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I. Introduction

In nuclear theory great success has been achieved in explaining the structure of low-lying nuclear excited states. The success is however impaired to a certain degree by the use of the phenomenological residual interaction operators. This hinders the comparison of the nuclear structure calculations with the results of more fundamental approaches to the finite nucleus problem, and makes it very difficult to connect the nuclear structure data with the properties of the real nucleon-nucleon interaction. The problem of restoring the interparticle forces by the known data on the properties of many-particle systems of extreme difficulty, and one may hardly hope to give its complete solution. In this paper a simple method is proposed to relate the data on nuclear matter properties to the residual interaction which should be used in the nuclear structure calculations. The accuracy of the method suggested can not exceed that of the method used to solve the many-body problem for a given residual interaction. Nevertheless it provides useful information both on the phenomenological residual interaction operators and on the properties of the nuclear single-particle potential.

The residual interaction as used in nuclear spectroscopy calculations is not a clear-cut concept. The ambiguity is introduced by the fact, that in practice for diagonalization of the Hamiltonian matrix one has to reduce its rank. The part of the Hamiltonian which is diagonalized in any particular calculation depends (i) on a choice of the single-particle functions (or the shell model potential) and (ii) on the type of correlations which can be accounted for by the method used to solve the many-body problem. In fact, the empirically determined shell model potential is used and the diagonalization is achieved within the (generalized) Hartree-Fock method. Such methods may in principle be cast in the form of a single-quasiparticle self-consistent problem, and they take into account only the so called "field producing" part of the nucleon-nucleon interaction. The important fact is, however, that even the self-consistent equations obtained in this way can be solved only approximately, and it is at this very point that the residual interaction ap-

appears. This interaction defines the difference between the self-consistent potential and that of the shell model, and does not lead to any configuration mixing if the two potentials coincide.

On the other hand, in the nuclear matter calculations similar methods are used, with the same type of the self-consistency conditions. Thus the obvious place to look for the points of contact between the nuclear structure theory and that of nuclear matter, are the self-consistency conditions for the nuclear potential. Such contacts can really be established, and were used in fact by some authors, e.g. to connect the strength of the quadrupole-quadrupole interaction with the deformability of the shell model potential^{1/}. The purpose of this paper is to analyse the properties of the residual interaction with the use of the information available on the nuclear potential and, in particular, on its density dependence.

In section 2 of this paper the self-consistency conditions are used to introduce the residual interaction. The explicit use is made of the chosen shell model potential. This eliminates the most important part of the above mentioned ambiguous nature of the residual interaction.

In section 3 the explicit form of the residual interaction is obtained for some realistic models giving the density dependence of the single-particle potential in finite nuclei. The pairing correlations, which are supposed to give only a slight correction to the results presented in this section are not taken into account here.

In section 4 the generalized Hartree-Fock method is used to obtain the residual interaction in the particle-particle channel. It is shown that in the absence of pairing correlations this part of the residual interaction reproduces the effective interaction between nucleons in the nuclear matter.

2. The Residual Interaction Derived from the Self-Consistency Conditions

The main features of the methods used in nuclear spectroscopy calculations can be cast into the form of one-body Schroedinger equation. In this equation the one-body Hamiltonian is self-consistent in the sense that it depends on the solutions of the equation itself. The nature of this dependence can be established for each of the specific model approaches to the finite nucleus problem, and will be discussed later on. In the density matrix formalism the equation reads^{2-4/}:

$$H(K)K - KH(K) = i \frac{\partial K}{\partial t} \quad (2.1)$$

Here K is the density matrix, and $H(K)$ is the one-body Hamiltonian. In the

usual Hartree-Fock formulation, which can be used when the pairing correlations are not important, K is just the Dirac's density matrix:

$$K = \rho = \sum_{\nu\nu'} |\nu\rangle \rho_{\nu\nu'} \langle \nu'|, \quad (2.2)$$

where $|\nu\rangle$ is a single-particle state, and the summation is over the complete set of $|\nu\rangle$. The matrix $\rho_{\nu\nu'}$ is Hermitian and satisfies the condition:

$$\rho^2 = \rho, \quad (2.3)$$

i.e. ρ is a projection operator.

In the generalized Hartree-Fock method, in which the pairing correlations can be taken into account, the K matrix assumes the form

$$K = \begin{pmatrix} \rho & X \\ -X^* & 1-\rho^* \end{pmatrix} \quad (2.4)$$

with ρ given by eq. (2.2) and

$$X = \sum_{\nu\nu'} X_{\nu\nu'} |\nu\rangle \langle \nu'| \quad (2.5)$$

$$(X_{\nu\nu'} = -X_{\nu'\nu}^*)$$

Instead of eq. (2.3) one has in this case:

$$K^2 = K = K^\dagger, \quad (2.6)$$

It is interesting to note, that still further generalization of the Hartree-Fock theory achieved in ref.^[5] can also be written in the form^{x)} of eq.(2.1). Finally eq.(2.1) is a somewhat awkward but nevertheless exact form of the Schroedinger equation for N interacting particles if

$$K = \Psi^*(\xi_1, \dots, \xi_N, t) \Psi(\xi_1', \dots, \xi_N', t)$$

($\Psi(\xi_1, \dots, \xi_N, t)$ being the N particle wave function).

Thus eq. (2.1) is general enough to be used in attempt to investigate the main features of the residual interaction in different nuclear models. In fact the two examples of the density matrix K given by eqs. (2.1) (2.4) are sufficient to connect the nuclear matter calculations with those of nuclear spectroscopy.

^{x)} See section 2 of ref.^[5] and particularly eq.(2.30). It is somewhat misleading to refer to the Hamiltonian in eq.(2.30) (ref.^[5]) as an one-body operator, since its matrix includes on the same footing both quasiparticle and collective degrees of freedom. However the use of such name in this paper will not give rise to any misunderstanding because only the models with the K defined as in eqs.(2.2)(2.4) are considered here.

As it was mentioned in the introduction, even when the self-consistency conditions for the one-body Hamiltonian H are known, eq. (2.1) can be solved only approximately. The aim and the nature of the approximations adopted distinguishes between the nuclear matter theory and the nuclear spectroscopy. The first starts from the realistic nucleon-nucleon potential but focuses attention on the gross features of nuclei which essentially involve only the density dependence of H . The nuclear spectroscopy theories try to satisfy the self-consistency by an empirical choice of the model Hamiltonian to explain the large number of the spectroscopic data.

