}$) in phase space it is necessary to know the probability ρ (q) and the function of the potential energy $U(q)$. The momentum distribution $f(p)$ with such an approach can be predicted basing on (6b) and (5b), where $\mathcal{E}_2$ should be taken equal to $\frac{p^2}{2m} + U_{min}$. Relation (7) implies that the equality $\mathcal{E} = \frac{p^2}{2m} + U(q)$ should be valid. Relations (5) and (6) can be used in statistical consideration of the object motion independently of the dynamical laws of classical mechanics. In this case all the statistical distributions refer to the Hibbs statistical assembly consisting of similar not interacting with each other physical systems suffering stochastic perturbations of motion from one and the same thermostat. The observed distributions $\rho(q)$ and $f(p)$ are sums taken with certain weights of statistical distributions corresponding to motion along separate trajectories in phase space. Due to stochastic perturbation of motion the system under study passes from a trajectory to another one , its micro-state being varied by step. If the independent measurements of q and p do not violate the motion of our system, statistical distributions can be obtained in the assembly of repeated independent measurements at stochastic moments of time using one physical system interacting with the thermostat. The probability distributions of measurement results are due in this case not only to stochastic choice of the time of measurement but to the stochastic character of the variation of dynamical motion of the object under study.

Accepting the canonical energy distribution $W(\theta)=\frac{1}{\theta}e^{-\frac{\varepsilon}{\theta}}$ for a one-dimensional classical oscillator which is a classical Brownian particle in the force field $m\omega_0^2 q$ and suffering independent stochastic perturbations one obtains from relation (5) for q and p the Gaussian law distributions in agreement with the results of ref./30/ Thus, Gaussian distributions are obtained as a result of summing the distributions

$$\rho(q,\mathcal{E})=\frac{1}{\pi\sqrt{q_0^2(\mathcal{E})-q^2}} \quad \text{and} \quad f(p,\mathcal{E})=\frac{1}{\pi\sqrt{p_0^2(\mathcal{E})-p^2}}, \tag{10}$$

where $q_0^2=\frac{2\mathcal{E}}{m\omega_0^2}$ and $p_0^2=2m\mathcal{E}$.

The particle moving in phase space along an elliptical trajectory corresponding to the total energy $\mathcal{E}$ at some stochastic time t at the point q(t) suffers a perturbation changing instantly its momentum P (t) by some stochastic quantity Δp_i. Hence, being at the same point q, having the previous potential energy $\frac{m\omega_0^2q^2}{2}$ at the time t + dt, the particle has now another value of kinetic energy $\frac{(p+\Delta p_i)^2}{2m}=\frac{p^2}{2m}+\Delta\mathcal{E}_i$. The total oscillator energy is varied by the same value $\Delta\mathcal{E}_i=\frac{2p\Delta p+(\Delta p_i)^2}{2m}$ according to which further particle motion in phase space occurs along a new elliptical trajectory.

5. Problem of Unique Statistical Description of Hidden Microparticle Motion in Phase Space

The hidden value of the probability density F (p, q) in phase space for quantum systems in a general form should be, of course, searched for proceeding from the wave function of the state, as it contains all the data on the statistical distributions of the measured quantities. However, the approach developed in the previous Section for the systems of classical mechanics seems to offer interesting possibilities for studying some typical quantum systems as well.

As has been already mentioned, for the classical oscillator having the definite energy $E_0=\frac{h\omega_0}{2}$, the dispersions for the q and p distributions coincide with those for the quantum oscillator in the zero state. The q and p distributions themselves considerably differ from quantum distributions. However, for the classical oscillator excited by Brownian perturbations we have the q and p distributions of the same type as in the case of the zero state of the quantum oscillator. This similarity of distributions allows to expect that information on the hidden motion can be obtained in analysing some quantum systems basing on the solution of inverse problem (9) obtained for a classical system.

To employ relation (9) for the determination of F ($\mathcal{E}$) on $\rho(q)$ for the quantum oscillator is to accept the hypothesis that oscillator motion between perturbations occurs according to the laws of classical dynamics, whereas perturbations of the known origin varying the state of hidden motion occur stochastically and independently of object position in phase space. In order to prove the validity of this unusual and simple hypothesis on the hidden motion of a quantum oscillator it is necessary to obtain the everywhere positive probability density $F(p,q)$ which would provide complete coincidence of quantum mechanical distributions for all the variables observed obtained by using that density.

