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ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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SPIN-QUADRUPOLE FORCES AND COLLECTIVE STATES IN DEFORMED NUCLEI

I. 0^+ States

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1. Introduction

The spin-dependent forces appear to be very important on analysis of a number of qualitative features of the nuclear structure.The spin dependent foroes, for example, define the coupling momentum rules in odd-odd nuclei and the spin-splitting of the two-quasiparticle states in even-mass nuolei[1]. These foroes affect significantly the odd--even shift in a rotational band with K=0 [1,2]. The spin-dependent forces are responsible for the magnetic polarization effects in odd-even nuclei [3].

It is natural also to attempt to clarify the spin interaction effects in the collective motion of nuclei.

The present paper is devoted to the study of the spin quadrupole interaction effect on the properties of collective 0^+ and 2^+ states. The pairing plus quadrupole as well as spin-quadrupole interactions model is developed using the approximate second quantization method [4]. It is shown that the introduction of the spin-quadrupole interactions does not lead to a simple renormalization of the constants in the microscopic theory of collective excitations developed in recent years (see, e.g. [5-11]).

The spin-quadrupole interactions affect significantly the properties of monopole exoitations in even-even nuclei. They affect noticeably the energy and E2-transition probability. It seems that the spin-quadrupole interactions may be responsible for the appearanoe of a new branch of colleotive excitations below (or near) the energy gap.

For a certain strength of the spin-quadrupole interactions the appearance of new 2^+ states below the two-quasi-partiole excitation energy is also possible.

The paper presents the numerical caloulations of energies, transition probabilities and wave functions of 0^+ states for nuclei in the region $150 \leq A \leq 174$, which were performed with an electronic oomputer. The numerical calculations for 2^+ states will be published as well.

II. Theory

The effect of the long-range spin- dependent forces on collective motion in single--closed shell nuclei has been first investigated by Kisslinger [12]. He studied the 2^+ vibrational states, using pairing plus quadrupole or spin-quadrupole forces. It turned out that the spin-quadrupole coupling parameter estimated from the empirical energies of 2^+-states must be chosen about five to ten times larger than the ordinary quadrupole one. From the comparison of the oalculated transition probabilities B(E2) as well as the energy levels in the odd-mass nuclei with experimental data it was concluded that the quadrupole force is the major component leading to the 2^+ collective vibrations. This is connected mainly with the importance of quadrupole interactions within a shell, while the spin-quadrupole effects may result from interaction between partioles filling different j-levels only.

But the large energy shifts in the odd-mass nuclei can result from an addition of spin-quadrupole force which has little effect on vibrational energies and the B(E2) values. The effects of quadrupole and spin-quadrupole interactions mixing appear to be small in these nuclei and have not been considered by Kisslinger. The single-particle levels are two-fold degenerated in deformed nuclei and the interaction between particles in different levels is expected to be more significant than in spherical nuclei.

1. The Interaction Hamiltonian

The spin-multipole interaction operator is extracted from the multipole expansion of the usual spin potential $V(|\vec{r}_1-\vec{r}_2|)\,\vec{\sigma}_1\vec{\sigma}_2$, where σ_i are the Pauli spin matrices and $\vec{r}_i$ are the radius-vectors of particles. For simplicity we have assumed the radial part to be separable (as in the case of the quadrupole force) and we get the spin-multipole force in the form:

$$-\varkappa_t^{(\ell)} \sum_{\kappa} r_1^{\kappa} r_2^{\kappa} \sum_{m} P_{\ell m}^{*}(\kappa,\sigma)\, P_{\ell m}(\kappa,\sigma) \tag{1}$$

where

$$P_{\ell m}(\kappa,\sigma) = \sum_{q,\tau} \langle \kappa\, 1\, q\, \tau | \ell m \rangle\, Y_{\kappa q}(\theta,\varphi)\, \sigma_{\tau} \tag{2}$$

