ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E4 - 3624

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

O.Dumitrescu

MANY PARTICLE TRANSFER REACTIONS

1967г

E4 - 3624

O.Dumitrescu*

MANY PARTICLE TRANSFER REACTIONS

* Permanent address: Bucharest University, Chemistry Faculty, Physics Department, Bucharest, Romania.

I. Introduction

The nuclear transfer reactions of the stripping or pick-up types are important because they determine the degree of the occupation of nuclear energy level by nucleons.

In the most of reactions initiated by composite particles such as deuterons, tritons, alpha particles or even complex nuclei, the excitation of compound nucleus is strong. However, some of the low-lying states are not populated strongly by the compound nucleus mode. Such states can be excited with an apreciable probability by incident partial waves of the high angular momentum. In these cases more frequently appear single or few-nucleon transfers.

Few nucleon transfer reactions have been made especially in the light nuclei region because of the small density level. The nuclei which take part in such reactions can be both spherical and deformed. In the region of nonspherical nuclei not many people have analysed this problem.

In this paper we try to give a method for finite range calculations in many particle transfer reactions by using the two-particle residual interaction of the Gaussian type without the spin or isotopic spin part.

2. General Part

In the following we shall discuss the stripping reaction $A(a,b)B$ of a particle a by a target nucleus A. The residual nucleus will be denoted by B and the emerged particle by b. Also we shall denote the captured nucleon system by c. The kinetic energy operator will be represented by the symbol K and the total Hamiltonian by H:

$$H = H_A(\xi_A) + \sum_{i=1}^{a} \left[K(i) + V(\vec{r}_i, \xi_A) \right] + V_a . \tag{1}$$

Here $H_A(\xi_A)$ is the Hamiltonian of the internal motion of the target nucleus considered to be at rest, $V(\vec{r}_i, \xi_A)$ is the nucleon-core interaction potential, where $\vec{r}_i$ is the coordinate for the nucleon i with respect to the center of the core and V_a is the internal interaction potential in the incoming particle .

The Hamiltonian of the final system in which the residual nucleus B and the particle b are at a great distance is

$$H_f = H_B(\xi_B) + H_b(\eta_b) + K_{bB}(\vec{r}_b), \tag{2}$$

where $H_B(\xi_B)$ and $H_b(\eta_b)$ have the same meaning as $H_A(\xi_A)$ and $\vec{r}_b$ is the coordinate of the center of mass of the emerged particle b. Here we make the approximation that the centers of mass ($\vec{r}_A$ and $\vec{r}_B$) of the target and residual nuclei coincide and they are equal to zero. This approximation is good for heavy A and B nuclei with respect to a and b particles. Some of authors take partially into account the difference between $\vec{r}_A$ and $\vec{r}_B$, namely, in the wave functions of the center of mass motion, but the error connected with the internal wave functions given by models may be much more in such a hypothesis.

By definition the transition matrix for the mentioned stripping reaction is

$$T_{ab} = \langle e^{i\vec{k}_b\vec{r}_b} \varphi_b(\eta_b)\varphi_B(\xi_B) | H - H_f | \Psi_i^{(+)} \rangle , \qquad (3)$$

where $\Psi_i^{(+)}$ is the stationary solution for the Hamiltonian H while by φ we have denoted the internal wave functions of the outgoing particle b and final nucleus B, respectively. The relative $B-b$ momentum is represented here by $\hbar \vec{k}_b$.

In the plane wave Born approximation (PWBA) the nucleon-core interaction potential is neglected and the matrix (3) is

$$T_{PWBA} \simeq \langle e^{i\vec{k}_b\vec{r}_b} \varphi_b \varphi_B | V_{bc} | \Psi_i^{(+)} \rangle \qquad (4)$$

with

$$\Psi_i^{(+)} \simeq \varphi_a \varphi_A \exp[i\vec{k}_a\vec{r}_a] , \qquad (5)$$

where φ_a, φ_A and $\vec{k}_a$ have a similar meaning as φ_b, φ_B and $\vec{k}_b$ and

$$V_{bc} = V_a - V_b - V_c . \qquad (6)$$

Taking into account a part from the nucleon-core interaction potential, the so-called optical potential $U_{opt}(\vec{r})$ we can develope the distored wave method (DWBA):

$$T_{DWBA} \simeq \langle \chi^{(-)}(\vec{k}_b,\vec{r}_b) \varphi_b \varphi_B | V_{bc} | \varphi_a \varphi_A \chi^{(+)}(\vec{k}_a,\vec{r}_a)\rangle , \qquad (7)$$

where

$$[K(\vec{r}) + U_{opt}(\vec{r}) - \epsilon] \chi^{(\pm)}(\vec{k},\vec{r}) = 0 \qquad (8)$$

in which ϵ is the energy in the relative motion.

The rest of the nucleon-core interaction potential

$$U_{rez}(\vec{r}_1, \vec{r}_2, \ldots \xi) = \sum_i V(\vec{r}_i, \xi) - U_{opt}(\vec{r}), \tag{9}$$

where $\vec{r}$ is the coordinate of the relative motion can be taken into account developing the generalized distorted wave method (GDWBA) by writing the coupled equations:

$$\begin{aligned}[K(\vec{r}) + U_{opt}(\vec{r}) - \epsilon + \epsilon_{\tau_1} + \epsilon_{\tau_2}]\, \chi^{(\pm)}_{\tau_1\tau_2}(\vec{k}, \vec{r}) = \\ = -\sum_{\tau_1'\tau_2'} \langle \varphi_{\tau_1} \varphi_{\tau_2} | U_{rez} | \varphi_{\tau_1'} \varphi_{\tau_2'} \rangle\, \chi^{(\pm)}_{\tau_1'\tau_2'}(\vec{k}, \vec{r}),\end{aligned} \tag{10}$$

where the symbol 1 is a or b, while 2 may be A or B, respectively.