Substituting into (9) the distribution for the zero state of the quantum oscillator $\rho_0(q) = |\varphi_0|^2 = \frac{1}{q_0\sqrt{\pi}} e^{-q^2/q_0^2}$, where $q_0^2 = \frac{h}{m\omega_0}$, we find the function of the probability density in phase space $F(p,q) = \frac{\omega}{2\pi E_0} e^{-\mathcal{E}/E_0}$, where

$$\mathcal{E} = \frac{p^2}{2m} + U(q),$$

and the energy distribution, respectively, coinciding with the Hibbs canonical distribution if the distribution module θ is taken to be $E_0 = \frac{h\omega_0}{2}$. The obtained function F(p,q) is everywhere positive. The distribution f(p) calculated on its basis exactly coincides with the quantum mechanical distribution $f_0(p) = \frac{1}{p_0\sqrt{\pi}} e^{-p^2/p_0^2}$ where $p_0^2 = \hbar m\omega_0$.

It might seem on the face of it that there is, nevertheless, an obvious contradiction of energy distribution for the considered oscillator with the total energy constancy of the quantum oscillator. However, in fact just the consistent use of quantum theory formalism shows the complete identity of these systems. Our variable $\mathcal{E} = \frac{p^2}{2m} + U(q)$ for fixing the positions of systems under study at a definite elliptical trajectory in phase space is first of all a hidden quantity in quantum mechanics as the sum of instant values of kinetic and potential energies. It is due to impossibility to measure simultaneously the momentum and coordinate that the impossibility of measuring the quantity $\mathcal{E}$ and of observing its distribution dispersion arises. In quantum mechanics one measures another quantity E corresponding to the operator of the total energy $\hat{H} = \frac{\hat{p}^2}{2m} + U(q)$, which is equal to the sum of operators of kinetic and potential energies, whereas the hidden energy value $\mathcal{E}$ is the sum of eigen-values of kinetic and potential energies. The mean value of this unobservable quantity is equal to total oscillator energy. Indeed, $\bar{\mathcal{E}} = \int_0^\infty \mathcal{E} W_0(\mathcal{E})\, d\mathcal{E} = \frac{h\omega_0}{2}$. It should be kept in mind that for the precision measurement of total energy infinitely large time is always required according to the uncertainty principle.

Only the fact that the values of the hidden quantity $\mathcal{E}$ exceed the total system energy $E_0 = \frac{h\omega_0}{2}$ might seems to be discrepant. However, this circumstance is, first of all, a direct consequence of the same seeming contradiction of the relation between energy variables in quantum mechanics itself. The values of kinetic and potential energies measured separately take, according to quantum theory, also eigen-values exceeding that of total energy. The present day quantum theory does not explain this unusual relation between the energy characteristics stressing only the absence of discrepancy in the frame work of the accepted formalism due to the simultaneous immesurability of these quantities.

Thus, it is natural that the unobservable quantity $\mathcal{E}$ which is the sum of the eigen-values of kinetic and potential energies takes values exceeding those of total energy. But physical theory trying to describe hidden motion cannot be limited only by formal clearing out the formal absence of discrepancy in this fact.

It is necessary to obtain physical explanation both of this fact and of the reasons of the uninsulated non-conservation character of quantum systems.

An attempt to describe a quantum oscillator as entirely identical to the above-considered Brownian one for which it would be impossible to measure the instant energy $\mathcal{E}$, confronts a difficulty of principle. Quantum particles released due to the tunnel effect have always energy corresponding to the total energy of the system having a negligible energy dispersion spread. Hence, perturbation experienced by quantum systems does not change the total energy of the system. This will be possible if one supposes that with each perturbation simultaneously energy should be varied by $\Delta\mathcal{E}_i$ and the potential energy should be varied by the value $\Delta U_i = -\Delta\mathcal{E}_i$, which keeps the unchanged value in the process of further particle motion in phase space till next stochastic perturbation. Consequently, the unobserved quantity $\mathcal{E} = \frac{p^2}{2m} + U(q)$ taking into account instant changes of kinetic energy $E_i = \mathcal{E}_i + \Delta U_i$ which remain constant. Such processes occurring with total energy conservation are called virtual in quantum mechanics.