$Y_{\kappa q}(\theta,\varphi)$ are the sperical harmonics, $\varkappa_t^{(\ell)}$ is the spin-multipole coupling constant. The collective states, formed by interaction (1) will have the angular momentum component along the nuclear symmetry axis and parity $K\pi = m(-1)^{\kappa}$. In particular for 0^+ and 2^+ states we take into account the contribution of the term with k=l=2. Taking into account the interaction (1), we write the Hamiltonian in the form:

$$H = H_{s.p.} + H_{pair} + \sum_{\ell} H_{coll}^{(\ell)}, \tag{3}$$

where

$$H_{s.p.} = \sum_{s,\rho} (E_s - \lambda)\, a_{s\rho}^{+} a_{s\rho}, \quad (\rho = \pm) \tag{4}$$

$$H_{pair} = \sum_{s,s'} (-G)\, a_{s+}^{+} a_{s-}^{+} a_{s'-} a_{s'+} \tag{5}$$

$$\begin{aligned} H_{coll}^{(\ell)} &= -\frac{\varkappa_q^{(\ell)}}{2} T_q^{+} T_q - \frac{\varkappa_t^{(\ell)}}{2} T_t^{+} T_t \\ T_q &= \sum_{s,s',\rho,\rho'} q_{s\rho;s'\rho'}\, a_{s\rho}^{+} a_{s'\rho'} \\ T_t &= \sum_{s,s',\rho,\rho'} t_{s\rho;s'\rho'}\, a_{s\rho}^{+} a_{s'\rho'} \end{aligned} \tag{6}$$

E_s are the single-particle energies; λ is the chemical potential; G is the pairing interaction constant; $a^+_{s\rho}$ and $a_{s\rho}$ are creation and annihilation operators. $q_{s\rho;s'\rho'}$ and $t_{s\rho;s'\rho'}$ are single-particle matrix elements of the quadrupole and spin-dependent interaction, respectively. The usual multipole and spin dependent interactions are involved with corresponding coupling parameters $\varkappa_q^{(\ell)}$ and $\varkappa_t^{(\ell)}$ in the collective part of the Hamiltonian (3). In all expressions the summation over the neutron and proton one-particle states is assumed. For the sake of simplicity, we assume further that $\varkappa_q^{(\ell)}$ ($\varkappa_t^{(\ell)}$) is the same for (n,n), (p,p) and (n,p) interactions.

The single particle matrix elements $q_{s\rho;s'\rho'}$ and $t_{s\rho;s'\rho'}$ has different properties under the index permutation and time reversal:

$$
\begin{aligned}
q_{ss'} &\equiv \langle s+|r^\ell(Y^*_{\ell m}+Y_{\ell m})|s'+\rangle = q_{s+;s'+} = q_{s-;s'-} = q_{s's} \\
\bar{q}_{ss'} &\equiv \langle s+|r^\ell(Y^*_{\ell m}+Y_{\ell m})|s'-\rangle = q_{s+;s'-} = -q_{s'+;s-} = -\bar{q}_{s's} \\
t_{ss'} &\equiv \langle s+|r^\ell(P^*_{\ell m}+P_{\ell m})|s'+\rangle = t_{s+;s'+} = t_{s-;s'-} = -t_{s's} \\
\bar{t}_{ss'} &\equiv \langle s+|r^\ell(P^*_{\ell m}+P_{\ell m})|s'-\rangle = t_{s+;s'-} = t_{s'+;s-} = \bar{t}_{s's}
\end{aligned}
\qquad (7)
$$

All the diagonal spin-multipole matrix elements are equal to zero.

Let us subject the operators $a_{s\rho}$ to the Bogolubov canonical transformation:

$$
a_{s\rho} = u_s \alpha_{s-\rho} + \rho v_s \alpha^+_{s\rho} \qquad (8)
$$

where $\alpha^+_{s\rho}$ and $\alpha_{s\rho}$ are the quasiparticle creation and annihilation operators, respectively, and u_s and v_s are the transformation parameters. For the operators T_q and T_t we get the expressions:

$$
\begin{aligned}
T_q = \sum_{s,s'} \{ & V_{ss'} (q_{ss'} B_{ss'} + \bar{q}_{ss'} \bar{B}_{ss'}) + \\
& + \frac{1}{\sqrt{2}} U_{ss'} [q_{ss'} (A^+_{ss'} + A_{ss'}) + \\
& + \bar{q}_{ss'} (\bar{A}^+_{ss'} + \bar{A}_{ss'})]
\end{aligned}
\qquad (9)
$$

$$T_t = \sum_{s,s'} \{ M_{ss'} (t_{ss'} B_{ss'} + \bar{t}_{ss'} \bar{B}_{ss'}) +$$

$$+ \frac{1}{\sqrt{2}} L_{ss'} [t_{ss'} (A^+_{ss'} - A_{ss'}) + \bar{t}_{ss'} (\bar{A}^+_{ss'} - \bar{A}_{ss'})] \tag{10}$$

$$V_{ss'} = u_s u_{s'} - v_s v_{s'} \ ; \quad U_{ss'} = u_s v_{s'} + u_{s'} v_s$$
$$M_{ss'} = u_s u_{s'} + v_s v_{s'} \ ; \quad L_{ss'} = u_s v_{s'} - u_{s'} v_s \tag{11}$$

