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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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SPIN-QUADRUPOLE FORCES AND COLLECTIVE STATES IN DEFORMED NUCLEI. II. 2^+ STATES

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Introduction

The model with pairing plus quadrupole as well as spin-dependent interactions for collective states in deformed nuclei was suggested earlier by the authors/1/. The numerical calculations for 0^+ states were performed within the framework of this model. It was shown, that the model predicts well the energy of the second 0^+ vibrational states. It was found that the spin-quadrupole interactions affect significantly the decay properties of 0^+ states as well.

The present work is devoted to the effect of the spin-quadrupole interactions on properties of 2^+ vibrational states. Preliminary results of this work were reported earlier/2/.

The properties of the gamma-vibrational states were studied in recent years, within the framework of the model with pairing plus quadrupole interactions (see, e.g./3,4/). The numerical calculations, as well as experimental studies (see, e.g./5-8/) have shown that this model gives a satisfactory description of gamma-vibrational states. But the model with pairing plus quadrupole interactions predicts the second 2^+-states around 2 MeV, while they were observed much more lower in a number of nuclei. Thus, the first and the second 2^+ states were found in heavy nuclei at energies/9/:

$$Th^{228} \qquad \omega_1 = 0,969 \text{ MeV}, \quad \omega_2 = 1,15 \text{ MeV};$$

$$Th^{230} \qquad \omega_1 = 0,783 \text{ MeV}, \quad \omega_2 = 1,013 \text{ MeV};$$

U^{234} $\omega_1 = 0,926$ MeV, $\omega_2 = 1,126$ MeV .

In all the nuclei the states indicated lie below the energy gap.

In the present work we will try to answer the next questions:

a) how strongly the spin-quadrupole interactions affect the properties of the gamma-vibrational states ?

b) do they lead to the significant lowering of the second 2^+ -states?

c) how the spin-quadrupole interactions affect the structure of the wave functions of the 2^+-states?

The main equations of the model are given in previous work /1/, so we confine ourselves to the discussion of the details of calculations and the results obtained.

Calculations and Discussion of the Results

The numerical calculations have been carried out for rare-earth nuclei in the region $150 \leq A \leq 174$. The same version of the Nilsson scheme and pairing interaction parameters were used, as in ref. /1/. The blocking effect as well as other corrections connected with the improving of the accuracy of the method were not taken into account.

To determine the quadrupole and spin-quadrupole coupling parameters κ_q and κ_t the particular cases of the equation for the energy of 2^+-states when κ_t or κ_q equal to zero were solved. The coupling parameter values were estimated from empirical energies of the first 2^+ states. The results are given in fig. 1. It turned out that to explain the 2^+ state energies the spin-quadrupole coupling parameter must be chosen about 1,5 time larger than the quadrupole one. One can plot the equipotential line κ_q as function of κ_t for the permanent empirical energy of 2^+ state, if the quadrupole as well as spin-quadrupole interactions are treated at the same time. The point of intersection of such lines corresponds to the best set of coupling parameters κ_q and

κ_t for the whole region of nuclei. The calculated equipotential lines for the first 2^+ states are plotted on fig. 2. It turned out that it is difficult to choose the single set for all the nuclei. The equipotential lines intersected between those for the boundary nuclei Nd^{150} and Yb^{174} A satisfactory fitting to empirical energies can be achieved with $\kappa_t \approx 6$–6.5 and $\kappa_q \approx 4.4$–4.7 (in un. of $A^{-4/3}\hbar\omega_0$). The best agreement for Yb isotopes can be obtained with $\kappa_t \approx 5$. In table 1 calculated energies for lowest 2^+-states as well as lowest two-quasiparticle energies (which are the poles of the function $F(\omega)$ [x]) and empirical data are listed. It is seen that the spin quadrupole interactions affect slightly the energy of the first 2^+ state. In the beginning of the deformation region the energy of the second 2^+ state also affected slightly by the spin-quadrupole interactions. But in Gd, Dy and Er isotopes the second 2^+ state lowered significantly. Unfortunately, there are not practically empirical energies to compare with our predictions.