The solution of eq. (2.1) is formulated here in terms of perturbation equations in the difference $H = H(K) - H^0$. The operators in such equations are determined with the aid of the empirical prescriptions for the shell model Hamiltonian H^0 and the data on the density dependence of $H(K)$ provided by the nuclear matter calculations. This procedure is similar to the reference spectrum method of Bethe, Brandow and Petschek^[6]. The solution of eq.(2.1) in the presence of an external perturbing field V is required for many physical problems. For this reason it is assumed that H contains an external field term $e_q V$, where the effective charge e_q is in principle determined within the nuclear matter approach. The introduction of the term $e_q V$ allows to confine oneself to time-independent solution^[3,7] of eq. (2.1).

Let us define by K^0 the solution of the equation:

$$[H^0, K^0] = 0 \quad (2.7)$$

with Hamiltonian H^0 taken from the spectroscopic studies, and referred to as a shell model Hamiltonian^{x/}.

From eq. (2.7) it follows that the two matrices H^0 and K^0 can be diagonalized simultaneously. It is convenient to have a special name for the representation where these matrices are diagonal. In the following this representation is called ν representation. In the time independent case eq. (2.1) has the form:

$$[H(K), K] = 0 \quad (2.8)$$

^{x/} Note, that H^0 operator in eq. (2.7) can be any spectroscopic model operator with or without pairing effects included in it.

Retaining only the terms linear in $\delta K = K - K^0$ one has:

$$[H^0, \delta K] = -[\delta H, K^0]. \quad (2.9)$$

The formal solution of eq. (2.9) may be given in terms of the Green-function two-body matrix g_{12} . The latter satisfies the equation

$$[H_1^0, g_{12}] = I_{12}, \quad (2.10)$$

where I_{12} is the two-body operator, such that

$$\text{tr}_2 I_{12} x_2 = x_1 \quad (2.11)$$

is valid for every one-body operator x . The explicit form of the operators I_{12} and g_{12} are given below for two different models.

If the diagonal matrix elements $\delta H_{\nu\nu} = 0$ in the ν representation the solution of eq.(2.9) assumes the form

$$\delta K_1 = -\text{tr}_2 g_{12} [\delta H_2, K_2^0]. \quad (2.12)$$

The diagonal part of δH can be included into H^0 . This is of particular importance for problems in which a time-dependent external field defines the eigenfrequencies of the system^[3,7].

If H is a known functional of the density matrix the quantity δH can be expressed in terms of δK :

$$\begin{aligned} \delta H_1 = H_1(K^0) - H_1^0 + \\ + \text{tr}_2 \left(\frac{\delta H_1}{\delta K_2} \right)_{K=K^0} \delta K_2' \end{aligned} \quad (2.13)$$

The substitution of eq. (2.13) into eq. (2.12) gives

$$\delta K_1 = \text{tr}_2 \{ [g_{12}, K_2^0] (\delta H_2 + \text{tr}_2' \frac{\delta H_2}{\delta K_2'} \delta K_2') \} \quad (2.14)$$

where

$$\delta H^0 = H(K^0) - H^0 \quad (2.15)$$

To obtain eq. (2.14) the property of traces

$$\text{tr} [b, c] = \text{tr} [c, a] = \text{tr} [a, b] \quad (2.16)$$

has been used. The residual interaction operator is given by

$$F_{12} = \left(\frac{\delta H_1}{\delta K_2} \right)_{K=K^0} \quad (2.17)$$

It will be shown later on that if the matrix K is given by eq.(2.2) or eq.(2.4) and $\delta H^0 = e_q V$ then eq.(2.14) can be reduced to the equations of ref.^[7] describing the polarization of a nucleus by an external field (see in particular eq. (2.28) and eqs. (4.25)-(4.30)). As a rule the right hand side of eq. (2.17) is a non-local and density-dependent operator. With K being defined by eqs. (2.2) and (2.4) it is a two-body operator in the sense that its matrix elements are specified by the set of quantum numbers of a two quasiparticle state. Further, if $\delta H^0 = 0$ the solution of eq.(2.14) is $\delta K = 0$. This means that there is no configuration mixing in this case. The equality $\delta H^0 = 0$ implies that H^0 is a self-consistent Hamiltonian. Thus F_{12} is really a residual interaction operator. It should be noted that the residual interaction operator of eq. (2.17) is meaningful even in this case. It describes the polarization of a nucleus by an arbitrary external field and plays an important role in calculation of both the transition probabilities and collective excitation energies^[7].

To make the above formalism clear it is applied to the Hartree-Fock method without pairing. The density matrix $K = \rho$ is now given by eq. (2.2) and the self-consistency condition reads

$$H = T + U, \quad (2.18)$$

where T is the kinetic energy of a nucleon and

$$U_1 = \text{tr}_2 V_{12}^{\text{A.S.}} \rho_2 \quad (2.19)$$

In the latter $V_{12}^{\text{A.S.}}$ is the antisymmetric part of the inter-nucleon force

$$\langle \nu_1 \nu_2 | V_{12}^{\text{A.S.}} | \nu'_1 \nu'_2 \rangle = \langle \nu_1 \nu_2 | V_{12} | \nu'_1 \nu'_2 \rangle - \langle \nu_1 \nu_2 | V_{12} | \nu'_2 \nu'_1 \rangle \quad (2.20)$$

and tr_2 means summation over indices ν_2, ν'_2 . The shell model operator H^0 is now given by

$$H^0 = T + U^0 \quad (2.21)$$

with empirically chosen (local) potential U^0 .

In the representation where the shell model operator H^0 is diagonal, the solution of eq. (2.7) is given by

$$\begin{aligned} H^0 &= \{ \dots \epsilon_\nu \dots \} \\ \rho^0 &= \{ \dots n_\nu \dots \} \end{aligned} \quad \text{with} \quad n_\nu = \begin{cases} 1 & \nu \leq N \\ 0 & \nu > N \end{cases}, \quad (2.22)$$

where $\{ \dots x_\nu \dots \}$ stands for a diagonal matrix with diagonal matrix elements x_ν . The small variations of ρ admitted by the supplementary condition (2.3) satisfy the equation

$$\delta\rho = (\rho - \rho^0) = (1 - \rho^0)\delta\rho\rho^0 + \rho^0\delta\rho(1 - \rho^0) \quad (2.23)$$

from which it follows

$$\delta\rho_{\nu\nu'} = [(1 - n_\nu)n_{\nu'} + n_\nu(1 - n_{\nu'})]\delta\rho_{\nu\nu'}. \quad (2.24)$$

In eq. (2.24) the term in square brackets vanishes if $\nu, \nu' \leq N$ or $\nu, \nu' > N$ and otherwise is equal to unity. This means that only particle-hole matrix elements of $\delta\rho$ remain in the first approximation where eq. (2.14) is valid.