The operators $B_{ss'}$, $\bar{B}_{ss'}$, $A_{ss'}$ and $\bar{A}_{ss'}$ are introduced in ref. [11]:

$$B_{ss'} = \sum_{\rho} \alpha^+_{s\rho} \alpha_{s'\rho} \ ; \quad \bar{B}_{ss'} = \sum_{\rho} \rho \, \alpha^+_{s-\rho} \alpha_{s'\rho} \tag{12}$$

$$A_{ss'} = \frac{1}{\sqrt{2}} \sum_{\rho} \rho \, \alpha_{s\rho} \alpha_{s'-\rho} \ ; \quad \bar{A}_{ss'} = \frac{1}{\sqrt{2}} \sum_{\rho} \alpha_{s\rho} \alpha_{s'\rho}$$

The quantities T_q and T_t contain various factors connected with the canonical transformation parameters (8). This is due to the different parities of the multipole and spin-multipole interaction under time reversal.

Let us introduce the phonon operators by the following transformation:

$$A^+_{ss'} + A_{ss'} = \sum_i g^i_{ss'} (Q^+_i + Q_i) \ ; \quad A^+_{ss'} - A_{ss'} = \sum_i w^i_{ss'} (Q^+_i - Q_i)$$
$$\bar{A}^+_{ss'} + A_{ss'} = \sum_i \bar{g}^i_{ss'} (Q^+_i + Q_i) \ ; \quad \bar{A}^+_{ss'} - \bar{A}_{ss'} = \sum_i \bar{w}^i_{ss'} (Q^+_i - Q_i) \tag{13}$$

In the general case $g^i_{ss'}$, $\bar{g}^i_{ss'}$, $w^i_{ss'}$, $\bar{w}^i_{ss'}$ are the matrix elements of rectangular transformation matrices. We determine the inverse transformation in the form:

$$Q^+_i + Q_i = \frac{1}{2} \sum_{s,s'} [w^i_{ss'} (A^+_{ss'} + A_{ss'}) + \bar{w}^i_{ss'} (\bar{A}^+_{ss'} + \bar{A}_{ss'})] \tag{14}$$

$$Q^+_i \quad Q_i - \frac{1}{2} \sum_{s,s'} [g^i_{ss'} (A^+_{ss'} - A_{ss'}) + \bar{g}^i_{ss'} (\bar{A}^+_{ss'} - \bar{A}_{ss'})]$$

From (13) and (14) and from the linear independence of the operators Q_i it is easy to obtain a condition on the transformation parameters:

$$\sum_{s,s'} \left(w^{i}_{ss'}\, g^{i'}_{ss'} + \bar{w}^{i}_{ss'}\, \bar{g}^{i'}_{ss'} \right) = 2\, \delta_{ii'} \tag{15}$$

From the index permutation symmetry for the operators $A_{ss'}$ and $\bar{A}_{ss'}$ we get also the conditions:

$$g^{i}_{ss'} = g^{i}_{s's} \; ; \; w^{i}_{ss'} = w^{i}_{s's} \; ; \; \bar{g}^{i}_{ss'} = - \bar{g}^{i}_{s's} \; ; \; \bar{w}^{i}_{ss'} = - \bar{w}^{i}_{s's} \tag{16}$$