Calculated E2-transition probabilities $B(E2,\ 0 \rightarrow 2)$ are listed in table 2. As a rule the spin-quadrupole interactions affect slightly the $B(E2)$ value for the first 2^+ state. If only the calculated energy lie near the spin pole /1/ the $B(E2)$ value can be reduced significantly due to the spin-quadrupole interactions.

But the spin-quadrupole interactions affect strongly the E2-transition probability for the next 2^+ states.

In fig. 3 the empirical $B(E2)$ values for the first 2^+ state are compared with the results of our calculations, utilizing the empirical energy and different κ_t values. It is seen that in general the $B(E2)$ value reduced somewhat due to the spin-quadrupole interactions.

Recently the new experimental data on 2^+ states in Yb isotopes were obtained by means of transfer and scattering reactions /6-8/ as well as in beta-decay studies /10/. The detailed calculations of the energy, $B(E2)$ value and neutron two-quasi-particle amplitudes in the wave function for Yb^{172} nucleus are listed in table 3. It turned out that the predictions of the pairing plus quadrupole model with $\kappa_q = 4{,}6$ (which gives the best fitting to experimental energies for all the nuclei of the region)

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[x] For determination see ref. /1/.

are in contradiction with the empirical two-quasiparticle amplitudes/7/. The experimental data indicate the first 2^+ state in Yb^{172} to be preferably of collective nature, while the second 2^+ state contains only small mixtures to the two-neutron amplitude $nn\,[521\downarrow - 512\uparrow]$. The agreement between empirical and predicted by pairing plus quadrupole model amplitudes may be achieved with $\kappa_\sigma = 4.9$. But with such a κ_σ value we obtain too small (and even imaginary for some nuclei) energy for nearly a half of the nuclei of the region. On the other hand all the empirical data can be fitted due to the introduction of the spin-quadrupole interactions. Similar calculations for other Yb isotopes are listed in table 4. In the last column of the table the calculations of Zheleznova et al./3/ are listed, which were performed taking the blocking effect into account. The calculations show that the two-quasi-particle amplitudes may change significantly due to the spin-quadrupole interactions, especially for high-lying 2^+ states. Such a strong effect on energy and amplitudes is due to the increasing role of the mixing term $X(\omega)$ with the increasing of the energy. The typical plots of functions $F(\omega)$, $S(\omega)$, $X(\omega)$ and $P(\omega,\kappa_t)$ are given in fig. 4 for Er^{164} nucleus.

Conclusion

Numerical calculations have shown that the spin-quadrupole interactions affect weakly the energy and $B(E2)$ value for the first 2^+ state (gamma-vibrational state). Nevertheless we obtain a more consistent description of this state, taking the spin-quadrupole interaction into account. Furthermore the introduction of the spin-quadrupole interactions leads to the significant lowering of the energy of the second 2^+ state. The κ_t - transition probability for this state are effected strongly as well as the two-quasi-particle amplitudes in the wave function. But very scarce empirical data about this state do not permit to extract a certain κ_t value.

It seems a very interesting data about spin-quadrupole force could be obtained through the study of odd-even nuclei, as well as from beta and alpha-decay studies of even nuclei.

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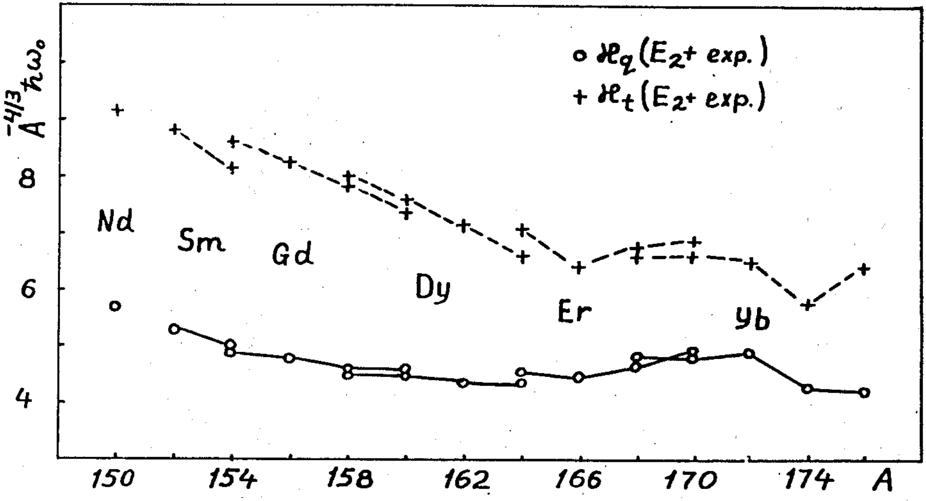

Fig. 1. The quadrupole and spin-quadrupole coupling parameters κ_q and κ_t estimated from empirical energies of the first 2^+ state.