Thus the matrix (3) becomes:

$$T_{GDWBA} = \sum_{\tau_a\tau_A} \sum_{\tau_b\tau_B} \langle \chi^{(-)}_{\tau_b\tau_B}(\vec{k}_b, \vec{r}_b)\, \varphi_{\tau_b} \varphi_{\tau_B} | V_{bc} | \varphi_{\tau_A} \varphi_{\tau_a}\, \chi^{(+)}_{\tau_a\tau_A}(\vec{k}_a, \vec{r}_a) \rangle. \tag{11}$$

The stationary waves

$$\begin{aligned}\Psi^{(+)} &= \sum_{\tau_a\tau_A} \chi^{(+)}_{\tau_a\tau_A}(\vec{k}_a, \vec{r}_a)\, \varphi_{\tau_a}(\eta_a)\, \varphi_{\tau_A}(\xi_A) \\ \Psi^{(-)} &= \sum_{\tau_b\tau_B} \chi^{(-)}_{\tau_b\tau_B}(\vec{k}_b, \vec{r}_b)\, \varphi_{\tau_b}(\eta_b)\, \varphi_{\tau_B}(\xi_B)\end{aligned} \tag{12}$$

have the following asymptotic behaviour:

$$\begin{aligned}\Psi^{(+)} &\sim \varphi_a\, \varphi_A \exp[\vec{k}_a \vec{r}_a] + \text{outgoing waves} \\ \Psi^{(-)} &\sim \varphi_b\, \varphi_B \exp[\vec{k}_b \vec{r}_b] + \text{incomming waves.}\end{aligned} \tag{13}$$

An example, of applying the formula (11) is given by Iano and Austern /1/ for (d,p) stripping reaction on deformed nuclei (see formula (4.19) from the mentioned paper).

3. Nuclear Model Treatment of the Matrix Element

In the following we shall study the matrix from the sum (11) omitting the signs τ_a, τ_A and τ_b, τ_B :

$$T_{ab} = D \langle \chi^{(-)}(\vec{k}_b, \vec{r}_b)\, \varphi_b\, \varphi_B \,|\, V_{bc} \,|\, \varphi_A\, \varphi_a\, \chi^{(+)}(\vec{k}_a, \vec{r}_a) \rangle , \qquad (14)$$

where $D = [A_a!\, A_B! / A_b!\, A_c!^2\, A_A!\,]^{1/2}$ due to the antisymmetrization /14,15/

a) Shell Model

The shell model wave functions for the nuclei A and B we can write in the following manner:

$$\varphi_A = |\gamma_A J_A M_A\rangle \quad ; \quad \varphi_B = |\gamma_B J_B M_B\rangle , \qquad (15)$$

where J is the total spin of the nuclear state, M is its projection on the z-axis and γ is the rest of the quantum numbers necessary for complete specification of the state.

Because of the physical picture which we have due to the stripping processes we expand the wave function for the residual nucleus in a basis exhibiting the target plus captured system of nucleons:

$$|\gamma_B J_B M_B\rangle = \sum_{\gamma_{A'} J_{A'}} \sum_{\gamma_c J_c} \beta^{\gamma_B}_{\gamma_{A'}\gamma_c}(J_{A'} J_c ; J_B)\, |\gamma_{A'} J_{A'}, \gamma_c J_c ; J_B M_B\rangle , \qquad (16)$$

where $|\gamma_{A'} J_{A'}, \gamma_c J_c ; J_B M_B\rangle$ is the wave function constructed by the vector coupling, the captured system described by C spin orbit states coupled to the given spin, to a target wave function with angular momentum $J_{A'}$:

$$|\gamma_{A'} J_{A'}, \gamma_c J_c \; ; J_B M_B\rangle = \sum_{M_{A'} M_c} C^{J_{A'}\, J_c\, J_B}_{M_{A'}\, M_c\, M_B} \; |\gamma_{A'} J_{A'} M_{A'}\rangle |\gamma_c J_c M_c\rangle . \qquad (17)$$

Other representations that the spin orbit one could have been used. They are all connected by a unitary transformation, and the choice in any situation could be governed by requiring that expansion (16) have a minimum of terms. The β -coefficients represent the degree to which the state of the residual nucleus has the configuration indicated by (17) and they are redated to the spectroscopic factor or relative reduced width of the state.

Inserting the expansions (16) and (17) in (14) we obtain:

$$T_{ab} = D \sum_{\gamma_c J_c} \beta^{\gamma_B}_{\gamma_A \gamma_c} (J_A J_c \; ; J_B) \; C^{J_A\, J_c\, J_B}_{M_A\, M_c\, M_B} \; T(a,c;b), \qquad (18)$$

where

$$T(a,c;b) = \langle \chi^{(-)}(\vec{k}_b, \vec{r}_b) \, \varphi_b \varphi_c \, | \, V_{bc} \, | \, \varphi_a \, \chi^{(+)}(\vec{k}_a, \vec{r}_a) \rangle \qquad (19)$$

and $\varphi_\tau \equiv |\gamma_\tau J_\tau M_\tau\rangle$ is the antisymmetric wave function for corresponding systems a, b or c . Taking into account that the interaction potential V_{bc} is the sum of two- particle potentials (these two particles being one from the system denoted here by b and another from c) and the antisymmetrization property of the internal wave functions we obtain:

$$T(a,c;b) = A_c A_b \, \langle \chi^{(-)}(\vec{k}_b, \vec{r}_b) \, \varphi_b \varphi_c \, / \, V_{ij} \, / \, \varphi_a \, \chi^{(+)}(\vec{k}_a, \vec{r}_a) \rangle , \qquad (20)$$

where $V_{ij} = V(r_{ij})$ i and j being fixed nucleons. Here A_b and A_c are the number of nucleons from the system b and c , respectively, for identical particles.