Kinetic energy fluctuations are responsible for the departure of the particle into a region remoted from the attraction centre where the potential energy determined by the classical function $U(q) = \frac{m\omega_0^2 q^2}{2}$ without taking into account stochastic perturbations of the potential by ΔU_i considerably exceeds the total energy of the system. The tunnel effect is explained by particle passage over the potential barrier due to the stochastic process of lowering the whole potential function. Employing the classical image of the force field potential quantum mechanics, in fact, takes into account in a hidden way the macroscopic nature of potential energy fluctuations. Without explaining the physical essence of the tunnel effect modern theory uses rather improper terminology (subbarier particles, the tunnel effect). The very fact that quantum mechanics predicts as large values of kinetic energy as desired makes the representation of particle passage under the barrier to be groundless.

Thus, for a typically quantum case of motion such as a quantum oscillator is at the lowest energy state, the positive distribution function in the mixed coordinate-momentum representation was determined for the first time. The same fact that this solution was found by basing on classical approach means that the quantum oscillator between separate perturbations makes a micromotion according to the dynamic laws of classical mechanics. Only the presence of outside stochastic perturbations of special type causes the specific properties of quantum mechanics. In classical physics there are no similar examples of uninsulated systems when outside perturbation leads to the contrary variations of kinetic and potential energies.

Earlier only the hypothesis and speculations were expressed on the nature of the statistical character of the quantum-mechanical description. Thus, according to D.I.Blokhintsev's opinion the quantum statistical character is due to the impossibility of insulating macrosystems from macroworld[26]. A more definite opinion on the possible nature of the uninsulated character of macrosystems was modestly expressed by the authors of the course of studies[27] as a preliminary untested hypothesis printed in small type letters. They called fluctuation perturbations from the virtual field of vacuum photons as a possible reason of stochastic effect on micro-object motion. The relation of uncertainty for a classical harmonic oscillator excited by fluctuations going from photon vacuum were laid in the basis of such an assumption[28]. The inadequacy of this basis becomes especially evident if one recollects the above-obtained result of the validity (at some energy) of the uncertainty relation also for a classical oscillator performing dynamic motion.

A convincing argument of the important role of vacuum fluctuations in quantum mechanics has been obtained by E.I.Adirovich and M.I.Podgoretsky[29]. It has been shown there that a classical harmonic oscillator excited by fluctuation perturbations with the virtual photons of vacuum has exactly the same coordinate and momentum distributions as a quantum oscillator in the zero state.

The authors have not found for the oscillator under consideration the energy distribution W($\mathcal{E}$) defining probability density in phase space. Having obtained the dispersion of energy distribution differing from zero and having not noticed that it was related to the hidden, directly not observed quantity they made a wrong conclusion on the incomplete adequacy of the considered classical and quantum oscillator.

Using the above proof of the absence of contradiction in the energy characteristics of these oscillators we can, basing on the results of ref. [29] make an unambiguous conclusion on the physical nature of fluctuation perturbations violating the insulated character of quantum systems. It is virtual processes of quantum system excitation with photon vacuum that cause simultaneous fluctuations of kinetic and potential energies, total energy being conserved.

Thus, a specific peculiarity of the presence of the zero energy $E_o = \frac{h\omega_o}{2}$ of quantum systems is completely due to their uninsulation of zero vacuum oscillations, i.e. to the effect of virtual photons of vacuum. Since the virtual process of kinetic energy variation due to the potential one can occur by definition only in coupled systems, only such systems have zero energy, and the value of zero energy turns out to be proportional to the frequency ω_o describing the degree of microparticle coupling. The existence of the finite value of zero energy of the coupled system in turn means the absence in nature of the assemblies having dispersions for q and p simultaneously equal to zero. Consequently, the uncertainty relation is due to the impossibility in principle to insulate the systems from zero vacuum oscillations but not to the properties of measurement. On the contrary, the impossibility to simultaneously measure the momentum and coordinate is a direct consequence of the absence of quantum assemblies corresponding to the state of rest of the localized particle and to the necessity in principle to violate the motion state under study in each separate act of measurement.

Thus, just interaction with vacuum taken into account by the theoretical formalism in a not obvious way makes all the difference in quantum mechanics from the statistical description of dynamical systems of classical physics. The impossibility in principle to insulate any material system from interaction with physical vacuum representing systems with infinite number of freedom degrees makes quantum laws universal and leads to not causal motion of microparticles in principle.