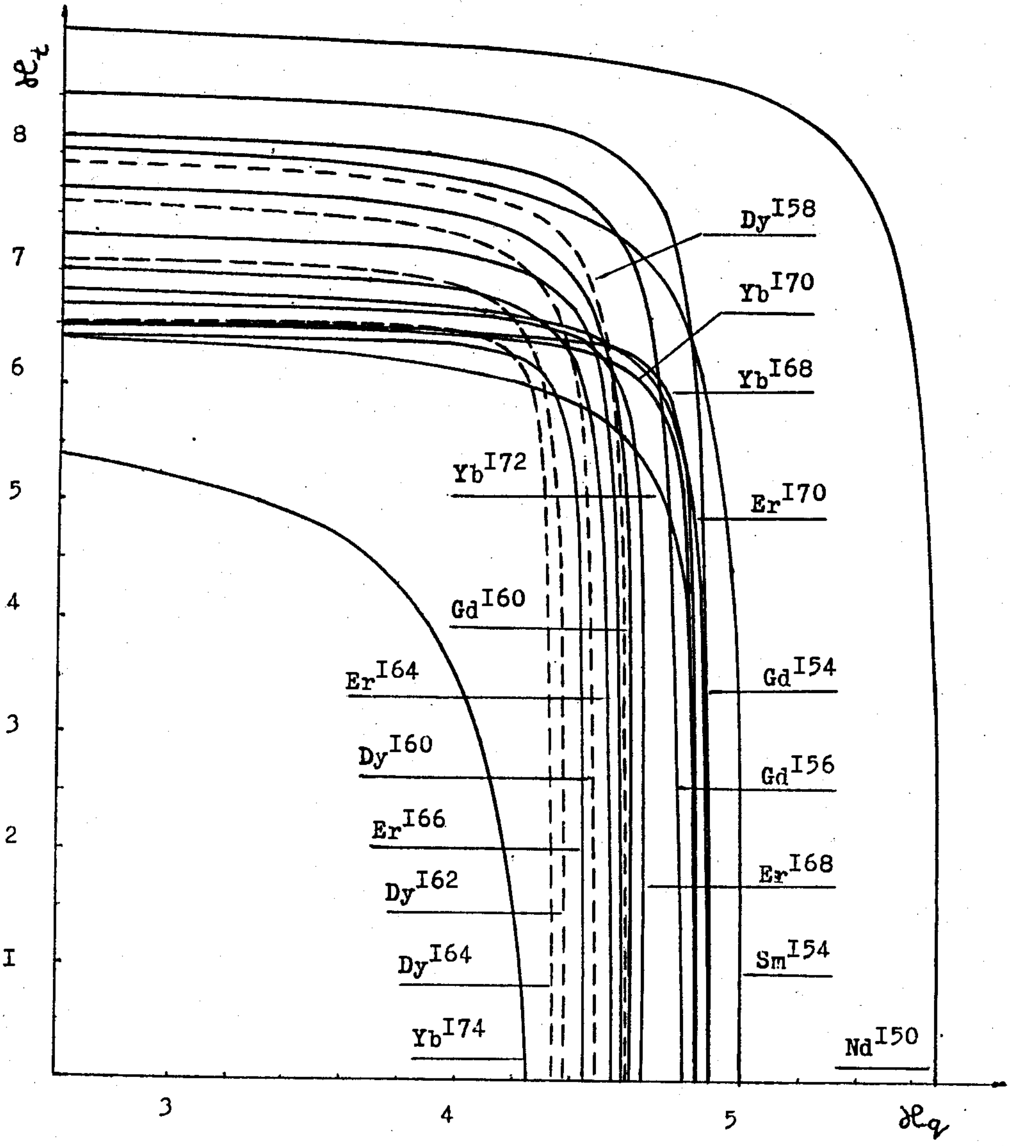

Fig. 2. κ_q as function of κ_t for the permanent energy of the first 2^+ state. The empirical energies are used in calculations.