The operators in eqs. (2.10)(2.11) have the matrix elements

$$[\nu_1 \nu_1'; \nu_2 \nu_2'] = \delta_{\nu_1 \nu_2} \delta_{\nu_1' \nu_2'} \quad (2.25)$$

$$g_{\nu_1 \nu_1'; \nu_2 \nu_2'} = \delta_{\nu_1 \nu_2} \delta_{\nu_1' \nu_2'} \frac{1}{\epsilon_{\nu_1} - \epsilon_{\nu_2'}} \quad (\epsilon_{\nu_1} \neq \epsilon_{\nu_1'}) \quad (2.26)$$

From eqs. (2.22), (2.26) it follows that the commutator $[g_{12}, \rho_2]$ is given by

$$[g_{12}, \rho_2]_{\nu_1 \nu_1'; \nu_2 \nu_2'} = -\delta_{\nu_1 \nu_2} \delta_{\nu_1' \nu_2'} \frac{n_{\nu_1} - n_{\nu_1'}}{\epsilon_{\nu_1} - \epsilon_{\nu_1'}} \quad (2.27)$$

Substituting eq. (2.27) into (2.14) one brings the latter to the form given in ref.^{7/}

$$\delta\rho_{\nu\nu'} = -\frac{n_\nu - n_{\nu'}}{\epsilon_\nu - \epsilon_{\nu'}} \{ (\delta H^0)_{\nu\nu'} + \sum_{\nu_2 \nu_2'} \langle \nu \nu_2 | F | \nu' \nu_2' \rangle \delta\rho_{\nu_2 \nu_2'} \}, \quad (2.28)$$

where in view of eq. (2.24) only the particle-hole matrix elements should be retained.

The self-consistency condition (2.19) implies that the matrix of the residual interaction is given by

$$\langle \nu_1 \nu_2 | F | \nu'_1 \nu'_2 \rangle = \langle \nu_1 \nu_2 | \frac{\delta U_1}{\delta \rho_2} | \nu'_1 \nu'_2 \rangle = \langle \nu_1 \nu_2 | V^{A.S.} | \nu'_1 \nu'_2 \rangle \quad (2.29)$$

Thus in this case the residual interaction reproduces essentially the real inter-nucleon force. This is a direct consequence of the fact, that in eq. (2.19) the real nucleon-nucleon force is used. Application of the real nucleon-nucleon force in the self-consistency condition of the generalized Hartree-Fock method leads to the same result, without restricting it to the particle-hole excitations (see section 4). The point is, however, that the self-consistency condition should properly account for the hard-core part of the nuclear interaction. This is usually achieved by substituting in eq. (2.19) the real inter-nucleon interaction by a density-dependent operator. The techniques of calculating such an operator (reaction matrix) have been developed by Brueckner and Gammel^[8], Moszkowski and Scott^[9] and Bethe, Brandow and Petschek^[6]. Instead of eqs. (2.19), (2.20) one then has

$$U_1 = \text{tr}_2 G_{12}(\rho) \rho_2 \quad (2.30)$$

with a density-dependent reaction matrix $\langle \nu_1 \nu_2 | G(\rho) | \nu'_1 \nu'_2 \rangle$. The residual interaction is now given by

$$F_{12} = G_{12}(\rho^0) + \text{tr}_3 \left(\frac{\delta G_{13}}{\delta \rho_2} \right)_{\rho=\rho^0} \rho_3^0 \quad (2.31)$$

where the density matrix ρ^0 is determined by eqs. (2.7) (2.22) and consequently depends on a particular choice of the shell model potential U^0 .

It is useful to examine the structure of the operator F in the coordinate representation for the case when the reaction matrix G is approximated by a delta-type local operator. In the above notation such an operator can be written in the form

$$\langle \vec{r}_1 \vec{r}_2 | G | \vec{r}'_1 \vec{r}'_2 \rangle = \delta(\vec{r}_1 - \vec{r}'_1) \delta(\vec{r}_2 - \vec{r}'_2) \delta(\vec{r}_1 - \vec{r}_2) \bar{G}(\vec{\rho}) \quad (2.32)$$

with the particle density at the point \vec{r}

$$\bar{\rho}(\vec{r}) = \rho(\vec{r}, \vec{r}) \quad (2.33)$$

In this case the single-particle potential is also local

$$\begin{aligned}
\langle \vec{r}_1 | \bar{U} | \vec{r}_1' \rangle &= \delta(\vec{r}_1 - \vec{r}_1') \bar{\rho}(\vec{r}_1) G(\bar{\rho}(\vec{r}_1)) = \\
&= \delta(\vec{r}_1 - \vec{r}_1') \bar{U}(\bar{\rho}(\vec{r}_1)).
\end{aligned}
\tag{2.34}$$

For the residual interaction of eq. (2.17) one has

$$\langle \vec{r}_1 \vec{r}_2 | \bar{F} | \vec{r}_1' \vec{r}_2' \rangle = \delta(\vec{r}_1 - \vec{r}_1') \delta(\vec{r}_2 - \vec{r}_2') \delta(\vec{r}_1 - \vec{r}_2) \bar{F}(\vec{r}_1), \tag{2.35}$$

where

$$\bar{F}(\vec{r}) = G(\bar{\rho}^0(\vec{r})) + \bar{\rho}^0(\vec{r}) \left(\frac{\partial G}{\partial \bar{\rho}} \right)_{\bar{\rho} = \bar{\rho}^0(\vec{r})}. \tag{2.36}$$

The density distribution is given by

$$\bar{\rho}^0 = \sum_{\nu=1}^N \psi_{\nu}^*(\vec{r}) \psi_{\nu}(\vec{r}), \tag{2.37}$$

where ψ_{ν} stands for the eigenfunction of the shell model Hamiltonian H^0 . The dependence of $\bar{\rho}^0$ on a particular choice of H^0 brings out the model-dependent nature of the residual interaction.

Note that eq. (2.36) can be written in the form

$$\bar{F}(\vec{r}) = \left(\frac{\partial \bar{U}}{\partial \bar{\rho}^2} \right)_{\bar{\rho}^0(\vec{r})} = \left(\frac{\partial^2 \bar{\epsilon}_{\text{pot}}}{\partial \bar{\rho}^2} \right)_{\bar{\rho}^0(\vec{r})}, \tag{2.36'}$$

where $\bar{\epsilon}_{\text{pot}}(\bar{\rho})$ is the potential energy density in the nuclear matter. Eq.(2.36) allows one to calculate the parameters of the residual interaction without any reference to the reaction matrix, provided that $\bar{\epsilon}_{\text{pot}}(\bar{\rho})$ is known.