These basic specific properties of hidden micro-particle motion were clarified in this study basing on the unique mixed coordinate-momentum statistical description of a quantum harmonic oscillator in a zero state. However, a further analysis shows that the stochastic effect (discovered in this particular case) of physical vacuum upon microparticles in a bound state does not describe the specific properties of the hidden motion of quantum particles. This stochastic effect of vacuum predetermines some similarity stressed by Shroedinger [34] and Fuert [2] of the basic equation of quantum mechanics and the classical equation of Brownian particle motion. But, on the other hand, the available principal difference of these equations fails to be fully explained by the difference of the origin of virtual perturbations going from physical vacuum, of molecular perturbations suffered by the Brownian particle.

Further, a short survey is made of investigations where some analogy available in the statistic laws of quantum mechanics and classical diffusion have been obviously overestimated. B.T.Gelikman[24] was the first to indicate the possibility of describing classical diffusion of a free Brownian particle with a system of two equations, the first of which is "the motion equation" for probability density (a statistical analog of Jacobi-Hamilton equation)

$$\frac{\partial \bar{S}}{\partial t} + \frac{(\nabla \bar{S})^2}{2m} + mD^2\left[\frac{\nabla^2 \rho}{\rho} - \frac{1}{2}\frac{(\nabla \rho)^2}{\rho^2}\right] = 0 , \tag{11}$$

and the second is the continuum equation in which probability density velocity $\vec{V}$ is expressed by the action function $\vec{V} = \vec{\nabla}\bar{S}$ which is average for the particle continuum. Comparing this system of equations with that obtained from the Shroedinger equation for the module squared $|\Psi|^2 = \rho$ and the phase S of the wave function, B.T.Gelikman has arrived at the conclusion of the coincidence of these equations basing on the fact that the quantum-mechanical analog of the Jacobi-Hamilton equation for a free particle

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} - \frac{h^2}{4m}\left[\frac{\nabla^2 \rho}{\rho} - \frac{1}{2}\frac{(\nabla \rho)^2}{\rho^2}\right] = 0 \tag{12}$$

differs from the equation for a Brownian particle (11) only by its sign of the statistical potential. However, the inverse sign of the statistical quantum-mechanical potential means, in fact, that free quantum particle motion is not similar to diffusion. Indeed, $\frac{\partial \rho}{\partial t} = iD\nabla^2\rho$ corresponds to equation (12) with D = h/2m which cannot be satisfied with any real ρ .

Quite a wrong conclusion on the total similarity of quantum particle motion and Brownian particle diffusion has been drawn by J.Fenyes[31]. He made an attempt to deduce the same new "motion equation" for the probability density of Brownian motion as B.T.Gelikman but in a general case, an external field being present. The errors made in the conclusion lead the author to an equation completely coinciding with the quantum-mechanical analog of the Jacobi-Hamilton equation

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} - \frac{h^2}{4m}\left[\frac{\nabla^2 \rho}{\rho} - \frac{1}{2}\frac{(\nabla \rho)^2}{\rho^2}\right] + U = 0 . \tag{12a}$$

The fact that the external field potential was included into the statistical equation of probability density motion in the same way as into quantum-mechanical equation (21a) appears to imply that the author had considered, in fact, an example of motion without friction which is entirely discrepant with the classical diffusion of a Brownian particle. The obtained inverse sign in front of the statistical potential

$$mD^2\left[\frac{\nabla^2 \rho}{\rho} - \frac{1}{2}\frac{(\nabla \rho)^2}{\rho^2}\right]$$

disagrees with ref.[24] .

Using the deduction method described in ref./24/ it is easy to obtain for conventional diffusion with friction, the external field being present, the following "equation of motion" for the probability density

$$\frac{\partial \bar{S}}{\partial t}+\frac{(\nabla \bar{S})^2}{2m}+mD^2\left[\frac{\nabla^2\rho}{\rho}-\frac{1}{2}\frac{(\nabla\rho)^2}{\rho^2}\right]+\frac{1}{\beta}\left[\frac{\partial U}{\partial t}+D\nabla^2 U-\frac{1}{m\beta}(\nabla U)^2\right]=0, \quad (13)$$

where $m\beta$ is the friction coefficient determining the velocity of macroscopic flux in the force field $\vec{u}=-\frac{\vec{\nabla} U}{m\beta}$. As is seen, our equation (13) considerably differs from quantum-mechanical equation (12a).