Now the operators T_q and T_t are of the form:

$$T_q = \sum_{s,s'} V_{ss'} \left(q_{ss'} B_{ss'} + \bar{q}_{ss'} \bar{B}_{ss'} \right) + \sum_{i} R^{q}_{i} \left(Q^{+}_{i} + Q_{i} \right) \tag{17}$$

$$T_t = \sum_{s,s'} M_{ss'} \left(t_{ss'} B_{ss'} + \bar{t}_{ss'} \bar{B}_{ss'} \right) + \sum_{i} R^{t}_{i} \left(Q^{+}_{i} - Q_{i} \right) \tag{18}$$

$$R^{q}_{i} = \frac{1}{\sqrt{2}} \sum_{s,s'} U_{ss'} \left(q_{ss'}\, g^{i}_{ss'} + \bar{q}_{ss'}\, \bar{g}^{i}_{ss'} \right) \tag{19}$$

$$R^{t}_{i} = \frac{1}{\sqrt{2}} \sum_{s,s'} L_{ss'} \left(t_{ss'}\, w^{i}_{ss'} + \bar{t}_{ss'}\, \bar{w}^{i}_{ss'} \right) \tag{20}$$

Considering $A_{ss'}$, $\bar{A}_{ss'}$ and Q_i to be approximately Bose operators (this approximation is discussed in detail, e.g. in ref.[11]) we determine the phonon vacuum Ψ by

$$Q_i \Psi = 0 \tag{21}$$

We assume the vacuum expectation values of the operators $B_{ss'}$ and $\bar{B}_{ss'}$ to be approximately zero.

To simplify the notations we further distinguish between the operators $A_{ss'}$ and $\bar{A}_{ss'}$, $B_{ss'}$ and $\bar{B}_{ss'}$ as well as between the matrix elements, $q_{ss'}$ and $\bar{q}_{ss'}$, $t_{ss'}$ and $\bar{t}_{ss'}$ etc. only when necessary.

The Hamiltonian (3) is expressed approximately in terms of the $B_{ss'}$ and Q_i operators (for a certain multipole state):

$$\begin{aligned}
H^{(\ell)} \approx & \sum_{s} \varepsilon_s B_{ss} + \frac{G}{\sqrt{2}} \sum_{i} \sum_{ss'} u_s v_s \left(u^2_{s'} - v^2_{s'} \right) g^{i}_{s's'} B_{ss} \left(Q^{+}_{i} + Q_i \right) - \\
& - \frac{1}{2} \varkappa^{(\ell)}_{q} \sum_{i} R^{q}_{i} \sum_{ss'} V_{ss'}\, q_{ss'} \left[B_{ss'} \left(Q^{+}_{i} + Q_i \right) + \mathrm{c.c} \right] + \\
& + \frac{1}{2} \varkappa^{(\ell)}_{t} \sum_{i} R^{t}_{i} \sum_{ss'} M_{ss'}\, t_{ss'} \left[B_{ss'} \left(Q^{+}_{i} - Q_i \right) + \mathrm{c.c.} \right] - \\
& - \frac{1}{2} \varkappa^{(\ell)}_{q} \sum_{ii'} R^{q}_{i} R^{q}_{i'} \left(Q^{+}_{i} + Q_i \right) \left(Q^{+}_{i'} + Q_{i'} \right) + \\
& \qquad + \frac{1}{2} \varkappa^{(\ell)}_{t} \sum_{ii'} R^{t}_{i} R^{t}_{i'} \left(Q^{+}_{i} - Q_i \right) \left(Q^{+}_{i'} - Q_{i'} \right)
\end{aligned} \tag{22}$$

Here $\varepsilon_s = \sqrt{(E_s-\lambda)^2 + C^2}$ are the quasipartiole energies, and $C = G \sum_s U_s V_s$. The parameters U_s and V_s are assumed to be determined previously by solving the pairing correlation problem. The pairing interaotion oonstants G_N and G_Z are determined from the calculations of the pairing energies.

The Hamiltonian (22) oontains terms whioh correspond to quasiparticle and phonon excitations as well as terms of the quasipartiole-phonon interaotion. The latter are taken into aooount only in the spectra of odd-mass nuolei.

2. Colleotive States of Even-Even Nuolei

We consider one-phonon exoitations of the type:

$$Q_i^+ \Psi = \frac{1}{4} \sum_{s,s'} \left\{ (g_{ss'}^i + w_{ss'}^i) A_{ss'}^+ - (g_{ss'}^i - w_{ss'}^i) A_{ss'} \right\} \Psi \tag{23}$$

By means of a variational prooedure we determine the parameters $g_{ss'}^i$ and $w_{ss'}^i$ and the one phonon energies ω_i (the latter are formally introduoed as the Lagrangian mutlitpliers):

$$\delta \left\{ (\Psi^+ Q_i H^{(\ell)} Q_i^+ \Psi) - \frac{\omega_i}{2} \left(\sum_{ss'} w_{ss'}^i g_{ss'}^i - 2 \right) \right\} = 0 \tag{24}$$