The antisymmetrized wave functions φ_τ can be expanded on a chosen basis of nonantisymmetrized functions

$$\varphi_\tau = |\gamma_\tau J_\tau M_\tau\rangle = \sum_{x_\tau} d^{\,shell}_{x_\tau} (\gamma_\tau J_\tau) \; |x_\tau J_\tau M_\tau\rangle . \qquad (21)$$

Thus the matrix (20) becomes:

$$T(a,c;b) = A_b A_c \sum_{\varkappa_a} \sum_{\varkappa_b} \sum_{\varkappa_c} d^{shell}_{\varkappa_a}(\gamma_a J_a)\, d^{shell}_{\varkappa_b}(\gamma_b J_b)\, d^{shell}_{\varkappa_c}(\gamma_c J_c)\, t(a,c,b)\,; \qquad (22)$$

where

$$t(a,c,b) = \langle \chi^{(-)}(\vec{k}_b,\vec{r}_b)\, f_b\, f_c \,|V_{ij}|\, f_a\, \chi^{(+)}(\vec{k}_a,\vec{r}_a)\rangle \qquad (23)$$

is the absorption amplitude and

$$f_\tau \equiv |\varkappa_\tau\, J_\tau\, M_\tau\rangle . \qquad (24)$$

b). Model of Axially Deformed Shape

The model wave functions for the nuclei A and B denoted here by ω we can write as /2/:

$$\varphi_\omega \equiv \mathcal{N}_\omega [\, \chi^{q_\omega}_{K_\omega} D^{J_\omega}_{M_\omega K_\omega} + R(0,\pi,0)\, \chi^{q_\omega}_{K_\omega} D^{J_\omega}_{M_\omega K_\omega}\,, \qquad (25)$$

where $\mathcal{N}_\omega = (4\pi)^{-1}\, \hat{J}_\omega\, (1+\delta_{K_\omega,0})^{-\frac{1}{2}}$ with $\hat{J}_\omega = \sqrt{2J_\omega+1}$ $R(0,\pi,0)$ is the operator for finite rotations with the Eulerian angles 0 , π , 0 . $\chi^{q_\omega}_{K_\omega}$ is the wave function of the nonrotational state, where q_ω is the parity and K_ω is the projection of the nuclear angular momentum on the nucleus symmetry axis.

The $D^{J_\omega}_{M_\omega K_\omega}$ functions are the matrix elements of finite rotations and they represent the amplitude for finding the bodyfixed (nuclear) system in the orientation described by the Eulerian angles θ_1 , θ_2 , θ_3

The second term from (25) was obtained for the axial symmetric nucleus by requiring that the state be an eigenfunction of parity and of time reversal. It can be written by using the time reversed $\chi_{K_\omega}^{q_\omega}$ function

$$\chi_{-K_\omega}^{q_\omega} = \mathcal{T}\chi_{K_\omega}^{q_\omega}, \tag{26}$$

where here $\mathcal{T} = i\,\sigma_y \times$ complex conjugation.
Thus

$$R(0,\pi,0)\,\chi_{K_\omega}^{q_\omega}\,D_{M_\omega K_\omega}^{J_\omega} = (-)^{J_\omega - K_\omega}\,q_\omega\,\chi_{-K_\omega}^{q_\omega}\,D_{M_\omega -K_\omega}^{J_\omega} \tag{27}$$

Taking into account the unitarity of the operator $R(0,\pi,0)$ and knowing that $\chi^{(-)}(\vec{k}_b,\vec{r}_b)$, $\chi^{(+)}(\vec{k}_d,\vec{r}_d)$, φ_d, φ_b and V_{bc} do not depend upon the Eulerian angles the matrix element (14) has two terms:

$$T_{db} = T_{K_A} + (-)^{J_A - K_A}\,q_A\,T_{-K_A}, \tag{28}$$

where

$$T_{K_A} = 2D_Z D_N \mathcal{N}_A \mathcal{N}_B \langle \chi^{(-)}(\vec{k}_b,\vec{r}_b)\,\varphi_b\,\chi_{K_B}^{q_B} D_{M_B K_B}^{J_B} | V_{bc} | D_{M_A K_A}^{J_A}\,\chi_{K_A}^{q_A}\,\varphi_d\,\chi^{(+)}(\vec{k}_d,\vec{r}_d)\rangle \tag{29}$$

in which D_Z, D_N are the proton and neutron antisymmetrisation factors like D from (14). Expanding the $\chi_{K_B}^{q_B}$ function in the same manner as (16) we obtain

$$\chi_{K_B}^{q_B} = \sum_{\Omega_1 \ldots \Omega_c}\sum_{K_{A'} q_{A'}} \beta_{\Omega_1 \ldots \Omega_c}(K_{A'} q_{A'}; K_B q_B)\,\chi_{K_{A'}}^{q_{A'}}\,\phi_c, \tag{30}$$

where

$$\phi_c = (Z_c!)^{-\frac{1}{2}}\det\left(\chi_{\Omega_{p_1}}(1)\ldots\chi_{\Omega_{p_{Z_c}}}(Z_c)\right).(N_c!)^{-\frac{1}{2}}\det\left(\chi_{\Omega_{n_1}}(1)\ldots\chi_{\Omega_{n_{N_c}}}(N_c)\right). \tag{31}$$

Using the second quantization notation the β - coefficients are of the form:

$$\beta_{\Omega_1 \ldots \Omega_c}(K_A q_A; K_B q_B) = \left[\binom{N_B}{N_c}\binom{Z_B}{Z_c}\right]^{\frac{1}{2}} \langle X^{q_A}_{K_A} | a_{\Omega_1} \ldots a_{\Omega_c} | X^{q_B}_{K_B} \rangle . \quad (32)$$

Here a_{Ω_i} is the absorption operator for a Nilsson orbital and $N(Z)$ is the neutron (proton) number of the nucleus. These β-coefficients can be evaluated for instance in the framework of the Soloviev's model /3/.

Expanding the Nilsson orbitals from (30) on the spin orbit basis:

$$\phi_c = \sum_{\varkappa_c J_c} d^{BM}_{\varkappa_c J_c} |\varkappa_c J_c K_c\rangle \quad (33)$$

and then inserting the expansion (30) in (29) the matrix element becomes

$$T_{K_A} = 2 \mathcal{N}_A \mathcal{N}_B D_Z D_N A_c A_b \sum_{\Omega_1 \ldots \Omega_c} \beta_{\Omega_1 \ldots \Omega_c}(K_A q_A; K_B q_B) \cdot$$
$$\cdot \sum_{\varkappa_a \varkappa_b} \sum_{\varkappa_c J_c} d^{shell}_{\varkappa_a}(\gamma_a J_a)\, d^{shell}_{\varkappa_b}(\gamma_b J_b)\, d^{BM}_{\varkappa_c J_c} \langle D^{J_B}_{M_B K_B} | D^{J_A}_{M_A K_A} D^{J_c}_{M_c K_c} \rangle t(a,c,b), \quad (34)$$

where $t(a,c,b)$ is given by (23) and

$$\langle D^{J_B}_{M_B K_B} | D^{J_A}_{M_A K_A} D^{J_c}_{M_c K_c} \rangle = \frac{8\pi^2}{\hat{J}_B^2} C^{J_A J_c J_B}_{M_A M_c M_B} C^{J_A J_c J_B}_{K_A K_c K_B} . \quad (35)$$