Recently E.Nelson/36/ poited out the possibility of deducing the Shroedinger equation from classical diffusion equations without friction. Equations given in ref./36/ are easily transformed into a statistical equation similar to that of Jacobi-Hamilton. Due to the absence of friction this equation differs from equation (13) by the fact that the external field potential enters into it directly as into quantum-mechanical equation (12a) or the true Jacobi-Hamilton equation. However, it differs from equation (13) also by the sign of the statistical potential. The latter circumstance means that in deducing the author retreated from classical diffusion not only in eliminating friction. By the way, the author pays no attention to the fact that the idea itself of diffusion without friction is already a considerable retreat from classical physics. Mathematical statistics allows to actually generalize the notion of diffusion for the case of friction absence as well. But this only step, despite E.Nelson's opinion, does not lead to quantum mechanics laws. This is easily seen if one considers the kinetic equation given, e.g., by Chandrasekhar/37/ for probability density in phase space. If the coefficient β is taken to be equal to zero, while q is considered to be constant, one obtains the description of inconventional diffusion without friction for which there is diffusion motion in velocity space similar to conventional diffusion, whereas in coordinate space distribution dispersion for free particles is increased proportionally to time cubed.

Such motion does not correspond obviously to free quantum particle motion.

The coefficient q appears to differ from zero only for bound quantum particles.It is not only friction but the peculiarities of perturbation suffered by a quantum particle due to the external field fluctuations that the future statistical theory of hidden micro-object motion should reflect.

G.Comisar/38/, another American physicist, has made an attempt to develop the theory of quantum particle motion causal at short time intervals which due to "false" microparticle interaction with vacuum turns into conventional quantum theory for long time intervals. However, if all the specific properties of the quantum particle could be explained by independent stochastic perturbations of vacuum violating causal motion according to classical laws, our solution (9) would provide for any quantum state the description of hidden microparticle motion in the form of the everywhere positive function of probability density in phase space. In order to clear out the limited possibilities of applying our solution of the inverse problem to quantum systems, it is sufficient to consider a quantum oscillator in an excited state.

By taking the coordinate distribution function for the first excited level of the quantum oscillator $\rho_1(q) = \frac{2}{\sqrt{\pi}} \frac{q^2}{q_0^3} e^{-q^2/q_0^2}$ one obtains the unique solution of equation (9) $F_1(\varepsilon) = \frac{\omega_0}{2\pi} 2\left(\frac{\varepsilon}{E_0} - \frac{1}{2}\right)\frac{1}{E_0} e^{-\varepsilon/E_0}$ and, respectively, the energy distribution which has the mean energy coinciding with the total energy of the oscillator in this state $\overline{\varepsilon} = \int_0^\infty \varepsilon W_1(\varepsilon) d\varepsilon = \frac{3}{2} h\omega_0$. The obtained momentum distribution on the basis of $F_1(\varepsilon)$ also coincides with the quantum-mechanical distribution. However, the obtained function $F_1(\varepsilon)$ does not provide the statistical description in phase space since with $\varepsilon < E_0/2$ it has a negative value and, hence, is not probability density in phase space. The same situation arises in considering other excited states as well. Nevertheless, it is also very important that on the basis of the classical statistical analysis of the quantum - mechanical probability density $\rho_n(q)$ in the coordinate space the function $F_n(\varepsilon)$ can be obtained which allows to calculate the momentum distribution $f_n(p) = |C(p)|^2$ and the values of the total energy $E_n = h\omega_0(n + \frac{1}{2})$ without resorting to the Shroedinger equation. The formal applicability of solution (9) in this case appears to be due to the fulfilment of the main assumption of calculation made which consists in particle motion occurring according to dynamic laws of classical mechanics in the intervals between fluctuation perturbations of vacuum. However, the fact that the obtained solution for the excited state of a quantum oscillator in some phase space region has a negative meaning inevitably means that the specific properties of quantum motion cannot be explained fully taking into account only independent stochastic violations of this motion by the virtual photons of vacuum*). Without considering the physical nature of these violations we can assume,

*) The author of ref./38/ could not find out the insufficiency of his diffusion idea to develop some new interpretation of quantum mechanics since he avoided solving the problem of statistical description of hidden microparticle motion in phase space.

in a general case, a more complex picture of their stochastic motion admitting, for instance, the existence of perturbation independency of micro-object state.

The appearence of the negative values of the function $F_n(\mathcal{E})$ for $n>0$ can be due to the fact that equation (9) determining a single solution with independent stochastic fluctuations of interactions takes into account perturbation correlations with the state of micromotion by the proper subtraction of some states. Indeed, a simple arithmetic addition of distributions (10) with statistical weights $W_n(\mathcal{E})$ cannot give from (5a) the quantum-mechanical distributions $\rho_n(q)$ with n nodal points. The change of sign of statistical weight of addition makes it necessary to subtract distributions corresponding to some value of $\mathcal{E}$, which provides the zeros of the function $\rho_n(q)$. All this means that distribution (10) in the state with the definite value of the hidden energy $\mathcal{E}$ is not fulfilled entirely and, hence, the condition of the independent character of fluctuation perturbations for the excited oscillator does not hold.