Using the variational prooedure we oan derive from (24) the following set of equations:

$$\begin{aligned} &\varepsilon_{ss'} g_{ss'}^i - \omega_i w_{ss'}^i - G (u_s^2 - v_s^2) \delta_{ss'} \sum_{\tau} (u_\tau^2 - v_\tau^2) g_{\tau\tau}^i - \\ &\quad - 2\sqrt{2}\, \varkappa_q^{(\ell)} R_i^q U_{ss'} q_{ss'} = 0 \\ &\varepsilon_{ss'} w_{ss'}^i - \omega_i g_{ss'}^i - G \delta_{ss'} \sum_{\tau} w_{\tau\tau}^i - \\ &\quad - 2\sqrt{2}\, \varkappa_t^{(\ell)} R_i^t L_{ss'} t_{ss'} = 0 \end{aligned} \tag{25}$$

where $\varepsilon_{ss'} = \varepsilon_s + \varepsilon_{s'}$

The diagonal terms are included to eliminate the spurious 0^+ state [6].

After combersome caloulations, which are omitted here, the equation for ω_i (index i stands for different solutions) oan be written in the form[x]

x) We omit the superscript of $\varkappa_q^{(\ell)}$ and $\varkappa_t^{(\ell)}$.

$$\left[1-\varkappa_q F(\omega_i)\right]\left[1-\varkappa_t S(\omega_i)\right]=\varkappa_q \varkappa_t X^2(\omega_i) \tag{26}$$

with the definitions:

$$F(\omega_i)=F_0(\omega_i)-\varphi_n(\omega_i)/\gamma_n(\omega_i)-\varphi_p(\omega_i)/\gamma_p(\omega_i)$$

$$F_0(\omega_i)=2\sum_{ss'}\frac{\varepsilon_{ss'}U^2_{ss'}q^2_{ss'}}{\varepsilon^2_{ss'}-\omega_i^2}$$

$$\varphi(\omega_i)=4C^2\sum_s\frac{q_{ss}\Gamma_s(\omega_i)}{\varepsilon_s(4\varepsilon_s^2-\omega_i^2)}\ ;\quad \Gamma_s(\omega_i)=\xi(\omega_i)-4(E_s-\lambda)\eta(\omega_i)$$

$$\xi(\omega_i)=\sum_{ss'}\frac{q_{ss}\left[4C^2-\omega_i^2+4(E_s-\lambda)(E_{s'}-\lambda)\right]}{\varepsilon_s\varepsilon_{s'}(4\varepsilon_s^2-\omega_i^2)(4\varepsilon_{s'}^2-\omega_i^2)}$$

$$\eta(\omega_i)=\sum_{ss'}\frac{q_{ss}(E_s-E_{s'})}{\varepsilon_s\varepsilon_{s'}(4\varepsilon_s^2-\omega_i^2)(4\varepsilon_{s'}^2-\omega_i^2)}.$$

$$\gamma(\omega_i)=\sum_{ss'}\frac{4C^2-\omega_i^2+4(E_s-\lambda)(E_{s'}-\lambda)}{\varepsilon_s\varepsilon_{s'}(4\varepsilon_s^2-\omega_i^2)(4\varepsilon_{s'}^2-\omega_i^2)}$$

$$S(\omega_i)=2\sum_{ss'}\frac{\varepsilon_{ss'}L^2_{ss'}t^2_{ss'}}{\varepsilon^2_{ss'}-\omega_i^2}$$

$$X(\omega_i)=2\sum_{ss'}\frac{\omega_i U_{ss'}q_{ss'}L_{ss'}t_{ss'}}{\varepsilon^2_{ss'}-\omega_i^2}$$

In $\varphi(\omega_i)$, $\xi(\omega_i)$, $\eta(\omega_i)$ and $\gamma(\omega_i)$ summation over the neutron or over the proton single-particle states is assumed. The sums in $F_0(\omega_i)$, $S(\omega_i)$ and $X(\omega_i)$ run over all two-quasi-particle states with the projections, which couple to a given $K\pi$. Eq.(26) is written for the 0^+ and 2^+ collective states [x]. Putting $\varkappa_t$ or $\varkappa_q$ equal to zero we obtain the equations for the case of pure quadrupole or pure spin-quadrupole interactions, respectively. In the general case in eq. (26) there is a term $X(\omega_i)$ corresponding to the quadrupole and spin-quadrupole interaction mixing. Note that $S(\omega_i)$ and $X(\omega_i)$ contain no diagonal terms, therefore the exclusion of the spurious state by no means affects them.

x) The spurious 0^+ state is eliminated and the solution with $\omega=0$ is excluded from eq. (26). The term $\varphi(\omega)/\gamma(\omega)$ is due to elimination of a spurious 0^+ state.