The second terms in eq. (2.31)(2.36) appreciably affect the residual interaction. It will be shown in section 4, that these terms appear only in the particle-hole part of the residual interaction. The particle-particle and hole-hole parts of it in nuclei without pairing simply reproduce the G operator of eq. (2.30) or (2.32). This in fact, proves the consistency of methods in finite nucleus theory in which the G matrix calculated for nuclear matter of a given density is used.

The appearance of the second term in the expression for the particle-hole interaction has a simple physical interpretation. When a particle is taken out of the system (i.e. when a hole is created) the change in ρ is negligible (it is of the order ρ/A). The particle-hole interaction is the interaction of the particle which

does not belong to the system and thus can be localized in space, with the particles of the system. In the case of short-range forces two particles interact provided that both of them are in a volume which contains only one particle on the average. The local density thus increases twice and the interaction should be considered with $G(2\rho)$ rather than with $G(\rho)$. The expansion of $G(2\rho)$ in powers of ρ gives eq. (2.37).

3. The Residual Delta-Interaction

The discussion of this section is stimulated by the success of the nuclear structure calculations in which the delta-type density-dependent residual interactions are used^{/7,10,11/}. The range of the residual interaction introduced in section 2 is closely connected with the non-locality of the self-consistent potential U . If the potential U is approximated by a function which depends only on the local value of the density ρ , then the residual interaction F of section 2 becomes a local delta-interaction with the density dependence given by

$$F(\vec{r}_1, \vec{r}_2) = \left(\frac{\partial U(\rho)}{\partial \rho} \right)_{\rho=\rho_0} \delta(\vec{r}_1 - \vec{r}_2) \quad (3.1)$$

It is important to note that the range of the interaction considered in this paper has little to do with the commonly used division of the interaction into a long-range and a short-range parts. The residual interaction (3.1) is in fact a pseudopotential and only the direct matrix elements of it are to be taken into account. The delta-function in eq. (3.1) may be expanded in the Legendre polynomial series and thus contains low multipoles which, as is well known, give rise to the correlations at large angular distances (see also reference^{/10/}).

The nonlocality of the potential U is partly caused by the finite range of the nucleon-nucleon interaction and is mainly displayed in the exchange part of the potential. The other reason for the nonlocality is a procedure by means of which the repulsive core is treated. The nonlocality of the potential may be taken into account approximately by adding to the local potential a term which depends on the space derivatives of the density ρ . The surface effects (the density distribution at the surface, the surface energy and so on) are dependent on a choice of such potentials. The calculations performed within the Thomas-Fermi approximation with potentials of this type^{/12,13/} give a good description of such quantities as the fall-off distance in the density distribution and the surface energy,

provided that the Hamiltonian gives correct estimates for the binding energy per particle and the saturation density. Thus one may think, that the local potential (but with a gradient term) is a good approximation for the nuclear self-consistent potential. The method developed in section 2 allows to find the residual delta-interaction which corresponds to this approximation.

Any self-consistent potential must obey the following obvious conditions. It should give simultaneously a correct value of the saturation density and the binding energy per particle. It is useful to examine the consequences of this condition disregarding for a moment the gradient term of the potential. The latter is sensitive mainly to the shape of the density distribution at the nuclear surface and will be discussed later on.

The power expansion of the potential U giving the right saturation properties must contain, in addition to the term linear in ρ at least one more term, e.g. the term ρ^2 with a sufficiently large coefficient and with an opposite sign. If

$$U = ay + \frac{1}{2}by^2, \quad (3.2)$$

where $y = \rho/\rho_0$ (ρ_0 being the nuclear density in the centre of the nucleus), eq. (3.1) gives the residual interaction

$$F = (a + by) \delta(\vec{r}_1 - \vec{r}_2). \quad (3.3)$$

Eq. (3.3) is just the interpolation formula of ref. [7] with $f_{ex} = a$ and $f_{in} - f_{ex} = b$. In the Thomas-Fermi approximation for the kinetic energy the coefficients in eq. (3.2) can be determined from the conditions imposed by the saturation properties. In table 1 the values of f_{in} and f_{ex} are given which correspond to the binding energy per particle $E_0 = 15$ MeV and the saturation density given by the Fermi kinetic energy $T_F = \frac{\hbar^2}{2M} \left(\frac{3\pi^2}{2}\right)^{2/3} \rho^{2/3} = 35$ MeV. The empirical values of these parameters [7] averaged over the spin and isospin quantum numbers are also given for comparison.

The potential (3.2) is very close to that of the semi-empirical model of Wilets [4] which have succeeded in explaining the surface nuclear parameters. The main difference between the present calculations and those of Wilets is that the kinetic energy is included in eq. (3.2) in the Wilets' model and is treated separately here. As a result of this the compressibility coefficient of the nuclear matter increases by about 50 MeV compared to the estimates by Wilets which are already large (302 MeV for the potential closest to eq. (3.2)). The large value of this coefficient is compatible with the strong repulsion between the nucleons inside the nuc-

lets which is displayed by the model and the empirical results of ref.^[7]. Such a repulsion means that much energy is needed to draw together the nucleons localized inside the potential well.

It should be noted that even the rough estimates exhibit the main feature of the empirical density-dependent interaction: the large change in its strength at the nuclear surface. It will be shown later on, that this property of the interaction is not affected by the inclusion of the surface term into the potential.

The spin and isospin dependence of the residual interaction can be determined if that of the nuclear potential is known. To determine this dependence the results of the nuclear matter calculations of ref.^[15] which give good saturation properties are used. The values given in table 1 are obtained by fitting the density dependence of the potential energy of ref.^[15] with expression (3.2). The coefficients a and b for the interaction in the spin-triplet and spin-singlet S -states are determined by the corresponding coefficients in the expression for \mathcal{E}_{pot} . In ref.^[15] the latter quantity is calculated separately for the two possible S -channels.

In the following discussion the two problems are studied: the density dependence of the residual interaction and the role of the surface terms in the potential. The data on the density dependence of the nuclear potential are given in ref.^[13] where the reaction matrix approach by Brueckner is applied to the finite nucleus problem. The additional approximations of ref.^[13] consist in adopting the Thomas-Fermi expression for the kinetic energy and retaining only the first derivatives of ρ . In ref.^[13] the functional dependence on ρ of the Hamiltonian is found and the parameters are fixed by the analysis of the data on the nuclei along the stability line. According to ref.^[13] the density of the potential energy is given by

$$\mathcal{E}_{\text{pot}} = -a_1 \rho^2 + \frac{2^{4/3}}{3} r_0 a_2 \rho^{8/3} + a_3 (\nabla \rho)^2 \quad (3.4)$$

for equal proton and neutron densities. In eq. (3.4) a_1, a_2, a_3 are in general density-dependent parameters related to the reaction matrix, $r_0 = 5.73$ and ∇ stands for the gradient.