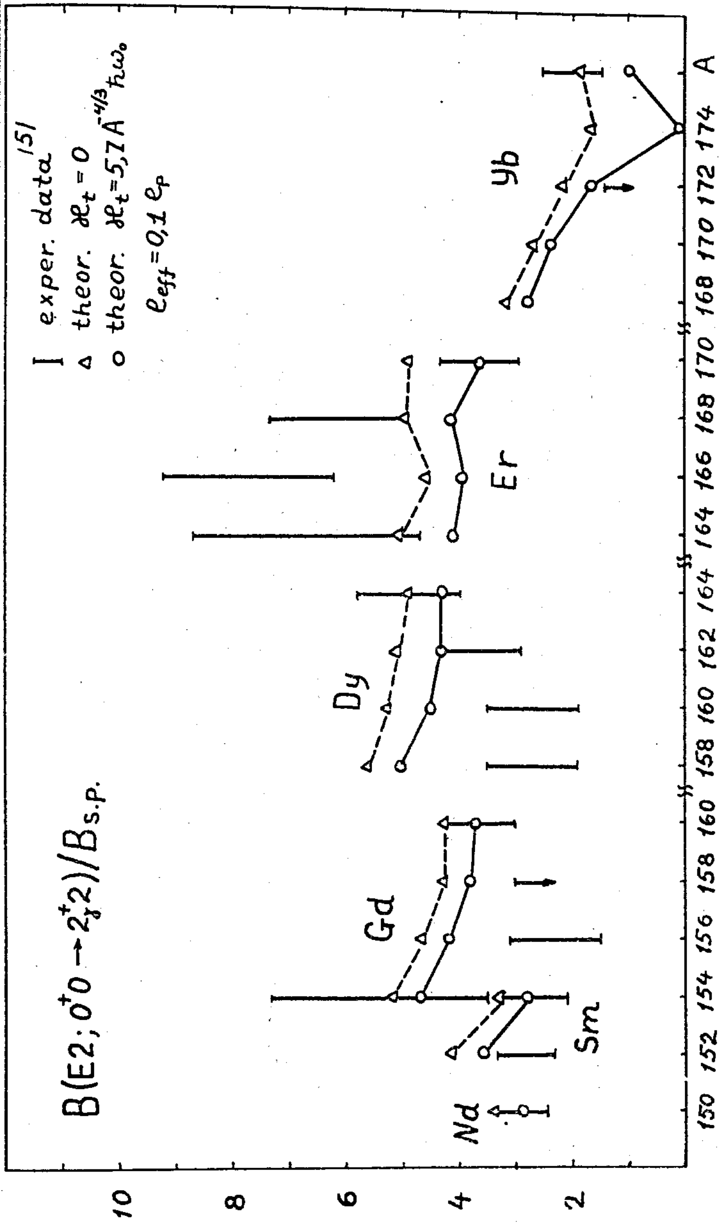

Fig. 3. The reduced E2-transition probability (in single particle un.) for the first 2^+ states. The calculations were performed for the empirical energy and taking the spin-quadrupole interactions into account as well as for pure quadrupole force.