Thus

$$T_{K_A} = A_b A_c D_Z D_N (\hat{J}_A / \hat{J}_B) \left[(1+\delta_{K_A 0})(1+\delta_{K_B 0})\right]^{-\frac{1}{2}} \cdot$$
$$\cdot \sum_{\Omega_1 \ldots \Omega_c} \sum_{\varkappa_a} \sum_{\varkappa_b} \sum_{\varkappa_c J_c} \beta_{\Omega_1 \ldots \Omega_c}(K_A q_A; K_B q_B)\, d^{shell}_{\varkappa_a}(\gamma_a J_a)\, d^{shell}_{\varkappa_b}(\gamma_b J_b) \cdot \quad (36)$$
$$\cdot\, d^{BM}_{\varkappa_c J_c} C^{J_A J_c J_B}_{M_A M_c M_B} C^{J_A J_c J_B}_{K_A K_c K_B} t(a,c,b) .$$

Here the multiplier $A_c A_b$ has the same meaning as in (20).

4. *The Absorption Amplitude*

As an example we shall study the $A(Li^6, p)B$ stripping reaction. In this case the factor $A_b A_c$ from (20) is equal to 5 and the stripping interaction potential is $V_{ij} = V(r_{65}) = V_{65}$. Now the problem is to evaluate the matrix (23) in which

$$f_b \equiv |\tfrac{1}{2}\,\sigma_p\rangle \tag{37}$$

$$f_c \equiv |L_c S_c; J_c M_c\rangle = \sum_{m_c \sigma_c} C^{L_c\, S_c\, J_c}_{m_c\, \sigma_c\, M_c}\, |L_c m_c\rangle\, |S_c \sigma_c\rangle, \tag{38}$$

where

$$|L_c m_c\rangle = \sum_{m_\alpha m_5} C^{l_\alpha\, l_5\, L_c}_{m_\alpha\, m_5\, m_c}\, \Psi_{l_\alpha m_\alpha}\, |n_5 l_5 m_5(\vec{r}_5)\rangle \tag{39}$$

$$|S_c \sigma_c\rangle = \sum_{\sigma_\alpha \sigma_5} C^{S_\alpha\, \frac{1}{2}\, S_c}_{\sigma_\alpha\, \sigma_5\, \sigma_c}\, \chi_{S_\alpha \sigma_\alpha}\, |\tfrac{1}{2}\,\sigma_5\rangle \tag{40}$$

are the spatial and spin parts of the model wave function of the captured particles, in which $|nlm\ (\vec{r})\rangle$ is the three dimensional isotropic harmonic oscillator wave function and $|\tfrac{1}{2}\,\sigma\rangle$ the one-nucleon spin function.

Consider then the wave functions $\Psi_{l_\alpha m_\alpha}$ and $\chi_{S_\alpha \sigma_\alpha}$ as coupled to a given angular momentum wave functions:

$$\Psi_{l_\alpha m_\alpha} = |n_1 l_1, n_2 l_2\,(l_{12})\, n_3 l_3, n_4 l_4\,(l_{34}); l_\alpha m_\alpha\rangle =$$

$$= \sum_{m_1 m_2} \sum_{m_3 m_4} C^{l_1\, l_2\, l_{12}}_{m_1\, m_2\, m_{12}}\, C^{l_3\, l_4\, l_{34}}_{m_3\, m_4\, m_{34}}\, C^{l_{12}\, l_{34}\, l_\alpha}_{m_{12}\, m_{34}\, m_\alpha} \prod_{i=1}^{4} |n_i l_i m_i(\vec{r}_i)\rangle \tag{41}$$

and

$$\chi_{S_\alpha \sigma_\alpha} = \left| \tfrac{1}{2}\tfrac{1}{2}(s_{12})\tfrac{1}{2}\tfrac{1}{2}(s_{34})\, ; S_\alpha \sigma_\alpha \right\rangle =$$

$$= \sum_{\sigma_1 \sigma_2 \sigma_3 \sigma_4} C^{\frac{1}{2}\frac{1}{2}S_{12}}_{\sigma_1 \sigma_2 \sigma_{12}} C^{\frac{1}{2}\frac{1}{2}S_{34}}_{\sigma_3 \sigma_4 \sigma_{34}} C^{S_{12} S_{34} S_\alpha}_{\sigma_{12} \sigma_{34} \sigma_\alpha} \prod_{i=1}^{4} \left| \tfrac{1}{2} \sigma_i \right\rangle . \qquad (42)$$

The last unknown function from (23) is the internal wave function of Li^6 :

$$f_d = \varphi_\alpha \chi_\alpha \left| 01, 01 ; 00 \right\rangle \left| \tfrac{1}{2} \tfrac{1}{2} ; 1 M_d \right\rangle \qquad (43)$$

taken as a product between the internal alpha particle wave function and the shell model wave function for two p-nucleons coupled to the total orbital angular momentum $L=0$ and the total spin $S=1$. The last two p-nucleons move around the alpha particle center of mass. Thus the stripping picture looks as in fig.1. The antisymmetrical properties of the internal wave function of Li^6 nucleus have been taken into account in the expansion (21). The internal alpha particle wave function, we shall use, is /4/

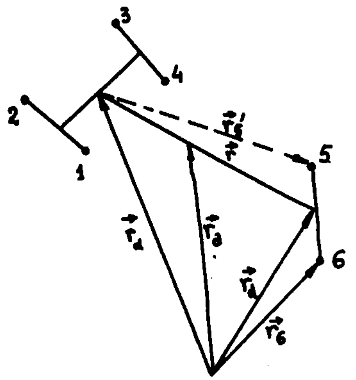

Fig.1.