The coordinate dependence of ρ is obtained in the Thomas-Fermi approximation by the variational procedure. Varying the density distribution so that to minimize the total energy, one obtains the differential equation for ρ with the binding energy per particle as a Lagrange multiplier. However, the solutions of this differential equation do not decrease exponentially at large distances. The latter property of the solutions is characteristic of quantum mechanics and the lack

of it is due to the break down of the Thomas-Fermi approximation at low densities.

The solutions of ref.^{/13/} obey the following boundary conditions. The density ρ and its derivative vanish at a distance R from the centre of the nucleus and remains equal to zero at larger distances. The quantity R appears to be only somewhat larger than the nucleus radius, and the solutions obtained in such a way give good estimates for the fall-off distance and the binding energy.

It is easily seen, however, that the above solutions can not be used in the present calculation. The boundary condition of ref.^{/13/} determines a priori the density dependence of the nuclear potential: it is equal to zero outside the nucleus, decreases by the binding energy per particle $E \approx 8$ MeV at the point R and differs from this value by the Thomas-Fermi kinetic energy inside the nucleus. Thus one has

$$U = -E_b - T_F y^{2/3} \quad (3.5)$$

The residual interaction is then given by

$$F = -y^{-1/3} \cdot \delta(\vec{r}_1 - \vec{r}_2) \quad (3.6)$$

(the units are the same as in table 1). The residual interaction of eq. (3.6) diverges at the nuclear surface where $y = 0$. In eq. (3.4) the Coulomb energy is not taken into account. Inclusion of the Coulomb energy in the Hamiltonian leads to a certain change in the values of the coefficients in eqs. (3.5), (3.6) but does not remove the divergency exhibited by eq. (3.6).

In order to calculate the residual interaction at the surface one has to avoid the use of the Thomas-Fermi method, or improve the boundary conditions. It is not the subject of this paper to solve this problem. In what follows the contribution of the volume part and surface (i.e. gradient) part of the residual interaction is estimated separately. To this end the latter is written in the form

$$F = \left(\frac{d}{d\rho} \right)^2 \bar{\epsilon}_{\text{pot}}(\rho) \cdot \delta(\vec{r}_1 - \vec{r}_2) \quad (3.7)$$

with $\bar{\epsilon}_{\text{pot}}(\rho)$ defined eq. (3.4). According to^{/13/} the density dependence of the parameters a_1 , a_2 , a_3 in eq. (3.4) is due to the repulsive core contribution to the reaction matrix and is essential only for the quantity a_1 . Using the empirical parameters of ref.^{/13/} and in particular

$$a_1 = 402(1 - 0.24y^{2/3}) \text{ MeV fm}^3 \quad (3.8)$$

one has

$$F_{\text{vol}} = [-5,6 + 5,9\gamma^{2/8}] \delta(\vec{r}_1 - \vec{r}_2) \quad (3,9)$$

In order to take into account the surface term in $\bar{\epsilon}_{\text{pot}}$ one can take the corresponding variational derivative of it. This gives a velocity-dependent delta-interaction, the effects of which depend on the density distribution at the nuclear surface and thus are difficult to be determined. To estimate this term one can guess a proper shape of the density distribution. Assuming it to be given by the Saxon-Woods formula one has

$$F_{\text{surf.}} = \frac{3a_8 \rho_0}{b^2 \cdot T_F} [1 - 6\gamma + 6\gamma^2] \delta(\vec{r}_1 - \vec{r}_2) \quad (3,10)$$

where b is the fall-off parameter of the Saxon-Woods distribution. The latter is surely a good approximation at large distances from the centre of the nucleus, but may lead to considerable errors inside it. The value of a_8 of ref.^[13] can not be used here since it is obtained in the framework of the Thomas-Fermi method while the object of the present discussion is to examine the surface term avoiding the use of this approximation. The value of this coefficient can be determined from the requirement that the residual delta-interaction should give a good approximation to the nucleon-nucleon interaction outside the nucleus. If the latter is assumed to be given by $f_{\text{ex}}^{\text{av}} = -4\delta(\vec{r}_1 - \vec{r}_2)$ one obtains $\frac{3a_8}{b^2} \cdot \frac{\rho_0}{T_F} = 1,6$. Then the residual interaction is

$$F = [-4 + 5,9\gamma^{2/8} + 9,6[\gamma^2 - \gamma]] \delta(\vec{r}_1 - \vec{r}_2) \quad (3,11)$$

It may be noted that the above value of the coefficient a_8 is close to that of ref.^[13]. With $b = 0,55$ fm the above procedure gives $a_8 = 34$ MeV fm⁵ which is close to the lowest limit for a_8 of ref.^[13] ($a_8 = 36$ meV fm⁵). On the other hand, the value of this parameter is proportional to the difference between the mass of a free nucleon and its effective mass in the nucleus, which is too large in the Brueckner approach used in ref.^[13].

The surface term in eq. (3,11) has a deep minimum at $\gamma = \frac{1}{2}$. Because of it the strength of the residual interaction changes at the surface much more rapidly than it follows from the interpolation formula of ref.^[7] or from eq. (3,3). The interaction given by eq. (3,11) is strongly attractive at the surface where the density is large ($\gamma \approx \frac{1}{2}$). This is the main quantitative result which the inclusion of

the surface terms leads to. The strong attraction between the nucleons which is displayed by the surface term in the residual interaction may be important for the nuclear structure calculations. The latter remark is justified by the results of refs.^{/10,11/} where the surface delta-interaction is considered.

The above considerations concern only the particle-hole part of the residual interaction. It was pointed out in section 2 that the residual interaction in the particle-particle and hole-hole channels coincide with the reaction matrix in eq. (2.19) (this will be proved in section 4). It implies a certain change in the above technique for the calculation of the particle-particle (hole-hole) part of the residual delta-interaction considered in this section. To calculate this part of the interaction one can use eq. (3.1) but in this equation the reaction matrix and the parameters depending on it should be considered as constants when the derivative is taken. The density dependence of these parameters should be introduced only in the final expression for F . In the calculations of Kumar et al.^{/13/} the density dependence of the reaction matrix is represented by the density dependence of the parameters a_1, a_2, a_3 in eq. (3.4). By analysing the data on the binding energies and the fall-off distances in the density distribution they found that only a_1 depends strongly on the density ρ while the ρ -dependence of the other two parameters can be neglected. From eq. (3.9) for a_1 it follows that the particle-particle (hole-hole) part of the residual delta interaction is given by

$$F^{\text{part-part}} = \{-4 + 4.3y^{2/3} + 9.6(y^2 - y)\} \delta(\vec{r}_1 - \vec{r}_2) \quad (3.12)$$

The coefficient for the delta-function in eq. (3.12) is very close to zero inside the nucleus where $y = 1$. The interaction (3.12) differs from the particle-hole one by 1.6 inside the nucleus and coincides with it outside the nucleus. For the empirically determined parameters of the residual interaction of ref.^{/7/} such a change means that in contrast to the interaction in the particle-hole channel, the interaction in the particle-particle channel remains attractive everywhere.

