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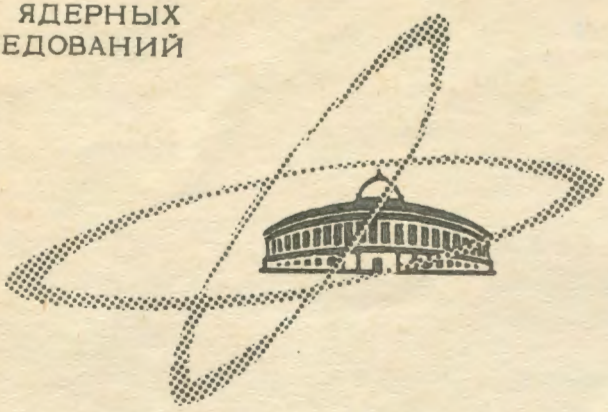

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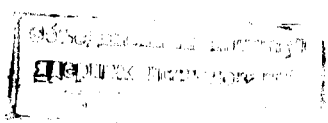

Introduction

The vibrational states of spherical even-even nuclei are usually considered in the framework of the approximate second quantization method. The calculations are performed within the pairing-plus-quadrupole force model[1]. However the energies of the two-phonon states and the relative magnitudes of E2 transition probabilities from them to the first 2^+ and the ground states cannot be explained in this way. Application of the Higher Random Phase Approximation (HRPA) improves the results to a certain degree[2]. But the large values of the static quadrupole moments of the first 2^+ states found in Cd isotopes[3] make this approach to the problem unsatisfactory. Indeed, none of the corrections gives sufficiently large value of Q_2 [4].

The simplest explanation of the large values of Q_2 consists in assuming the wave function of the first 2^+ state to be given by a linear combination of the one- and two-phonon harmonic-vibrational states. It is known that the other anharmonic effects can be explained in this manner[4]. The interaction which mixes the one- and two-phonon states is presented in the pairing plus quadrupole force model. But this interaction has been taken into account only in odd nuclei[5]. The phonon in the harmonic approximation is described by the sum of the diagrams of the type "A" (Fig. 1). The interaction which is used in odd nuclei has the form "B" (Fig. 1). The interaction of the type "B" mixes the one- and two-phonon states and we get the diagrams of the type "C" (Fig. 1).

We are in need of the mathematical apparatus only to take into account this interaction in a sufficiently simple manner. The perturbation theory cannot be used in all cases. Such mathematical apparatus has

been suggested in ref.[6] and used in refs.[7,8]. The idea consists in replacing pairs of fermion operators by an expansion in powers of a pure boson operator. The boson expansions must satisfy the commutation relations for pairs of fermion operators.

I.

The hamiltonian is written in the form:

$$H = \sum_{jm} \epsilon_j \, a^+_{jm} \, a_{jm} - \kappa \sum_{\mu} (-)^{\mu} \hat{Q}_{2\mu} \hat{Q}_{2-\mu}$$

$$\hat{Q}_{2\mu} = \hat{Q}^{\circ}_{2\mu} + \hat{Q}^{s}_{2\mu}$$

$$\hat{Q}^{\circ}_{2\mu} = \sum_{\substack{jj' \\ mm'}} u_{jj'} (-)^{j'-\frac{1}{2}} (C^{2\mu}_{jm\, j'm'} a^+_{jm} a^+_{j'm'} + (-)^{\mu} C^{2-\mu}_{jm\, j'm'} a_{j'm'} a_{jm})$$

$$\hat{Q}^{s}_{2\mu} = \sum_{\substack{jj' \\ mm'}} v_{jj'} (-)^{j'-\frac{1}{2}} \sqrt{2j'+1} \; C^{jm}_{j'm'\, 2\mu} \, a^+_{jm} a_{j'm'}$$

$$u_{jj'} = \frac{1}{2} \sqrt{\frac{(2j+1)(2j'+1)}{5}} (u_j v_{j'} + u_{j'} v_j) q_{jj'}$$

$$v_{jj'} = (u_j u_{j'} - v_j v_{j'}) q_{jj'}$$

$$q_{jj'} = -\frac{1}{2\sqrt{\pi}} < r^2 > C^{20}_{j'-\frac{1}{2}\, j\frac{1}{2}}$$