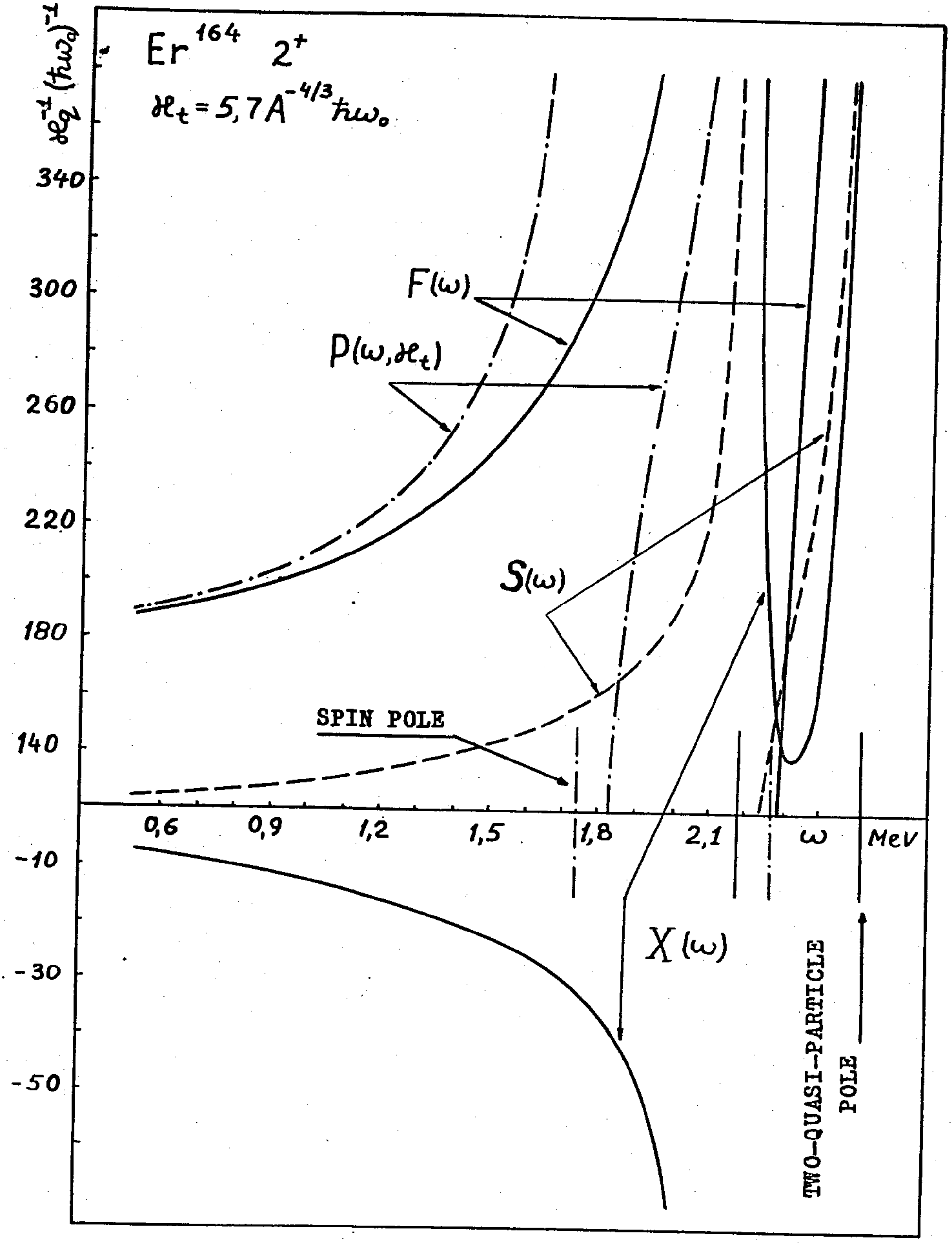

Fig. 4. Typical plots on functions F (ω), S (ω), X (ω) and P (ω, κ_t) for Er^{164} nucleus.

Table 1

2^+ state energies, calculated with different κ_t and κ_q values (in un. of $A^{4/3}\hbar\omega_o$). Experimental data are from the papers [5-7, 10-13]. In the last column of the table the lowest two-quasi-particle energies are listed.