4. Residual Interaction in the Particle-Particle Channel

As it was shown in Section 2 it is impossible to get any information on the particle-particle part of the residual interaction if the density matrix K is given by eq. (2.2) and only the linear terms in δK are retained. This is because

the only nonvanishing matrix elements of δK are in this case of the particle-hole type. Such a property of the density matrix (2.2) is closely connected with the fact that the number of particles in the system does not change when the density matrix varies. This remark alone suggests that to obtain the particle-particle part of the residual interaction it is necessary to pass from eq. (2.2) to eq. (2.4) defining the structure of the density matrix in the generalized Hartree-Fock theory. Though the generalization of this type appears to be especially important when the pairing of the superconductive type is strong, the authors confine themselves to the case when no pairing is included in H_0 i.e. when the matrix is taken from eq. (2.4) but the block χ_0 in it is equal to zero. The blocks $\delta\rho$ and $\delta\chi$ of the matrix $\delta K = K - K_0$ in eq. (2.14) are at the same time treated on the same basis, and this constitutes the generalization in the procedure of this section compared to that of section 2.

The equation for K now reads

$$[H(K), K] = 0 \tag{4.1}$$

with K given by eq. (2.4) and

$$H(K) = \begin{pmatrix} h & \mu \\ -\mu^* & -h^* \end{pmatrix}. \tag{4.2}$$

The operators h and μ depend on K according to equations

$$h_1 = T_1 + \text{tr}_2 G_{12} \rho_2 + (e_q V_1)_d \tag{4.3}$$

$$\mu_1 = \text{tr}_2 G_{12} \chi_2 + (e_q V_1)_{\text{off}},$$

where T_1 is the kinetic energy of an i -th particle, from which the chemical potential is subtracted. The operator G is the reaction matrix. For the same reason as in section 2 an external field is included in the self-consistent potential too. The term $(e_q V)_d$ represents a field conserving the number of particles and the term $(e_q V)_{\text{off}}$ describes the changes in energy induced by an extra pair of particles (or holes).

The operator G depends in general on the density distribution and on the correlations between the nucleons in the state determined by the density matrix K . From the discussion of section 3 it follows that the density dependence of G is

essential and strongly affects different properties of nuclei. On the other hand, one cannot expect that the pairing correlations in the nuclear matter produce noticeable effects on G . Indeed, only a small fraction of the nucleons in the single-particle states ν near the top of the Fermi sea contribute to the correlations of this type. From these remarks one may assume that the operator $G = G(\rho)$ depends only on the matrix ρ which determines uniquely the density of the nuclear matter. This assumption is supported by the calculations one has referred to in section 3 in which the reaction matrix depending only on the local density is used (see, in particular^[13]).

To write eq. (2.14) explicitly with K given by eq. (2.4) it is convenient to change the representation for arbitrary matrices which occur in the equations for K and $H[K]$. A matrix A of the rank twice as big as that of matrices in the Hartree-Fock theory, described in section 2, can be written in the form

$$A = A^{(0)} \sigma^{(0)} + A\sigma \equiv \sum_{l=0}^3 A^{(l)} \sigma^{(l)}, \quad (4.4)$$

where $A = (A^{(1)}, A^{(2)}, A^{(3)})$, $A^{(0)}$ are matrices of the rank equal to that in the Hartree-Fock theory, and the "quasi-spin" matrices $\sigma^{(l)}$ are:

$$\begin{aligned} \sigma^{(0)} &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & \sigma^{(1)} &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ \sigma^{(2)} &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} & \sigma^{(3)} &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} . \end{aligned} \quad (4.5)$$

The new representation for the matrices is determined by the following formulae. The matrix of the shell model Hamiltonian H_0 which is supposed to be invariant under the time inversion and the density matrix K_0 of eq. (2.7) are given by

$$\begin{aligned} H_0 &= \{ \dots \epsilon_\nu, \epsilon_{\bar{\nu}}, \dots \} \sigma^{(3)}, \quad \epsilon_\nu = \epsilon_{\bar{\nu}} \\ K_0 &= \frac{1}{2} I (\sigma^{(0)} - \sigma^{(3)}) + \{ \dots n_\nu, n_{\bar{\nu}}, \dots \} \sigma^{(3)} \\ n_\nu = n_{\bar{\nu}} &= \begin{cases} 1 & \nu \leq \frac{N}{2} \\ 0 & \nu > \frac{N}{2} \end{cases} \end{aligned} \quad (4.6)$$

In the latter equation $\{ \dots n_\nu \dots \}$ stands for a diagonal matrix, and $I = \{ \dots 1, \dots \}$.

The indices $\nu, \bar{\nu}$ designate the two conjugated in time eigenstates of H_0 . As it is shown in eq. (4.6) these indices label adjacent columns and rows of matrices in the generalized Hartree-Fock method.

The matrix product of two matrices A and B in the new notation is given by

$$AB = (A^{(0)} B^{(0)} + \vec{A} \cdot \vec{B}) \sigma^{(0)} + [(A^{(0)} \vec{B} + \vec{A} B^{(0)}) + (\vec{A} \times \vec{B})] \sigma . \quad (4.7)$$

The trace of a matrix A is equal to

$$\text{tr} A = 2 \sum_{\nu\nu'} A_{\nu\nu'}^{(0)} \equiv 2 \text{Sp} A^{(0)} , \quad (4.8)$$

where Sp stands for a sum over the single-particle quantum numbers, while tr designates additionally the summation over the extra indices inherent in the generalized Hartree-Fock theory.

From eqs. (4.6) (4.7) it follows, in particular, that

$$\text{tr} AB = 2 \text{Sp} (A^{(0)} B^{(0)} + \vec{A} \cdot \vec{B}) . \quad (4.9)$$

An arbitrary two-body operator A_{12} in this notation has the form

$$A_{12} = \sum_{i,j=0}^8 A_{12}^{(i,j)} \sigma^{(i)}(1) \sigma^{(j)}(2) . \quad (4.10)$$

Matrix elements of it are specified by two sets of single-particle quantum numbers $\nu_1 \nu'_1, \nu_2 \nu'_2$ and by two additional indices i, j ; i.e. in eq. (4.10) $A_{12}^{(ij)}$ stands for the matrix $A_{\nu_1 \nu'_1, \nu_2 \nu'_2}^{(ij)}$.