Let us analyse in more detail the structure of eq. (26). We write it in the form:

$$1/\varkappa_q = P(\omega_i, \varkappa_t) \, ,$$

$$P(\omega_i, \varkappa_t) = F(\omega_i) + \varkappa_t X^2(\omega_i) / [1 - \varkappa_t S(\omega_i)] \tag{26'}$$

As it was shown previously [11] the function $F(\omega_i)$ has the first order poles, when $\gamma(\omega) = 0$ and for $\omega = \mathcal{E}_{ss'}$ (the former for 0^+ states only). The functions $S(\omega_i)$ and $X(\omega_i)$ have poles of the first order only for $\omega = \mathcal{E}_{ss'}$. The total function $P(\omega_i, \varkappa_t)$ has poles at $\gamma(\omega) = 0$ and at points where $1 - \varkappa_t S(\omega) = 0$ (we call them spin poles). The position of the spin poles depends on the choice of $\varkappa_t$. Therefore for a certain choice of $\varkappa_t$ the first spin pole may lie at a lower energy than the first pole of $\gamma(\omega) = 0$. This makes it possible to obtain two collective 0^+ states below the energy gap.

For the 2^+ states the function $P(\omega_i, \varkappa_t)$ has no poles of the type $\gamma(\omega) = 0$. The solution of eq. (26') depends on the position of spin poles only (instead of the former $\omega = \mathcal{E}_{ss'}$ poles of $F_0(\omega)$). Now each spin-pole appears at the energy $\omega < \mathcal{E}_{ss'}$, if $\varkappa_t \neq 0$. It means, that for a certain $\varkappa_t$ the second collective 2^+ state may appear below the boundary of two-quasiparticle excitations.

For $\varkappa_t$ larger than $1/S(\omega=0)$ the lowest spin pole becomes imaginary as is seen from the equation

$$1 - \varkappa_t S(\omega) = 0$$

The experimental observation of the second collective 0^+ and 2^+ states below (or near) the two-quasiparticle excitation boundary give a good support to the existence and significance of the spin-quadrupole interactions. In principle, it is possible to determine the spin-quadrupole coupling constant $\varkappa_t$ by the use of empirical energies of these levels.

Using (15) and the variational equations (25) we can derive the following expressions for $g'_{ss'}$ and $w'_{ss'}$:

$$g'_{ss'} = \sqrt{2/Z(\omega_i, \varkappa_t)} \left\{ \frac{\mathcal{E}_{ss'} U_{ss'} q_{ss'}}{\mathcal{E}^2_{ss'} - \omega_i^2} + \right.$$

$$\left. + D(\omega_i, \varkappa_t) \frac{\omega_i L_{ss'} t_{ss'}}{\mathcal{E}^2_{ss'} - \omega_i^2} - \delta_{ss'} \frac{2 C \Gamma_s(\omega_i)}{\gamma(\omega_i)(4\mathcal{E}_s^2 - \omega_i^2)} \right\} \tag{27}$$

$$w'_{ss'} = \sqrt{2/Z(\omega_i, \varkappa_t)} \left\{ \frac{\omega_i U_{ss'} q_{ss'}}{\mathcal{E}^2_{ss'} - \omega_i^2} + D(\omega_i, \varkappa_t) \frac{\mathcal{E}_{ss'} L_{ss'} t_{ss'}}{\mathcal{E}^2_{ss'} - \omega_i^2} - \right.$$

$$- \delta_{ss'} \frac{C}{\mathcal{E}_s \gamma(\omega_i)} \left[\frac{\omega_i \Gamma_s(\omega_i)}{4\mathcal{E}_s^2 - \omega_i^2} + \frac{\xi(\omega_i)}{\omega_i} \right] \Big\} ; \quad D(\omega_i, \varkappa_t) - \frac{\varkappa_t X(\omega_i)}{1 \quad \varkappa_t S(\omega_i)} , \quad (28)$$