Nucleus	ω_{2^+} exper. MeV		Calculated energy, MeV					First pole of $F(\omega)$
		κ_t	0	6,6	5,4	5,7	6,0	
		κ_q	4,6	0	4,6	4,5	4,5	
Nd^{150}	1.06 –		1.85 2.31	2.12 2.42	1.73 2.24	1.76 2.24	1.74 2.23	2.30
Sm^{152}	1.086 – –		1.67 2.38 2.52	2.14 2.44 –	1.56 2.31 2.52	1.61 2.30 2.52	1.58 2.29 2.52	2.38
Sm^{154}	1.444 –		1.77 2.38	2.13 2.52	1.65 2.30	1.69 2.30	1.67 2.28	2.36
Gd^{154}	0.998 –		1.27 2.46	2.08 2.45	1.21 2.36	1.28 2.32	1.26 2.26	2.43
Gd^{156}	1.156 –		1.32 2.75	2.06 –	1.27 2.36	1.33 2.32	1.31 2.26	2.63
Gd^{158}	1.188 –		1.22 2.82	1.93 –	1.16 2.32	1.21 2.26	1.20 2.18	2.63
Gd^{160}	1.010 –		0.99 2.56	1.59 2.56	0.95 2.08	1.02 2.02	1.01 1.92	2.53
Dy^{158}	0.945 –		0.94 2.46	1.76 2.33	0.90 2.12	0.96 2.08	0.95 2.05	2.25
Dy^{160}	0.966 –		0.85 2.44	1.65 2.31	0.81 2.08	0.89 2.03	0.88 1.96	2.23
Dy^{162}	0.890 –		0.60 2.32	1.33 2.28	0.57 1.92	0.70 1.85	0.69 1.75	2.22
Dy^{164}	0.770 1.987 –		0.31 2.15 2.33	0.77 2.17 2.30	0.30 1.58 2.30	0.53 1.47 2.29	0.52 1.32 2.29	2.11
Er^{164}	0.861 –		0.80 2.30	1.27 2.26	0.75 1.96	0.82 1.87	0.80 1.78	2.17
Er^{166}	0.788 – –		0.59 2.13 2.34	0.64 2.14 2.29	0.56 1.57 2.31	0.66 1.45 2.30	0.64 1.31 2.30	2.10
Er^{168}	0.822 – –		0.89 1.84 2.20	0.99 2.20 2.36	0.83 1.76 1.86	0.90 1.66 1.85	0.87 1.58 1.85	1.83

Table I (continued)

Er^{170}	0.930 – –		1.16 1.62 2.32	1.20 2.24 2.52	1.04 1.60 2.02	1.07 1.60 1.98	1.02 1.58 1.91	1.61
Yb^{168}	0.986 –		1.13 2.24	0.86 2.22	1.09 1.66	1.13 1.56	1.09 1.47	2.10
Yb^{170}	1.220 – –		1.38 1.84 2.37	1.20 1.84 2.36	1.32 1.81 1.90	1.35 1.77 1.86	1.29 1.69 1.85	1.83
Yb^{172}	1.468 1.610		1.57 1.72	1.41 1.64	1.49 1.60	1.47 1.59	1.40 1.59	1.60
Yb^{174}	1.630 – –		1.45 1.97 2.52	1.01 1.98 –	1.17 1.96 2.28	1.14 1.96 2.25	1.05 1.95 2.19	1.96

Table 2

Calculated E2-transition probability for the lowest 2^+ states. The corresponding energies of the states are listed in table 1. The experimental B(E2) values are from the paper 5

Nucleus	B(E2, 0→2)/B(E2)s.p. $e_{eff.} = 0.1$			B(E2) exp .
κ_t	0	5.4	5.7	
κ_q	4.6	4.6	4.5	
Nd^{150}	1.31 6.10^{-6}	1.35 0.02	1.27 0.03	2.9±0.5
Sm^{152}	2.20 4.10^{-4} 0.11	2.17 0.02 0.11	2.0 0.03 0.11	2.8±0.5
Sm^{154}	2.5 0.14	2.4 0.03	2.2 0.05	2.7±0.6
Gd^{154}	3.9 0.13	3.8 0.07	3.5 0.08	5.3±2.0
Gd^{156}	4.0 0.10	3.9 0.04	3.6 0.06	2.3±0.8
Gd^{158}	4.2 0.11	4.0 0.05	3.7 0.08	≤3.0
Gd^{160}	4.4 0.2	4.2 0.03	3.7 0.06	3.6±0.6
Dy^{158}	5.7 0.01	5.4 0.09	4.9 0.12	2.7±0.8
Dy^{160}	6.1 0.01	5.7 0.10	5.0 0.14	2.7±0.8
Dy^{162}	7.8 0.01	7.2 0.03	5.6 0.12	3.6±0.7
Dy^{164}	13.1 0.27 0.01	12.4 5.10^{-4} 0.34	7.0 0.04 0.37	4.9±0.9
Er^{164}	5.6 0.01	5.1 0.09	4.3 0.16	6.7±1.9
Er^{166}	6.6 0.25 5.10^{-3}	6.0 0.01 0.18	4.7 0.08 0.21	7.7± 1.5