The traces in eqs. (2.11)(2.14) are given by

$$\text{tr}_2 A_{12} B_2 = 2 \sum_{i=0}^8 \left(\sum_{j=0}^8 \text{Sp}_2 A_{12}^{(ij)} B_2^{(j)} \right) \sigma^{(i)}(1) . \quad (4.11)$$

From eqs. (4.10) and (4.11) it follows in particular that the "delta-function operator" I_{12} in eq. (2.11) has the form

$$I_{12} = \frac{1}{2} \delta_{\nu_1 \nu_2} \cdot \delta_{\nu'_1 \nu'_2} \sum_{i=0}^8 \sigma^{(i)}(1) \sigma^{(i)}(2) . \quad (4.12)$$

The above formulae are sufficient to write the expression for the commutator $[\xi_{12}, K_0]$ in eq. (2.14):

$$[\xi_{12}, K_0] = \frac{1}{2} \delta_{\nu_1 \nu_2} \delta_{\nu'_1 \nu'_2} \sum_{l=0}^8 A_{\nu_1 \nu'_1}^{(l)} \sigma^{(l)} (1) \sigma^{(l)} (2) \quad (4.13)$$

with

$$\begin{aligned} A_{\nu \nu'}^{(0)} &= A_{\nu \nu'}^{(3)} = -\frac{n_{\nu} - n_{\nu'}}{\epsilon_{\nu} - \epsilon_{\nu'}} \\ A_{\nu \nu'}^{(1)} &= A_{\nu \nu'}^{(2)} = \frac{1 - n_{\nu} - n_{\nu'}}{\epsilon_{\nu} + \epsilon_{\nu'}} \end{aligned} \quad (4.14)$$

To write eq. (2.14) in matrix elements it is necessary to analyse the structure of the matrix δK in the "quasi-spin" notation. The variation δK is now written in the form

$$\delta K = \delta K_d + \delta K_{off} \quad (4.15)$$

where subscripts d and off stand for the diagonal and off-diagonal in quasispin indices components of δK . The latter are given by

$$\begin{aligned} \delta K_d &= \delta \rho^{(-)} \sigma^{(0)} + \delta \rho^{(+)} \sigma^{(8)} \\ \delta K_{off} &= \delta \chi^{(-)} \sigma^{(1)} + i \delta \chi^{(+)} \sigma^{(2)} \end{aligned} \quad (4.16)$$

In eq. (4.16) the matrices $\delta \rho^{(\pm)}$, $\delta \chi^{(\pm)}$ are the time-even and time-odd components of those in eq. (2.4):

^{x/} For the matrix ξ_{12} one has

$$\xi_{12} = \delta_{\nu \nu} \delta_{\nu' \nu'} \sum_{l,l=0}^8 \xi_{\nu_1 \nu'_1}^{(l,l)} \sigma^{(l)} (1) \sigma^{(l)} (2)$$

with the only nonvanishing components

$$\xi_{\nu \nu'}^{(08)} = \xi_{\nu \nu'}^{(8,0)} \frac{1}{\epsilon_{\nu} - \epsilon_{\nu'}}$$

$$\xi_{\nu \nu'}^{(12)} = -\xi_{\nu \nu'}^{(21)} \frac{1}{\epsilon_{\nu} + \epsilon_{\nu'}}$$

$$\delta\rho_{\nu\nu'}^{(\pm)} = \frac{1}{2}(\delta\rho_{\nu\nu'} \pm \delta\rho_{\nu'\nu}) = \pm \delta\rho_{\nu'\nu}^{(\pm)}$$

$$\delta\chi_{\nu\nu'}^{(\pm)} = \frac{1}{2}(\delta\chi_{\nu\nu'} + \delta\chi_{\nu'\nu}) = +\delta\chi_{\nu'\nu}^{(\pm)} = -\delta\chi_{\nu\nu'}^{(\pm)}$$
(4.17)

Because of the supplementary condition (2.6) the variations $\delta\rho$ and $\delta\chi$ (or $\delta\rho^{(\pm)}$ and $\delta\chi^{(\pm)}$) are not generally independent. However in the particular case considered here ($\chi_0 = 0$) the restrictions imposed by the condition (2.6) are very simple. These can be found from the equation analogous to eq. (2.23)

$$\delta K = (K_0 - 1) \delta K K_0 + K_0 \delta K (K_0 - 1).$$
(4.18)

The latter implies that

$$(\delta K_d)_{\nu\nu'} = (n_\nu - n_{\nu'})^2 (\delta K_d)_{\nu\nu'}$$
(4.19)

$$(\delta K_{off})_{\nu\nu'} = (n_\nu + n_{\nu'} - 1)^2 (\delta K_{off})_{\nu\nu'}$$

The first of the two equations (4.19) shows in agreement with eq. (2.24) that the variations of $\delta\rho$ are restricted to the particle-hole excitations of the system. From the second of eqs. (4.19) it follows that the nonvanishing matrix elements of δK_{off} (or $\delta\chi$) are confined to the excitations of particle-particle and hole-hole types.

To write the final expressions for the residual interaction it remains to determine the variation of the self-consistent Hamiltonian $\delta H(K)$. This is given by

$$\delta H = \delta H_d + \delta H_{off}$$
(4.20)

with the diagonal (δH_d) and off-diagonal (δH_{off}) parts defined by analogy with eq. (4.16):

$$\delta H_d = \delta b \sigma^{(-)} + \delta b \sigma^{(+)}$$

$$\delta H_{\text{off}} = \delta \mu_{\sigma}^{(-)} \sigma^{(1)} + i \delta \mu_{\sigma}^{(+)} \sigma^{(2)} \quad (4.21)$$

If the reaction matrix is invariant under the time inversion and antisymmetric in indices $\nu_1 \nu'_1$ ($\nu_2 \nu'_2$), then for the quantities $\delta h^{(\pm)}$, $\delta \mu^{(\pm)}$ one has

$$(\delta h^{(\pm)})_1 = \delta(h_0^{(\pm)})_1 + S p_2 F_{12}^{(\pm)} \delta \rho_2^{(\pm)} \quad (4.22)$$

$$(\delta \mu^{(\pm)})_1 = (\delta \mu_0^{(\pm)})_1 + S p_2 G_{12} \delta \mu_2^{(\pm)},$$

where $\delta h_0^{(\pm)}$ and $\delta \mu_0^{(\pm)}$ are time-even (+) and time-odd (-) components of the quantities

$$\delta(h_0)_1 = S p_2 G_{12} (\rho_0)_2 + (e_q V_d)_1 \quad (4.23)$$

$$\delta(\mu_0)_1 = (e_q V_{\text{off}})_1$$

and, finally,

$$F_{12}^{(\pm)} = G_{12} (\rho_0) + S p_3 \left(\frac{\delta G_{12}}{\delta \rho_2^{(\pm)}} \right)_{\rho=\rho_0} (\rho_0)_3 \quad (4.24)$$