Thus, there are all the grounds to expect the solution of a more complicated problem of describing hidden motion in phase space in the most general case taking into account the correlation of the fluctuation perturbations of vacuum. Solving this problem one can proceed from the quantum-mechanical density $\rho_n(q)$ only in the case of real wave function. In a general case of the complex wave function the solution of the problem should originate from the wave function, as only it has the full information on the quantum system state including also information on the fluctuation perturbation correlation. And if stochastic fluctuation perturbations experienced by the quantum system from physical vacuum turn out to be statistically related, i.e., they depend upon the state of microparticle motion, wave processes in vacuum whose generalized information is contained in the wave function, can be a single physical reason for regulating virtual processes of affecting on mechanical systems. The hidden de Broigle waves can correspond to the real physical process in vacuum which is related to the motion of the mechanical micro-object motion and manifesting its existence only by affecting the virtual process of vacuum effects on the same mechanical system. Attention should be paid to the principal difference of this assumption from the earlier discussed ideas of the wave-pilot or an idea of a double solution according to which the wave controls microparticle motion in space and time but not the stochastic process of virtual interaction with vacuum.

It should be mentioned that the specific properties of quantum laws can hardly be fully explained by the effect suffered by a bound microparticle from physical vacuum for the not classical properties of microparticle behaviour are clearly manifested also for free motion described by flat wave superposition. The attempt to interprete this motion also yield negative "probabilities". However, all the statistical analyses of quantum mechanics made up to now in order to analytically determine not measurable probability density in phase space proceed from one insufficiently grounded assumption on measurements in a quantum assembly. In classical statistical physics measurements result in the separation of the initial statistical body into such fragments having certain constant values of measured quantities which in mixing provide a statistical body entirely identical to the initial one. In the statistical analyses of quantum mechanics measurements are also assumed as the separation of the initial statistical body into fragments with constant values of the measurable which are proportional in volume to the probability that the systems of the initial assembly should have the given values of the measurable quantity.This is just the meaning of conventional relations (6a) and (6b). The same fact that the mixed statistical body obtained after measuring turns out in a general case to be not equivalent to the initial one is explained as follows: the very procedure of separation of the initial assembly into systems having the given values of the observable, using macroscopic means,results in varying additional quantities owing to system transition into another state of motion, the so-called eigen-state for the given measurable.

However, in principle, it cannot be excluded that in fact not only additional quantities but the measurables themselves suffer certain changes in the process of transition. Then the statistical distributions of probability densities given by quantum mechanics for various physical quantities should be treated as the probability of transition from one initial state into eigen-states directly not related to the fact that the systems of the initial assembly have the given value of the variable.

Just the wave process in vacuum can be, quite probably, the reason of the fact that in order to determine the probability of quantum system transition from the initial state of motion into an eigen-state one should take into account(for a certain quantity) the phase volume of the initial state corresponding not only to the given eigen-value of the variable but to other eigen-values.

As has been shown in ref./39/ , the necessity of treating measurements in this way results from the statistical analysis of the simplest case of free microparticle motion described by the superposition of flat waves. Attention was paid in the same ref. to the possibility of eliminating negative "probabilities" in describing this motion by introducing states in phase space differently responding to the observation means of additional quantities. Unfortunately, ref./39/ presents only a particular case of this more general approach considered in which, however, condition for normalizing the total probability to unity has been fulfilled.

Nobody has carried out the statistical analysis of quantum mechanics taking into account possible correlations in the interaction of microsystems with vacuum and the possibilities of transition of the initial state into the given eigen-state from any point of phase volume. And just these yet not taken into account specific properties of quantum systems can prove to be an essential addition to the general property of the absence of insulation of quantum systems from the zero oscillations of vacuum. These new additions (new properties) can turn out to be responsible for the above-discussed difference in principle of quantum equations from the classical diffusion equation or that of Brownian particle motion.