$$\varphi_\alpha = N_\alpha \exp\left[-\tfrac{1}{2}\beta^2 \sum_{i>j=1}^{4} r_{ij}^2\right], \qquad (44)$$

$$\chi_\alpha = \left|\tfrac{1}{2}\tfrac{1}{2}, 00\right\rangle_p \left|\tfrac{1}{2}\tfrac{1}{2}, 00\right\rangle_n , \qquad (45)$$

where

$$\left|\tfrac{1}{2}\tfrac{1}{2}, 00\right\rangle = \sum_{\sigma\sigma'} C^{\frac{1}{2}\frac{1}{2}0}_{\sigma\sigma' 0} \left|\tfrac{1}{2}\sigma\right\rangle \left|\tfrac{1}{2}\sigma'\right\rangle . \qquad (46)$$

Inserting the expression (31-46) in (23) we select the following integral [5]

$$\langle \psi_{l_\alpha m_\alpha} | \varphi_\alpha \rangle = \left(\tfrac{1}{2}!\right)^{-\frac{3}{2}} \left(\frac{2\alpha\beta}{\alpha^2+\beta^2}\right)^{9/2} \sum_{n_\alpha} \left(\frac{\beta^2-\alpha^2}{\beta^2+\alpha^2}\right)^{\mathcal{N}-n_\alpha} X_{n_\alpha}(n_1 l_1, n_2 l_2, n_3 l_3, n_4 l_4, l_{12} l_{34}) \langle n_\alpha l_\alpha m_\alpha | \tag{47}$$

with

$$2\mathcal{N} = \sum_{i=1}^{4} (2n_i + l_i) - l_\alpha .$$

The last oscillator wave function from (47) depends upon the alpha particle center of mass coordinate defined as:

$$\vec{r}_\alpha = 4^{-1} (\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4). \tag{48}$$

Further it is suitable to expand the DWBA solutions in the following way:

$$\chi^{(\pm)}(\vec{k}_s, \vec{r}_s) = \sum_{nlm} \phi^{*(\pm)}_{nlm}(\vec{k}_s, \alpha_s) \, |nlm(\alpha_s \vec{r}_s)\rangle \exp\left[\tfrac{1}{2}\alpha_s^2 r_s^2\right], \tag{49}$$

where $\alpha_s = \sqrt{m_s\omega/\hbar}$, ω being the frequency of the oscillator.

Bearing in mind the usual expansion for the plane wave function:

$$\exp[i\vec{k}_s \vec{r}_s] = \sum_{nlm} \psi^*_{nlm}(\vec{k}_s, \alpha_s) \, |nlm(\alpha_s \vec{r}_s)\rangle \exp\left[\tfrac{1}{2}\alpha_s^2 r_s^2\right] \tag{50}$$

with

$$\psi^*_{nlm}(\vec{k}_s, \alpha_s) = 4\pi \, i^l \, \alpha_s^{-\frac{3}{2}} \frac{\sqrt{\pi}}{2} \left(\frac{k_s}{2\alpha_s}\right)^{l+2n} \cdot \left[2n!\,(n+l+\tfrac{1}{2})!\right]^{-\frac{1}{2}} \exp\left[-\tfrac{1}{2}\frac{k_s^2}{\alpha_s^2}\right] Y^*_{lm}(\hat{k}_s) \tag{51}$$

we can get easily the expansion coefficients from (49) (see for example ref. [13] formula 5.42)

$$\phi_{nlm}(\vec{k}_s, \alpha_s) = \int d\vec{k}_s' \, \psi^*_{nlm}(\vec{k}_s', \alpha_s) \, \varphi(\vec{k}_s, \vec{k}_s'), \tag{52}$$

where $\varphi(\vec{k}_s, \vec{k}_{s'})$ is the solution of the following integro- differential equations:

$$\left[\frac{\hbar^2 k_s'^2}{2m} - E\right] \varphi(\vec{k}_s, \vec{k}_{s'}) - \int d\vec{k}_s'' \, U_{opt}(\vec{k}_s', \vec{k}_s'') \, \varphi(\vec{k}_s', \vec{k}_s'') = 0 \qquad (53)$$

in which

$$U_{opt}(\vec{k}_s', \vec{k}_s'') = \int d\vec{r}_s \, e^{-i \vec{k}_s' \vec{r}_s} \, U_{opt}(\vec{r}_s) \, e^{i \vec{k}_s'' \vec{r}_s} . \qquad (54)$$

With new obtained expressions (46) and (48) the absorption amplitude becomes:

$$t(a,c,b) = \left(\tfrac{1}{2}!\right)^{-\frac{3}{2}} \left(\frac{2\alpha\beta}{\alpha^2+\beta^2}\right)^{9/2} C^{L_c \ \frac{1}{2} \ J_c}_{m_c \ \sigma_n \ M_c} \, C^{\frac{1}{2} \ \frac{1}{2} \ 1}_{\sigma_p \ \sigma_n \ M_B} \cdot$$

$$\cdot \sum_{n_6 l_6 m_6} \sum_{n_a m_a l_a} \sum_{n_\alpha} \phi^{(-)}_{n_6 l_6 m_6}(\vec{k}_6, \alpha_6) \, \phi^{(+)}_{n_a l_a m_a}(\vec{k}_a, \alpha_a) \cdot \qquad (55)$$

$$\cdot \left(\frac{\beta^2-\alpha^2}{\beta^2+\alpha^2}\right)^{N-n_\alpha} X_{n_\alpha}(n_1 l_1, n_2 l_2, n_3 l_3, n_4 l_4, l_{12}, l_{34}) \; t_{abc}(6,5),$$

where

$$t_{abc}(6,5) = \sum_{m_\alpha m_5} C^{l_\alpha \ l_5 \ L_c}_{m_\alpha \ m_5 \ m_c} \int \langle n_6 l_6 m_6(\vec{r}_6)| \langle n_5 l_5 m_5(\vec{r}_5)| . \qquad (56)$$

$$. \langle n_\alpha l_\alpha m_\alpha(\vec{r}_\alpha)| \, V_{65} \, g(r_6, r_a) \, \varphi_{00} \, | n_a l_a m_a(\vec{r}_a) \rangle$$

with

$$\varphi_{00} \equiv |01,01;00\rangle = \sum_M C^{1\,1\,0}_{M\,-M\,0} \, |0\,1\,M(\vec{r}_5')\rangle \, |0\,1\,-M(\vec{r}_6')\rangle$$
$$\vec{r}_a = 6^{-1} (\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4 + \vec{r}_5 + \vec{r}_6) \qquad (57)$$
$$g(r_6, r_a) = \exp\left[\tfrac{1}{2}\alpha_6^2 r_6^2 + \tfrac{1}{2}\alpha_a^2 r_a^2\right].$$

Here $\vec{r}_5{}'$ and $\vec{r}_6{}'$ are the position vectors of the p-nucleons with respect to the center of the core of the L_i^6 nucleus (the alpha particle center of mass).