Table 2 (continued)

Er^{168}	4.6 7.10^{-3} 0.11	4.3 0.02 0.01	3.7 0.10 0.01	6.2± 1.1
Er^{170}	3.7 0.05 0.02	3.3 3.10^{-6} 0.29	2.9 6.10^{-3} 0.43	3.6± 0.7
Yb^{168}	2.5 0.22	2.5 5.10^{-6}	2.2 0.04	
Yb^{170}	3.5 0.02 0.34	2.3 3.10^{-3} 4.10^{-4}	2.0 0.02 0.02	
Yb^{172}	0.5 1.8	1.9 0.05	1.7 5.10^{-4}	≤1.4
Yb^{174}	2.1 8.10^{-3} 0.23	1.5 0.01 0.6	1.2 0.02 0.7	

Table 3

Calculated energy, E2-transition probability and two-quasiparticle amplitude nn(521↓ − 512↑) in the one-phonon wave function for two low-lying 2^+ vibrationalstates in Yb^{172} nucleus. The calculations are made, using the different quadrupole and spin-quadrupole coupling parameters κ_q and κ_t.

$\varkappa_t(\bar{A}^{4/3}\hbar\omega_0)$ $\kappa_q(\bar{A}^{4/3}\hbar\omega_0)$	0 4.6	0 4.9	5.4 4.4	5.4 4.5	5.4 4.6	5.7 4.5	6.0 4.5	7,10 exp.
$\omega_{2^+}^{(1)}$ MeV	1.57	1.47	1.56	1.53	1.49	1.47	1.40	1.468
$B(E2)_{s.p.u.}$	0.50	2.22	1.35	1.71	1.87	1.66	1.47	≤ 1.4
$\frac{1}{4}(g+w)^2$	0.815	0.237	0.263	0.090	0.051	0.010	$1.7.10^{-3}$	~0.2–0.3
$\omega_{2^+}^{(2)}$ MeV	1.72	1.64	1.60	1.60	1.60	1.59	1.59	1.61
$B(E2)_{s.p.u.}$	1.80	0.60	0.40	0.10	0.05	$0.5 \cdot 10^{-3}$	0.02	
$\frac{1}{4}(g+w)^2$	0.196	0.757	0.715	0.903	0.960	0.980	0.990	0.7–0.8

Table 4

Calculated energy and neutron two-quasiparticle amplitudes $\Psi_{ss'} = \frac{1}{2}(g_{ss'} + w_{ss'})$ for different values of coupling parameters κ_t and κ_q (in un. of $\bar{A}^{4/3}\hbar\omega_0$). The experimental data are given from Burke and Elbek's paper. The calculations of Zheleznova et al.[3] are listed in the last column.

Nucleus	Configuration	$\kappa_t = 0$ $\kappa_q = 4.6$	5.4 4.6	5.7 4.5	Exp.[7] ω_{2^+} and $\lvert\Psi\rvert$	Zheleznoga et al.[3]
Yb^{168}	523↓− 521↓	−.654	−.498	−.400		−.993
	521↓− 512↑	.030	.025	.0.20		.029
	512↑− 510↑	−. 097	−.107	−. 107		−.117
	521↑+ 521↓	−. 528	−.598	−.624		−.678
	Energy, MeV	1.13	1.09	1.13	0.986	1.31
Yb^{170}	523↓− 521↓	−.620	−.447	−.358	0.05÷0.20	−.908
	521↓− 512↑	.124	.079	.062		.224
	512↑− 510↑	−.192	−.221	−.225		−.259
	521↑+ 521↓	−.447	−.500	−.528	~0.53	−.514
	Energy, MeV	1.38	1.32	1.35	1.24	1.59
Yb^{174}	523↓− 521↓	−.128	−0.91	−.050		−.171
	521↓− 512↑	.090	.042	−.023		.075
	512↑− 510↑	−.705	−.738	−.736	~0.56	−.129
	521↑+ 521↓	−.296	−.111	−.109		−.145
	Energy, MeV	1.45	1.17	1.14	1.63	1.54