$$Z(\omega_i, \varkappa_t) = Y(\omega_i, \varkappa_t) + \frac{2C^2 \omega_i}{\gamma^2(\omega_i)} \sum_s \frac{\Gamma_s^2(\omega_i)}{\mathcal{E}_s (4\mathcal{E}_s^2 - \omega_i^2)^2}$$

$$- \frac{4C^2 \omega_i}{\gamma(\omega_i)} \sum_s \frac{q_{ss} \Gamma_s(\omega_i)}{\mathcal{E}_s (4\mathcal{E}_s^2 - \omega_i^2)^2} \quad (29)$$

$Y(\omega_i, \varkappa_t)$ is expressed in terms of the derivatives of $F_o(\omega_i)$, $S(\omega_i)$ and $X(\omega_i)$:

$$Y(\omega_i, \varkappa_t) = \frac{1}{4} \left\{ \frac{\partial F_o(\omega_i)}{\partial \omega} + 2\, D(\omega_i, \varkappa_t) \frac{\partial X(\omega_i)}{\partial \omega} + \right.$$

$$\left. + D^2(\omega_i, \varkappa_t) \frac{\partial S(\omega_i)}{\partial \omega} \right\} \quad (30)$$

For the 2^+ states all the diagonal matrix elements vanish and the above formulas are much simplified.

The diagonal two-quasiparticle amplitudes g_{ss}^i and w_{ss}^i in one phonon wave function of a 0^+ state are affected only via $Z(\omega_i, \varkappa_t)$, namely:

$$\frac{g_{ss}(\varkappa_t \neq 0)}{g_{ss}(\varkappa_t = 0)} = \frac{w_{ss}(\varkappa_t \neq 0)}{w_{ss}(\varkappa_t = 0)} = \left[\frac{Z(\omega_i, \varkappa_t = 0)}{Z(\omega_i, \varkappa_t \neq 0)} \right]^{1/2} \quad (31)$$

For $\varkappa_t > 0$ one can show that for the first 0^+ state $Z(\omega, \varkappa_t \neq 0) > Z(\omega, \varkappa_t = 0)$ (because the $Z(\omega, \varkappa_t)$ is proportional to the $\partial P(\omega, \varkappa_t)/\partial \omega$). This means that the spin-quadrupole interactions reduce the contribution of diagonal amplitudes to the wave function of this state. The amplitudes of quasiparticles in different levels may increase significantly at the expense of the diagonal configurations. For the nondiagonal amplitudes one can obtain the ratio :

$$\frac{w_{ss'}^i(\varkappa_t \neq 0)}{w_{ss'}^i(\varkappa_t = 0)} = \left[\frac{Z(\omega, \varkappa_t = 0)}{Z(\omega, \varkappa_t \neq 0)} \right]^{1/2} \times$$

$$\times \left[1 + D(\omega, \varkappa_t) \frac{\mathcal{E}_{ss'} L_{ss'} t_{ss'}}{\omega_i U_{ss'} q_{ss'}} \right] \quad (32)$$

It is seen that the contribution of nondiagonal two-quasiparticle configurations may dominate for the state near spin pole energy, when $|D(\omega, \varkappa_t)| \gg 1$.
Pairing interactions increased (through the $\mathcal{E}_{ss'}$, $L_{ss'}$ and $U_{ss'}$ factors) the role of the particle-hole configurations far off the Fermi surface.

Thus the spin-quadrupole interactions can change noticeably the spectroscopic factor values, which are propertional to the factor $\frac{1}{4}\left(g_{ss'} + w_{ss'}\right)^2$. This change is to be observed by means of transfer reactions.