By means of three successive transformations we can pass to one kind of coordinates in the integral (56) and then this integral can be easily performed.

These transformations are:

$$\vec{r}_d = \frac{\vec{r}_5 + \vec{r}_6}{2} \qquad \vec{\zeta} = \vec{r}_5 - \vec{r}_6 \tag{60}$$

$$\vec{r} = \frac{\vec{r}_5{}' + \vec{r}_6{}'}{2} \qquad \vec{\zeta} = \vec{r}_5{}' - \vec{r}_6{}' \tag{61}$$

$$\vec{r}_d = \vec{r}_\partial + \frac{2}{3}\vec{r} \qquad \vec{r}_\alpha = \vec{r}_\partial - \frac{1}{3}\vec{r} \,. \tag{62}$$

The transformation brackets of the corresponding oscillator wave functions may be obtained from the following equations

$$|n_5\ell_5, n_6\ell_6; \ell_{56} m_{56}\rangle = \sum_{n\ell NL} \langle n\ell, NL, \ell_{56} | n_5\ell_5, n_6\ell_6, \ell_{56}\rangle |n\ell, NL, \ell_{56} m_{56}\rangle \tag{63}$$

with the selection rule

$$2n_5 + \ell_5 + 2n_6 + \ell_6 = 2n + \ell + 2N + L \tag{64}$$

for the equations (60)

$$|01, 01; 00\rangle = \sum_{\nu n'\ell'} \langle n'\ell', \nu\ell', 0 | 01, 01, 0\rangle \, |n'\ell', \nu\ell'; 00\rangle \tag{65}$$

with the selection rule

$$n' + \nu + \ell' = 1 \,. \tag{66}$$

for the transformation (61) and

$$|\nu\ell', n_a\ell_a, \lambda\mu\rangle = \sum_{N'L'}\sum_{n'_\alpha \ell'_\alpha} \langle N'L', n'_\alpha\ell'_\alpha, \lambda | \nu\ell', n_a\ell_a, \lambda\rangle \; |N'L', n'_\alpha\ell'_\alpha, \lambda\mu\rangle \qquad (67)$$

with the selection rule

$$2N' + L' + 2n'_\alpha + \ell'_\alpha = 2\nu + \ell' + 2n_a + \ell_a \qquad (68)$$

for the last (62) one.

Inserting the equations (63) (65) and (67) in the integral (56) we obtain the matrix

$$t_{abc}(6,5) = \sum_{\substack{l_{56}\\ \lambda}} \sum_{\substack{nNn'N'n_\alpha \\ \ell L \ell' L' \ell_\alpha}} \sum_{\substack{m_\alpha m\\ m'_\alpha M'}} \langle n\ell, NL, l_{56} | n_5\ell_5, n_6\ell_6, l_{56}\rangle \cdot \qquad (69)$$

$$\cdot\, Y_\lambda\begin{pmatrix} 01,01,n_a\ell_a \\ n\ell', N'L', n'_\alpha\ell'_\alpha\end{pmatrix} C^{\ell_\alpha l_5 L_c}_{m_\alpha m_B m_c} C^{l_5 \ell_6 l_{56}}_{m_5 m_6 m_{56}} C^{\ell L l_{56}}_{m M m_{56}} C^{\ell'\ell' 0}_{m' -m' 0} C^{\ell' \ell_a \lambda}_{-m' m_a \mu} C^{L' \ell'_\alpha \lambda}_{M' m'_\alpha \mu} d_{56},$$

where

$$Y_\lambda\begin{pmatrix} 01,01,n_a\ell_a \\ n'\ell', N'L', n'_\alpha\ell'_\alpha\end{pmatrix} = \sum_\nu \langle n'\ell', \nu\ell', 0 | 01,01,0\rangle \langle N'L', n'_\alpha\ell'_\alpha, \lambda | \nu\ell', n_a\ell_a, \lambda\rangle \qquad (70)$$

with the selection rule

$$2n' + \ell' + 2N' + L' + 2n'_\alpha + \ell'_\alpha = 2n_a + \ell_a + 2 \qquad (71)$$

and

$$d_{56} = \int \langle n\ell m(\vec{\gamma})| \langle NLM(\vec{r}_\alpha)| \cdot \langle n_\alpha \ell_\alpha m_\alpha(\vec{r}_\alpha)| \cdot \qquad (72)$$

$$V_{65}(\gamma)\; g(r_6, r_a)\; |n'\ell' m'(\vec{\gamma})\rangle |N'L'M'(\vec{r}_\alpha)\rangle | n'_\alpha \ell'_\alpha m'_\alpha(\vec{r}_\alpha)\rangle .$$

The brackets from (63, 65, 67) are given in ref. /6/. The two-particle stripping interaction $V_{65}(\gamma)$ we shall use, is the central part of the phenomenological Gaussian interaction of Sack et al. /7/, namely

$$V_{65} = -V_0 \exp[-\gamma^2 \eta^2]\,. \tag{73}$$

The function $g(r_6, r_a)$ we can express in new coordinates, as follows:

$$g(r_6, r_a) = \exp\left[\frac{1}{2}\alpha^2\left(\vec{r}_d - \frac{1}{2}\vec{\eta}\right)^2 + \frac{1}{3}\alpha^2\left(\vec{r}_d + 2\vec{r}_\alpha\right)\right]. \tag{74}$$