ϵ_j is the quasiparticle energy, u_j, v_j are the coefficients of the $u-v$ transformation, κ is the quadrupole-quadrupole interaction constant, $C^{2\mu}_{jm\, j'm'}$ is the Clebsh-Gordan coefficient. In the harmonic approximation the phonon operator is written[5]:

$$b^+_{\mu} = \frac{1}{2} \sum_{\substack{jj' \\ mm'}} (\psi_{jj'} C^{2\mu}_{jm\, j'm'} a^+_{jm} a^+_{j'm'} - \phi_{jj'} (-)^{\mu} C^{2-\mu}_{jm\, j'm'} a_{j'm'} a_{jm}),$$

where

$$\psi_{jj'} = \frac{u_{jj'} (-)^{j'-\frac{1}{2}}}{\sqrt{Y}(\epsilon_{jj'} - h\omega)} \qquad \phi_{jj'} = \frac{u_{jj'} (-)^{j'-\frac{1}{2}}}{\sqrt{Y}(\epsilon_{jj'} + h\omega)}$$

$$Y = 2h\omega \sum_{jj'} \frac{\epsilon_{jj'} u^2_{jj'}}{(\epsilon^2_{jj'} - h^2\omega^2)^2}$$

$h\omega$ is the first root of the secular equation

$$1 = 8\kappa \sum_{jj'} \frac{\epsilon_{jj'} u^2_{jj'}}{\epsilon^2_{jj'} - h^2\omega^2} . \qquad (1)$$

Besides we suppose that

$$[b_{\mu}, b^+_{\mu'}] = \delta_{\mu\mu'} .$$

Then

$$\tilde{H} = \sum_{jm} \epsilon_j \, a^+_{jm} a_{jm} - \kappa \sum_{\mu} (-)^{\mu} \hat{Q}^{\circ}_{2\mu} \hat{Q}^{\circ}_{2-\mu} = h\omega \sum_{\mu} b^+_{\mu} b_{\mu}$$

$$\hat{Q}^{\circ}_{2\mu} = \frac{1}{4\kappa\sqrt{Y}} (b^+_{\mu} + (-)^{\mu} b_{-\mu}) .$$

In this approximation we did not take into account the diagrams which mix the one and two-phonon states. These diagrams are contained in the omitted terms of the hamiltonian:

$$-\kappa \sum_{\mu} (-)^{\mu} (\hat{Q}^{\circ}_{2\mu} \hat{Q}^{s}_{2-\mu} + \hat{Q}^{s}_{2\mu} \hat{Q}^{\circ}_{2-\mu}) .$$

In order to take them into account, we must replace $\hat{Q}^{*}_{2\mu}$ by a boson expansion. In the first approximation

$$[\hat{Q}^{*}_{2\mu}, b^{+}_{\mu'}] = K C^{2\mu+\mu'}_{2\mu' 2\mu} b^{+}_{\mu+\mu'} + \frac{1}{2} L (-)^{\mu} C^{2\mu}_{2\mu-\mu' 2\mu'} b_{\mu-\mu'} \qquad (2)$$

where

$$K = 2Y^{-1} \sum_{jj'j_1} (-)^{j+j'+j_1-\frac{1}{2}} v_{jj'} \sqrt{5(2j+1)(2j'+1)} \begin{Bmatrix} j' & j & 2 \\ 2 & 2 & j_1 \end{Bmatrix} u_{jj_1} u_{j'j_1} \times$$

$$\times \frac{\epsilon_{jj_1} \epsilon_{j'j_1} + \hbar^2\omega^2}{(\epsilon^2_{jj_1} - \hbar^2\omega^2)(\epsilon^2_{j'j_1} - \hbar^2\omega^2)}$$

$$L = Y^{-1} \sum_{jj'j_1} (-)^{j+j'+j_1-\frac{1}{2}} v_{jj'} \sqrt{5(2j+1)(2j'+1)} \begin{Bmatrix} j' & j & 2 \\ 2 & 2 & j_1 \end{Bmatrix} u_{jj_1} u_{j'j_1} \times$$

$$\times \frac{\epsilon_{jj_1} \epsilon_{j'j_1} - \hbar^2\omega^2}{(\epsilon^2_{jj_1} - \hbar^2\omega^2)(\epsilon^2_{j'j_1} - \hbar^2\omega^2)}$$