6. Conclusion

Of course, only the strict mathematical solution of the problem of describing hidden motion in a general case can confirm or reject the assumption made on the effect of some wave process on virtual process of mechanical system interaction with vacuum and the transitions of these systems into new states of motion. However, the very idea of separating quantum-mechanical object both into the micro-particle itself carrying real energy and capable of performing real (not virtual) effects used in measurements and into the hidden directly not observed wave process in vacuum producing stochastic effects on the micro-particle, the very idea is still the only possibility to uniquely explain without any logical and physical contradiction the interference effects of a single particle. This idea of explanation simply has no competitor if the idea predominating now of refusing from the explanation of interference in general is not taken into account. Discussing the contradiction of conventional attempts to explain two sets of experiments with single photons passed through a semi-transparent mirror N.Bohr writes: "In fact, after a preliminary measurement of the momentum of the diaphragm, we are in principle offered a choice, when an electron or photon has passed through the slit, either to repeat the moment measurement or to control the position of the diaphragm and, thus, to make predictions pertaining to alternative subsequent observations." From the unacceptability of each of the hypotheses for explaining the results of both additional experimental runs N.Bohr arrives at the conclusion of impossibility to anyhow explain photon motion in space after passing through a semi-transparent mirror. Thus, Bohr practically converts the principle of additivity into that of discrimination. However, the separation of an object into a descrete particle going only along one of the two possible ways and into a continuous wave process in vacuum going on both the ways and then statistically affecting the micro-particle allows one to give a single, unambiguous explanation of effects observed. Unfortunately, this possibility of explaining has not been discussed seriously either by

N.Bohr or any other physicists. In O.Frish's paper /4/ (p.382) among other really logically contradictive or physically groundless casual attempts of explanation there is also an idea against which only one inconvincing objection is given on the uselessness of wave process existing independently of a particle. But physicists have already acknowledged vacuum to be specific medium carying infinite energy in the bound form from which we cannot take a single calorie without spending the same amount of energy. Then why should we reject the possibility of the wave process in vacuum bound with energy concealed in vacuum? Of course, the wave function used in quantum mechanics can express only properties common for wave processes related to an assembly of microparticles separated macroscopically.

All the difficulties of interpreting modern quantum theory are due to the undiscovered peculiarities of micro-particle interaction with physical vacuum and of the effect on this interactions and the process of measurement produced by certain wave processes. It is easy to imagine how incomprehensible and formal the theory of precise description of ship motion would be, if it only inobviously took into account the real process of wave generation and the ship hitting its own reflected wave.

Therefore, one should not be surprised with the unusualness of the laws of the modern phenomenological theory of quantum phenomena where one can speak not of a wave in liquid or gas but of the directly unobserved waves of physical vacuum,not taken into account in an obvious way,affecting rather an unclassical form of fluctuation perturbations on micro-objects and on the process of measuring physical quantities defining their motion. Physical theory will become maximally clear and vivid after obvious presenting absolutely different aspects of quantum phenomena. The separation of a micro-object moving in space and time according to the laws of classical mechanics as an indicator of really completely unusual properties of physical vacuum can not only help to understand in full measure a contradictive picture of using classical images for describing not classical properties of the behaviour of the micro-object itself but will also allow to approach in a new way to the investigation of the physical properties of the deeper region of real world. If the statistical substantionation of the phenomenological thermodynamics 'has opened the way' for investigations of the world of "invisible" particles, the analogous substantiation of the phenomenological theory describing the behaviour of these particles will open up new vistas for studying entirely new properties of the directly unobserved universal physical medium - vacuum. The knowledge of these properties can prove extremely necessary for solving the fundamental problems of elementary particle theory.

Acknowledgement

The author is grateful to B.S.Neganov and Yu.I.Ivanshin for interest to the work and numerous valuable discussions.