Taking into account the following integrals [8]:

$$\int d\Omega\, Y^*_{lm}(\Omega)\, Y_{l'm'}(\Omega)\, e^{a\cos\theta} = (-)^m \delta_{mm'} \sqrt{\frac{\pi}{2a}}\, \hat{l}\hat{l}' \sum_L C^{ll'L}_{000}\, C^{ll'L}_{-mm0}\, I_{L+\frac{1}{2}}(a), \tag{75}$$

where $I_s(a)$ is the Bessel function of imaginary argument,

$$\begin{aligned}
&\langle nlm(\vec{r})|\exp[\beta^2 r^2 + \delta\vec{r}\vec{a}]|n'l'm'\rangle = (-)^m \delta_{m,m'}\, \hat{l}\hat{l}' \cdot \\
&\cdot \sum_L C^{ll'L}_{000}\, C^{ll'L}_{-mm0} \sqrt{\frac{2\pi\; n!\; n'!}{(n+l+\frac{1}{2})!\,(n'+l'+\frac{1}{2})!}} \sum_{k=0}^{n} \sum_{k'=0}^{n'} \binom{n+l+\frac{1}{2}}{n-k} \cdot \\
&\cdot \binom{n'+l'+\frac{1}{2}}{n'-k} \frac{(-)^{k+k'}}{(L+\frac{1}{2})!\; k!\,k'!} \left[\tfrac{1}{2}(l+l'+L+2k+2k'+1)\right]! \left(\frac{\alpha_r}{\delta a}\right)^{\frac{3}{2}} \cdot \\
&\cdot \left(1-\frac{\beta^2}{\alpha_r^2}\right)^{-\frac{1}{2}(l+l'+2k+2k'+\frac{3}{2})} \exp\left[\frac{\delta^2 a^2}{8(\alpha_r^2-\beta^2)}\right] M_{-\frac{1}{2}(l+l'+2k+2k'+\frac{3}{2}),\,\frac{1}{2}L+\frac{1}{4}}\left(\delta^2 a^2/4(\alpha_r^2-\beta^2)\right),
\end{aligned} \tag{76}$$

where $M_{\lambda\mu}$ is the Whittaker function [8] and $\alpha_r = \sqrt{m_r \omega/\hbar}$, where m_r is the corresponding mass of the coordinate $\vec{r}$, we can easily perform the integral (72) (see ref. [8] formula 7.622.3). Thus the explicit expression of the absorption amplitude of the (Li^6, p) stripping reaction is:

$$\begin{aligned}
t(a,c,b) = {}& \pi \left(\tfrac{1}{2}!\right)^{-\frac{3}{2}} \left(\frac{2\alpha\beta}{\alpha^2+\beta^2}\right)^{9/2} \left(\frac{288}{343}\right)^{\frac{1}{2}} \left(\frac{\alpha^2}{3\alpha^2+8\gamma^2}\right)^{\frac{1}{4}} \cdot \\
&\cdot \sum_{\substack{n_6 n_a\\ l_6 l_a}} \sum_{\substack{nN\\ lL}} \sum_{\substack{n'N'\\ l' l_{56}}} \sum_{\substack{n_\alpha n_\alpha'\\ \lambda l_\alpha'}} \left(\frac{\beta^2-\alpha^2}{\beta^2+\alpha^2}\right)^{N-n_\alpha} X_{n_\alpha}(n_1 l_1, n_2 l_2, n_3 l_3, n_4 l_4, l_{12}, l_{34}).
\end{aligned} \tag{77}$$

$$\cdot \langle n\ell\, NL\, \ell_{56} | n_5 \ell_5 n_6 \ell_6\, \ell_{56} \rangle \quad Y_\lambda \begin{pmatrix} 01, 01, n_a \ell_a \\ n'\ell', N'L', n'_\alpha \ell'_\alpha \end{pmatrix} .$$

$$\cdot \sum_{L_1 L_2} Q_{L_1 L_2} (n\ell, n'\ell', n_\alpha \ell_\alpha, n'_\alpha \ell'_\alpha, NL, N') \sum_{m_6 m_a} \phi^{(-)}_{n_6 \ell_6 m_6} (\vec{k}_6, \alpha) .$$

$$\cdot \phi^{*(+)}_{n_a \ell_a m_a} (\vec{k}_a, \alpha\sqrt{6}) \quad B_{m_6 m_a} (\ell_5 \ell_6 \ell_a\, \ell \ell' \ell_\alpha \ell'_\alpha L_c L\, \lambda\, \ell_{56})$$

where

$$2\mathcal{N} = \sum_{i=1}^{4} (2n_i + \ell_i) - \ell_\alpha , \tag{78}$$

$X_{n_\alpha} (n_1\ell_1, n_2\ell_2, n_3\ell_3, n_4\ell_4, \ell_{12}, \ell_{34})$ is given in ref./5/ (formulae 74, 78),

$Y_\lambda \begin{pmatrix} 01, 01, n_a \ell_a \\ n'\ell', N'L', n'_\alpha \ell'_\alpha \end{pmatrix}$ is given by expression (70) of this paper

$$Q_{L_1 L_2} (n\ell, n'\ell', n'_\alpha \ell'_\alpha, NL, N') = V_0\, \hat{\ell}\, \hat{\ell}'\, \hat{\ell}_\alpha\, \hat{\ell}'_\alpha .$$

$$\cdot \left[\frac{n!\, n'!\, n_\alpha!\, n'_\alpha!\, N!\, N'!}{(n+\ell+\frac{1}{2})!\, (n'+\ell'+\frac{1}{2})!\, (n_\alpha+\ell_\alpha+\frac{1}{2})!\, (n'_\alpha+\ell'_\alpha+\frac{1}{2})!\, (N+L+\frac{1}{2})!\, (N'+L'+\frac{1}{2})!} \right]^{\frac{1}{2}} .$$

$$\cdot \sum_{k=0}^{n} \sum_{k'=0}^{n'} \sum_{K_\alpha=0}^{n_\alpha} \sum_{K'_\alpha=0}^{n'_\alpha} \sum_{S=0}^{N} \sum_{S'=0}^{N'} C^{\ell \ell' L_1}_{000}\, C^{\ell_\alpha \ell'_\alpha L_2}_{000} \begin{pmatrix} n+\ell+\frac{1}{2} \\ n-k \end{pmatrix} .$$

$$\cdot \begin{pmatrix} n'+\ell'+\frac{1}{2} \\ n'-k' \end{pmatrix} \begin{pmatrix} n_\alpha+\ell_\alpha+\frac{1}{2} \\ n_\alpha-K_\alpha \end{pmatrix} \begin{pmatrix} n'_\alpha+\ell'_\alpha+\frac{1}{2} \\ n'_\alpha-K'_\alpha \end{pmatrix} \begin{pmatrix} N+L+\frac{1}{2} \\ N-S \end{pmatrix} \begin{pmatrix} N'+L+\frac{1}{2} \\ N'-S' \end{pmatrix} .$$