Since

$$[b_{\mu}, b^{+}_{\mu'}] = \delta_{\mu\mu'}$$

$$\hat{Q}^{*}_{2\mu} = \sum_{\nu\nu'} K C^{2\nu}_{2\nu' 2\mu} b^{+}_{\nu} b_{\nu'} + \sum_{\nu\nu'} L (C^{2\mu}_{2\nu 2\nu'} b^{+}_{\nu} b^{+}_{\nu'} + (-)^{\mu} C^{2-\mu}_{2\nu 2\nu'} b_{\nu'} b_{\nu}).$$

Substituting (2) to the hamiltonian and preserving the most important terms we get

$$H = \hbar\omega \sum_{\mu} b^{+}_{\mu} b_{\mu} - \frac{K+L}{2\sqrt{Y}} \sum_{\mu\nu'} (C^{2\mu}_{2\nu 2\nu'} b^{+}_{\nu} b^{+}_{\nu'} b_{\mu} + C^{2\mu}_{2\nu 2\nu'} b^{+}_{\mu} b_{\nu'} b_{\nu}) -$$

$$- \frac{L}{2\sqrt{Y}} \sum_{\mu\nu\nu'} (-)^{\mu} (C^{2-\mu}_{2\nu 2\nu'} b^{+}_{\mu} b^{+}_{\nu} b^{+}_{\nu'} + C^{2\mu}_{2\nu 2\nu'} b_{\nu} b_{\nu'} b_{-\mu}).$$

Thus, we have the hamiltonian with the anharmonic terms. In contrast to the phenomenological models [9] all the parameters of the hamiltonian are fixed in the harmonic approximation. Then we can check how the model, which has a wide applicability in explaining the first 2^{+} states, describes the anharmonic effects.

The interaction hamiltonian mixes states with different number of phonons. Therefore we must know how many phonons are to be taken into account. It depends on the magnitudes of K and L formally. But a lot of other anharmonic terms has not been taken into account. That is why the calculations are performed in the first order of perturbation theory.

Only the quantities connected with first and second 2^{+} states are calculated in this paper.

II.

The calculations were made with the parameters used in ref. [5]. We assume that $\kappa = k \cdot A^{-\frac{5}{6}}$. The constant k is chosen to obtain the best agreement of the calculated energies for the first 2^{+} states with the corresponding experimental data. The results of the calculations are given in Tables I, II, III. The experimental data are taken from ref. [10].

As is seen from Tables I, II, III it is impossible to use the same value of k for all nuclei. But it is interesting that in Ru and Pd

isotopes where the neutron shell only begin to fill k decrease with the mass number. In Cd isotopes where the neutron shell is half-filled k is a constant. In Te isotopes where the neutron shell is almost filled k increases with the mass number.

In order to clear up how strongly the results of calculations depend on the change in the single-particle energy levels the calculations were performed with the different schemes of levels. The obtained values of Q_2 differ little from those calculated with the scheme of single-particle levels used in ref.[5] with the exception of Q_2 in ^{102}Ru, ^{116}Cd, ^{130}Te.

For example, in ^{116}Cd we get:

1) $Q_2 = 0.304$ e barn, if all the parameters are taken from ref.[1].

2) $Q_2 = -0.429$ e barn, if $G_n = 23 A^{-1}$ MeV, but other parameters are the same as in ref.[1].

3) $Q_2 = -0.784$ e barn, if the following neutron scheme of levels is used:

$E(d_{5/2}) = 0.00$ MeV $E(s_{1/2}) = 1.42$ MeV $E(g_{7/2}) = 1.48$ MeV

$E(h_{11/2}) = 2.60$ MeV $E(d_{3/2}) = 3.08$ MeV

and other parameters are the same as in ref.[1]. The experimental value of $Q_{2^+_1}$ in ^{116}Cd is[3] (-0.93 ± 0.26) e.barn.

The value of Q_2 in ^{130}Te changes from 0.050 e barn to 0.450 e.barn for different schemes of levels. Only when the parameters of ref.[11] are used we get $Q_2 = -0.340$ e.barn.

It is impossible to say something about the sign of Q_2 in ^{102}Ru. In this nucleus we get $Q_2 = -0.8$ e.barn for the parameters of ref.[1] instead of $Q_2 = 0.6$ e.barn in Table III.