References*

1. Л.И.Мандельштам. Полное собрание трудов, т.5, 345, АН СССР, I950.
2. В. Гейзенберг. Физика и философия ИЛ.М., II6, I963.
   W.Heisenberg. Physik und Philosophie. Stuttgart, Herzel, 1959.
3. М.Борн. УФН 59, I3I, I956.
   M.Born. Naturwissenschaften, 42, 425, 1955.
4. О.Фриш. УФН, 90, 379, I966.
   O.R.Frisch. Contemporary Phys., 7, 45, 1965.
5. Н.Бор, УФН, 66, 585, I958
   N.Bohr. The Library of Living Philosophers. Albert Einstein: Philosopher-Sciensit, p. 201, 1949.
6. Д.И.Блохинцев. Принципиальные вопросы квантовой механики. Изд. "Наука", Москва, I966.
7. И.Фон Нейман. Математические основы квантовой механики, "Наука" I964.
   Johann Von Neumann. Matematische Grundlagen der Quantenmechanik, Berlin, 1932.
8. П.А.М.Дирак. Принципы квантовой механики. Физматгиз, I960.
   P.A.M.Dirac. Principles of Quantum Mechanics. Oxford, 1958, p.145.
9. Луи де Бройль. Вопросы причинности в квантовой механике. II, ИЛ, I955.
   L. de Broigle. La physique quantique restera-t-elle indeterministe? Paris, 1953, p.1.
10. Ж.-П.Вижье. Вопросы причинности в квантовой механике, I945, ИЛ, I955.
    J.-P. Vigier. La physique quantique restera-t-elle indeterministe? Paris, 1953, p.89.
11. Д.Бом. Вопросы причинности в квантовой механике.34, ИЛ, I955.
    D.Bohm. Phys.Rev., 85, 166, 1952.
12. Л.Яноши. Вопросы причинности в квантовой механике. 289, ИЛ, I955.
    L.Janossy. Acta Physica, 1, 423, 1952.
13. E.Wigner. Physical Review, 40, 749, 1932.
14. Г. Вейль. Теория групп и квантовая механика, ОНТИ, М.-Л., I936.

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* The pages of the references in the text are given to Russian editions.

15. Я.П.Терлецкий. ЖЭТФ, 7, 1290, 1937.

16. D.I.Blokhintsev. J. of Phys. (USSR), 2, 71, 1940.

17. P.A.Dirac. Rev. of Mod. Phys., 17, 195, 1945.

18. Дж.Мойэл. Вопросы причинности в квантовой механике, 208, ИЛ, 1955.
J.E.Moyal. Proc.Cambr.Phil.Soc., 45, 99, 1949.

19. Р.Л.Стратанович. ЖЭТФ, 32, 1483, 1957.

20. Г.В.Рязанов. ЖЭТФ, 35, 121, 1958.

21. Р.Фейман. Вопросы причинности в квантовой механике. 167, ИЛ, 1955.
R.Feynman. Rev.Mod.Phys., 20, 367, 1948.

22. К.В.Никольский. Квантовые процессы, Гостехиздат, 1940.

23. Д.И.Блохинцев. "Основы квантовой механики", "Высшая школа", 1963.

24. Б.Т.Геликман. ЖЭТФ, 17, 830, 1947.

25. В.И.Смирнов. Курс высшей математики, т.2, 246, Гостехиздат, М.-Л., 1952.

26. Д.И.Блохинцев. В сб. "Философские вопросы современной физики", 376, М.,1952.

27. А.А.Соколов, Ю.М.Лоскутов, И.М.Терпов. Квантовая механика, 145, М., 1962.

28. А.А.Соколов, В.С.Туманов. ЖЭТФ, 30, 802, 1956.

29. Э.И.Адирович и М.И.Подгорецкий, ЖЭТФ, 26, 150, 1954.

30. Н.Н.Боголюбов. О некоторых статистических методах в математической физике. Изд. АН СССР, 1945.

31. И.Феньеш. Вопросы причинности в квантовой механике, 244, ИЛ. 1955.
J.Fenyes. Zs.f.Phys., 132, 81, 1952.

32. М.Борн. УФН, 69, 173, 1959.
M.Born. Zeit. f. Physik, 153, 372, 1958.

33. М.А.Леонтович. Статистическая физика. ГТИ, 1944.

34. E.Schroedinger. Berlin, 1931, 144.

35. F.Fuerth. Zeit. f. Phys., 81, 143, 1933.

36. E.Nelson. Phys.Rev., 150, 1079, 1966.

37. С.Чандрасекар. Стохастические проблемы в физике и астрономии. ИЛ. М.1947. (Формула 249)
S.Chandrasekhar. Rev.Mod.Phys., 15, 1, 1947.

38. G.G.Comisar. Phys.Rev., 138, 5B, 1332, 1965.

39. А.А.Тяпкин. Философские проблемы физики элементарных частиц. Изд. АН, М., 1963.

40. В.Гейзенберг. Физика и философия. ИЛ., М., II6. I963.
W.Heisenberg. Physik und Philosophie. Stuttgart, Herzel, 1959.
41. М.Борн. УФН, 59, I3I, I956.
M.Born. Naturwissenschaften, 42, 425, 1955.

Received by Publishing Department
on January 26, 1968