$$\cdot \frac{(-)^{\ell'+K+K'+K_\alpha+K'_\alpha+S+S'} \quad (L+S+S'+\frac{L_1}{2}+\frac{L_2}{2}+\frac{1}{2})!}{K!\, K'!\, K_\alpha!\, K'_\alpha!\, S!\, S'! \quad (L_1+\frac{1}{2})! \quad (L_2+\frac{1}{2})!} .$$

$$\cdot \left[\tfrac{1}{2}(l+l'+2k+2k'+1)!\right]!\ \left[\tfrac{1}{2}(l_\alpha+l_\alpha'+2K_\alpha+2K_\alpha'+1)\right]!\ .$$

$$\cdot \left(\frac{4\alpha^2}{3\alpha^2+8\gamma^2}\right)^{\frac{1}{2}(l+l'+2k+2k')} \left(\frac{3}{2}\right)^{\frac{1}{2}(l_\alpha+l_\alpha'+2k_\alpha+2k_\alpha')}$$

$$\cdot \left(\frac{\alpha^2}{4\left[3\alpha^2+8\gamma^2\right]}\right)^{\frac{L_1}{2}} \left(\frac{12}{7}\right)^{L+S+S'+\frac{L_1}{2}} \left(\frac{1}{7}\right)^{\frac{L_2}{2}} .$$

$$\cdot F\left(L+S+S'+\frac{L_1}{2}+\frac{L_2}{2}+\frac{3}{2};\ \tfrac{1}{2}(l+l'+L_1+2k+2k'+3),\ \tfrac{1}{2}(l_\alpha+l_\alpha'+L_2+2k_\alpha+2k_\alpha'+3);\ \frac{L_1}{2}+\frac{3}{2},\ \frac{L_2}{2}+\frac{3}{2};\ z_1, z_2\right),$$

where $$z_1 = \frac{3\alpha^2}{7(3\alpha^2+8\gamma^2)} \quad \text{and} \quad z_2 = \frac{1}{7} \qquad (79)$$

and

$$B_{m_6 m_d}(l_5 l_6 l_d l l' l_\alpha l_\alpha' L_c L \lambda l_{56}) =$$

$$= \sum_{m\, m_\alpha} (-)^{l'+m_\alpha+1}\ \hat{l}^{-1}\ C^{l l' L}_{-m\, m\, 0}\ C^{l_\alpha l_\alpha' L_2}_{-m_\alpha m_\alpha 0}\ . \qquad (80)$$

$$\cdot C^{l_\alpha l_5 L_c}_{m_\alpha m_5 m_c}\ C^{l_5 l_6 l_{56}}_{m_5 m_6 m_{56}}\ C^{l' l_d \lambda}_{-m' m_d \mu}\ C^{L\, l_\alpha' \lambda}_{M\, m_\alpha \mu}\ .$$

$$\cdot C^{L_c \frac{1}{2} J_c}_{m_c \sigma_n M_c}\ C^{\frac{1}{2}\frac{1}{2} 1}_{\sigma_p \sigma_n M_d}\ .$$

In the expression of $Q_{L_1 L_2}$, $F(\alpha, \beta_1 \beta_2, \gamma_1 \gamma_2; z_1 z_2)$ is the hypergeometric function of two arguments /8/.

The formulae (18), (27), (77) for spherical and (28) (35) (77) for deformed nuclei add to the nuclear structure analysis the finite range effect. The result takes into account the complex structure of the nuclear energy levels and can give a similar information about the nuclear structure as the explanation of the hindrance factors from the alpha decay theory /10/. The numerical calculations for a particular reaction will give the first answer in this direction. Similar results for (α,n) and (α,d) reactions are obtained in refs. /11/ and /12/, respectively.

Acknowledgments are due to Prof. A.Sandulescu, Prof. V.G.Soloviev Dr.V.K.Lukianov and Dr.I.J.Petkov for many constructive discussions and to Prof.V.G.Soloviev for the kind hospitality at the Joint Institute for Nuclear Research in Dubna (USSR).

References:

1. P.J.Iano, N.Austern. Phys.Rev., 151, 853 (1966).
2. A.Bohr, B.R.Mottelson. Mat.Fys.Medd. Dan.Vid.Selsk., 27, No.16 (1953).
3. V.G.Soloviev, Nuclear Physics 69, 1 (1965).
4. R.Hofstadter. Rev.Mod.Phys., 28, 214 (1956).
5. O.Dumitrescu, A.Sandulescu. Rev.Roum.Phys., 10, 77 (1965).
   A.Sandulescu,. Nucl.Phys., 37, 332 (1962).
6. M.Moshinski. Nucl.Phys.., 13, 104 (1959).
   I.Talmi. Helv. Phys. Acta., 25, 185 (1952).
   A.Sandulescu, O.Dumitrescu. Rev. Roum. Phys., 9, 87 (1964).
   Y.F.Smirnov. Nucl. Phys., 27, 177 (1961).
7. S.Sack, L.C.Biedenharn, G.Breit. Phys. Rev., 93, 321 (1954).
8. I.S.Gradstein, I.M.Risik (in russian) Moscow, Fizmatgiz, 1963 .
9. E.T.Whittaker, G.N.Watson. Modern Analysis, 4-th edition, Cambridge, 1946.
10. A.Sandulescu, O.Dumitrescu. Phys. Letters., 19, 404 (1965) and 24B, 212 (1967).
    O.Dumitrescu, A.Sandulescu, Nucl. Phys., A100, 456 (1967).
11. O.Dumitrescu, A.Sandulescu. To be published.
12. I.Brandus, O.Dumitrescu , A.Sandulescu. To be published.
13. A.R.Curtis , Coulomb Wave Functions, University Press, Cambridge 1964.

14. M.L.Goldberger, K.M.Watson "Collison Theory" John Wiley Sons. Inc. N.Y. - London - Sydney, 1964.

15. N.K.Glendenning. Ann. Rev. Nucl. Sci., 13, 191 (1963).

Received by Publishing Department
on December 14, 1967.