In consequence, we can say that the static quadrupole moment of the first 2^+ state in ^{98,100}Ru, ^{104,106,108,110}Pd, ^{110,112}Cd is negative and in ^{122,124,126,128,130}Te is positive and decreases with the mass number. (It is possible that in ^{130}Te Q_2 has a negative small value).

The results of ref.[4] also indicate that Te isotopes must have positive values of Q_2.

We get for all nuclei that

$$|Q_2| < |Q_{2'}| < |Q_4| \qquad (3)$$

and

$$\text{sign } Q_2 = -\text{sign } Q_{2'} = \text{sign } Q_4 \qquad (4)$$

It is interesting to compare (3) and (4) with the results of application of the formula[13]

$$Q_s = \frac{3K^2 - I(I+1)}{(I+1)(2I+3)} Q_0 \qquad (5)$$

If we suppose that

2^+ is the state with $K = 0$; $I = 2$

$2'^+$ is the state with $K = 2$; $I = 2$

4^+ is the state with $K = 0$; $I = 4$

and put (6) into (5) we get (4). But K is not a good quantum number in spherical nuclei.

The anharmonic term in hamiltonian has influence on the values of E_2 and $B(E2; 0^+ \to 2^+)$ lowering E_2 below $\hbar\omega$. We have, usually, $\hbar\omega - E_2$ (100-300) KeV. Therefore, we must use the smaller value of κ than in the harmonic approximation. It is the reason for which we get smaller values of $B(E2; 0^+ \to 2^+)$ and better agreement with the experimental data for this quantity, than in ref.[5] (especially for ^{104,106,108,110}Pd and ^{122,124}Te).

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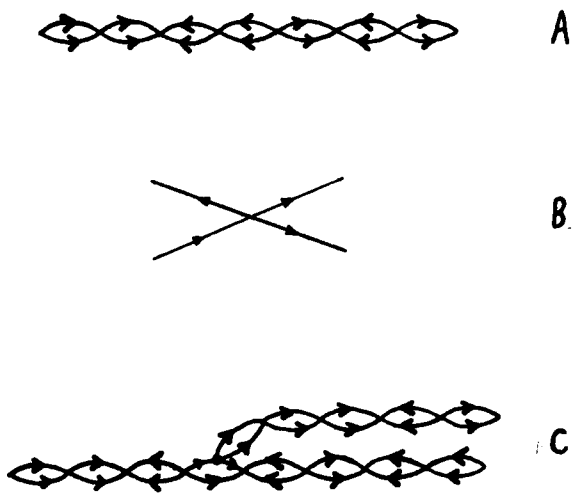

Fig. 1.

Table I

Calculated and experimental values of the energies of the first 2^+ states (E_2) and the B(E2)

Isotope	χ (mev)	$\hbar\omega$ (kev)	E_2 (kev)		$B(E2; 0^+ \to 2^+)$ (e·barn)^2		
			theor	exp	theor	exp [10]	[5]
Ru^{98}	2.5	819	557	654	0.387	0.475	0.563
Ru^{100}	2.5	636	513	543	0.687	0.572	0.947
Ru^{102}	2.35	613	554	474	0.937	0.733	1.424
Pd^{104}	2.20	812	597	556	0.561	0.547	1.006
Pd^{106}	2.05	824	679	512	0.639	0.645	1.261
Pd^{108}	2.0	736	580	434	0.776	0.742	1.603
Pd^{110}	2.0	564	405	374	1.059	0.860	2.009
Pd^{110}	1.95	677	576	374	0.927	0.860	2.009
Cd^{110}	2.1	928	612	656	0.457	0.504	0.687
Cd^{112}	2.1	864	582	615	0.530	0.542	0.758
Cd^{114}	2.1	814	651	558	0.618	0.584	0.799
Cd^{116}	2.1	760	717	513	0.731	0.600	0.809
Te^{122}	2.0	857	541	565	0.584	0.652	1.307
Te^{124}	2.0	811	576	603	0.532	0.610	1.080
Te^{126}	2.0	891	806	668	0.517	0,532	0.729
Te^{128}	2.2	863	825	751	0.479	0.412	0.468
Te^{130}	2.3	1002	986	846	0.319	0,340	0.289
Ba^{130}	I.84	458	368	357	I.78		