Using the commutator $[g, K_0]$ given by eqs. (4.13)(4.14) and the above expressions for δH (see eqs. (4.20)-(4.24)) eq. (2.14) splits into four uncoupled sets of equations for the matrix elements of $\delta \rho^{(\pm)}$ and $\delta \chi^{(\pm)}$. These equations read

$$\begin{aligned} (\delta \rho^{(\pm)})_{\nu \nu'} &= - \frac{n_{\nu} - n_{\nu'}}{\epsilon_{\nu} - \epsilon_{\nu'}} [(\delta h_0^{(\pm)})_{\nu \nu'} + \sum_{\nu_2 \nu'_2} F_{\nu \nu', \nu_2 \nu'_2} (\delta \rho^{(\pm)})_{\nu_2 \nu'_2}] \\ (\delta \chi^{(\pm)}) &= \frac{1 - n_{\nu} - n_{\nu'}}{\epsilon_{\nu} + \epsilon_{\nu'}} [(\delta \mu_0^{(\pm)})_{\nu \nu'} + \sum_{\nu_2 \nu'_2} G_{\nu \nu', \nu_2 \nu'_2} (\delta \chi^{(\pm)})_{\nu_2 \nu'_2}] \end{aligned} \quad (4.25)$$

The first of eqs. (4.25) coincides with eq. (2.28). When

$$- \frac{n_{\nu} - n_{\nu'}}{\epsilon_{\nu} - \epsilon_{\nu'}} (\delta h_0)_{\nu \nu'} = (\delta \rho_0)_{\nu \nu'} = \delta_{\nu \nu'} \delta_{\nu \nu_2} \quad (4.26)$$

it represents the changes in the nuclear state (or the polarization of the nucleus) which take place if a particle from an orbital ν_1 is transferred to another orbital ν_2 . It is seen that these changes are represented by a two-body residual interaction operator $F_{12}^{(\pm)}$ of eq. (4.24). In the same way the polarization of a nucleus produced by an extra pair of particles (or by two holes) occupying two states $\nu_1 \nu_2$ above the Fermi surface (or below it) is given by the second of eqs. (4.25) with

$$\frac{1 - n_{\nu} - n_{\nu'}}{\epsilon_{\nu} + \epsilon_{\nu'}} \delta \mu_{\nu \nu'} = (\delta \chi_0)_{\nu \nu'} = \delta_{\nu \nu_1} \delta_{\nu' \nu_2} \quad (4.27)$$

Again the polarization is accounted for by a two-body operator. In this case the residual interaction is just the reaction matrix G_{12} . The two different expressions for the particle-particle and particle-hole components of the residual interaction operator can be combined in a formula

$$F_{12} = \frac{\delta H_1}{\delta K_2} = F_{12}^{(d)} + F_{12}^{(off)} \quad (4.28)$$

where

$$F_{12}^{(d)} = \frac{\sigma^{(0)}(1) \sigma^{(0)}(2)}{2} \left\{ G_{12} + \text{Sp}_3 \left(\frac{\delta G_{12}}{\delta \rho_2} \right)_{\rho = \rho_0} \rho_3^0 \right\} + \quad (4.29)$$

$$+ \frac{\sigma^{(8)}(1) \sigma^{(8)}(2)}{2} \left\{ G_{12} + \text{Sp}_3 \left(\frac{\delta G_{12}}{\delta \rho} \right)_{\rho = \rho_0} \rho_3^0 \right\}$$

and

$$F_{12} = \frac{1}{2} (\sigma^{(1)}(1) \sigma^{(1)}(2) + \sigma^{(2)}(1) \sigma^{(2)}(2)) G_{12} \quad (4.30)$$

The effective delta-interaction can be obtained in this case in the same way as in section 2. It corresponds again to the approximation for the self-consistent potential in which the latter is determined by the local value of the particle density. The discussion following eq. (4.26) clarifies the physical meaning of the approximation used in transition from the self-consistent eqs. (2.1), (2.8), (4.1) to the perturbation expansion, based on the shell model Hamiltonian H_0 of eq. (2.7). The perturbation theory is valid if $\delta K_{\nu \nu'} \ll 1$ for all ν and ν' . The necessary conditions for that read $(\delta \rho_0)_{\nu \nu'} \ll 1$. Now, the latter inequalities mean that the probability to reach any of excited states of H_0 acting on the shell model ground state K_0 by the operator $\delta H_0 = \epsilon_0 V + H(\rho_0) - H_0$ is small.

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Table 1

Particle-hole part of the residual delta interaction

	1	2	3	4
f_{ex}^{av}	-3	-5	-2.2	-2.8
f_{in}^{av}	+1	+0.9	+0.8	+0.3
f_{ex}^{pp}	-3		-1	-1.6
f_{in}^{pp}	+1.2		+0.2	-0.2
f_{ex}^{pn}	-3		-3.3	-4.1
f_{in}^{pn}	+0.6		+1.5	+0.7
g_{ex}^{pp}	+1		+1.2	+1.5
g_{in}^{pp}	+1		0	+0.1
g_{ex}^{pn}	0		-0.4	-0.3
g_{in}^{pn}	0		+0.4	+0.3

The parameters of the residual interaction in the form

$$F_{12} = [f_{ex} + (f_{in} - f_{ex})\gamma + [g_{ex} + (g_{in} - g_{ex})\gamma]\vec{\sigma}_1 \cdot \vec{\sigma}_2] \delta(\vec{r}_1 - \vec{r}_2)$$

are given. The empirical values of ref.^{/7/} are given in the column (1). The calculated values are obtained by fitting the data on the nuclear potential with eq. (3.2). In the latter the parameters are fixed: (i) by the conditions $E_0 = 15$ MeV, $T_F = 35$ MeV (column 2), (ii) by the results of ref.^{/15/} where two different expressions for the nucleon-nucleon interaction force were used. Column 3 corresponds to the Reid's (hard core) and column (4) to the Bressel-Kerman's Revised (soft-core) nucleon-nucleon interaction. The quantities f and g are given in units of $\frac{2}{3} \frac{T_F}{\rho_0} = 130$ MeV. The quantity $\gamma = \rho(\vec{r})/\rho_0$ being the density at the point $\vec{r}_1 = \vec{r}_2$ related to its magnitude at the centre of the nucleus.