Table II

Calculated and experimental values of the quantities

$$\frac{E_{2'}}{E_2}\ ,\quad \frac{B(E2; 2^+ \to 2^+)}{B(E2; 2^+ \to 0^+)}\ ,\quad \frac{B(E2; 2'^+ \to 0^+)}{B(E2; 2^+ \to 0^+)}$$

Isotope	χ (MeV)	$E_{2'}/E_2$		$B(E2; 2' \to 2)/B(E2; 2 \to 0)$			$B(E2; 2' \to 0)/B(E2; 2 \to 0)$		
		theor.	exp.	Theor.	exp [14]	exp [15]	exp [16]	theor	exp [15]	exp [16]
Ru^{98}	2.5	2.89	2.16	0.69	> 0.25			0.26	⩾ 0.010	
Ru^{100}	2.5	2.45	2.52	1.11	0.48± 0.08	0.80 ±0.24		0.19	0.025±0.009	
Ru^{102}	2.35	2.20	2.33	1.44		0.62± 0.19		0.082	0.025±0.007	
Pd^{104}	2.20	2.68	2.41	0.77			2.3	0.199		
Pd^{106}	2.05	2.40	2.20	1.06	1.09± 0.25	1.0± 0.37	I.08; I,55	0.114	0.022±0.008	0.025

Table II. Continued

Pd^{108}	2.0	2.51	2.17	0.95			0.55;0.85	0.170	⩾ 0.010	0.021	
Pd^{110}	2.0	2.75	2.17	0.77			0.76;1.25	0.229	⩾ 0.011	0.021	
Pd^{110}	1.95	2.33		1.19			0.76;1.25	0.124	⩾ 0.011		
Cd^{110}	2.1	2.98	2.25	0.48	1.47±0.29	1.30±0.25		0.220	0.045±0.014		
Cd^{112}	2.1	2.92	2.10	0.54	1.74±0.40	1.50±0.32		0.220	0.019±0.007		
Cd^{114}	2.1	2.47	2.17	0.91		1.23±0.25		0.150	0.015±0.005		
Cd^{116}	2.1	2.11	2.17	1.58	0.33	0.74±0.17		0.380	⩾ 0.020		
Te^{122}	2.0	3.11	2.22	0.52	3.0± 0.7	2.7± 1.2		0.280	0.033±0.015		
Te^{124}	2.0	2.78	2.20	0.70			1.64	0.23		0.025	
Te^{126}	2.0	2.20	2.14	1.39			2.8	0.078		0.0085	
Te^{128}	2.2	2.08	2.03	1.68				0.036			
Te^{130}	2.3	2.03	1.93	1.84				0.013			
Ba^{130}	1.84	2.46	2.58	1.02				0.15			

Table III

Caloulated and experimental values of the quadrupole moments

Isotope	k (MeV)	Q_2 (e·barn) theor.	Q_2 (e·barn) experim.[3]	$Q_{2'}$ (e·barn)	Q_4 (e·barn)
Ru^{98}	2.5	-0.600		0.721	-1.398
Ru^{100}	2.5	-0.662		0.887	-1.609
Ru^{102}	2.35	+0.616		-0.838	1.692
Pd^{104}	2.2	-0.703		0.839	-1.715
Pd^{106}	2.05	-0.657		0.824	-1.687
Pd^{108}	2.0	-0.764		0.952	-1.900
Pd^{110}	2.0	-0.963		1.135	-2.287
Pd^{110}	1.95	-0.735		0.951	-1.914
Cd^{110}	2.1	-0.716		0.764	-1.779
Cd^{112}	2.1	-0.753		0.826	-1.858
Cd^{114}	2.1	-0.702	-0.6±0.2; -0.72±0.18	0.834	-1.811
Cd^{116}	2.1	-0.479	-0.93±0.26	0.593	-1.443
Te^{122}	2.0	0.792		-0.926	1.871
Te^{124}	2.0	0.739		-0.905	1.785
Te^{126}	2.0	0.482		-0.633	1.350
Te^{128}	2.2	0.339		-0.443	1.040
Te^{130}	2.3	0.207		-0.244	0.687
Ba^{130}	I.84	-I.II	-I.I ± 0.3	I.4I	-2.78