The spin-quadrupole interactions can affect the structure of the 2^+ state wave function, especially in the case when one (or several) two-quasiparticle configuration is dominant, if $\varkappa_1 \quad 0$

3. Decay Properties of 0^+ and 2^+ States

Let us consider the probabilities for $E\lambda$ transitions from 0^+ and 2^+ states. The electrical multipole transition operator

$$\mathfrak{M}(\lambda\mu) = \sum_{sp,\, s'p'} f^{(\lambda\mu)}_{sp,\, s'p'}\, a^{+}_{sp}\, a_{s'p'} \tag{33}$$

utilizing the canonical transformation and separating the collective part may be written in the form

$$\begin{aligned}\mathfrak{M}(\lambda\mu) = {} & \sum_{ss'} 2\, v_s^2\, \delta_{ss'}\, f^{(\lambda\mu)}_{ss'} + \sum_{ss'} V_{ss'}\, f^{(\lambda\mu)}_{ss'}\, B_{ss'} + \\ & + \frac{1}{\sqrt{2}} \sum_{i} \sum_{ss'} U_{ss'}\, g^{i}_{ss'}\, f^{(\lambda\mu)}_{ss'} \left(Q^{+}_{i} + Q_{i}\right)\end{aligned} \tag{34}$$

where $f^{(\lambda\mu)}_{ss'}$ are the matrix elements of the $E\lambda$ transition between the one-particle states. The sums in (34) run over all the proton states. Utilizing the second quantization approximation one can derive the expression the for the $E\lambda$ transition matrix element as follow

$$M_i(E\lambda) = \frac{e_p}{\sqrt{2}} \sum_{ss'} U_{ss'}\, g^{i}_{ss'}\, f^{(\lambda\mu)}_{ss'} \tag{35}$$

where e_p is the proton charge.

In cutting off the summation it is necessary to introduce the effective charge and take the contribution of neutron states into account [13]. We introduce the effective charge in the following manner :

$$e_p' = e_p + e_{eff}\,, \quad e_n = e_{eff} \tag{36}$$

Consider E2-transitions of the type $0^+ \to 2^+$ (excitation of 2^+ state or decay of 0^+ state t o the rotational 2^+ state). Using (35) and (36) we get the reduced probability in single-particle units [x]:

x) $B(E2)_{s.p.} = 0.3\, A^{4/3}\, e^2\, fm^4$

$$B(E2, 0^+ \to 2^+)/B(E2)_{s.p.} = 0{,}84\, A^{-2/3} [Z(\omega, \varkappa_t)]^{-1} \times$$
$$\times \left| e_{eff} P(\omega, \varkappa_t) + F_{prot.}(\omega) + D(\omega, \varkappa_t) X_{prot.}(\omega) \right|^2 \qquad (37)$$

Here in $F_{prot.}(\omega)$ and $X_{prot.}(\omega)$ the summation is over the proton single particle states. The E0-transition reduced matrix element may be written in the form:

$$\rho_i^2 = \frac{1}{2R_0^4} \Big| e_p' \sum_{ss'(prot)} U_{ss'} g^i_{ss'} f^{(E0)}_{ss'} +$$
$$+ e_n \sum_{ss'(neut.)} U_{ss'} g^i_{ss'} f^{(E0)}_{ss'} \Big|^2 \qquad (38)$$

where R_0 is the nuclear radius. The single particle matrix elements $f^{(E0)}_{ss'}$ are derived in [14].

The spin-quadrupole interactions affect significantly B(E2) quantity near the spin poles of $P(\omega, \varkappa_t)$. Then $D(\omega, \varkappa_t)$ sharply increases and $Z(\omega, \varkappa_t) \sim D^2(\omega, \varkappa_t) \frac{\partial S(\omega)}{\partial \omega}$. The B(E2) quantity can be roughly estimated as

$$B(E2) \sim \left[\frac{\partial S(\omega)}{\partial \omega} \right]^{-1} \left| e_{eff} X(\omega) + X_{prot.}(\omega) \right|^2$$

Since the contributions of the two-quasiparticle states to $X(\omega)$ are non-coherent a sharp decrease of B(E2) value may be expected. From the physical point of view this corresponds to the switching off of the quadrupole interactions, i.e. to a collective state formed only by the spin-quadrupole interactions.

For the beta-transition probabilities to the 0^+ and 2^+ collective states the same formulae as in ref.[11], are obtained in which two-quasiparticle amplitudes $\Psi_{ss'}$ are to be replaced by $1/2\, (g^i_{ss'} + w^i_{ss'})$.

The reduction of diagonal amplitudes due to the spin-quadrupole interactions may affect the favoured beta-transition rate to the lowest 0^+ state. One can expect the transition rate to this state to be hindered as compared to the ground state, if the contribution of non-diagonal amplitudes become significant.

III. Calculations and Discussion of the Results

For some time past great attention was devoted to the properties of collective 0^+ states in deformed nuclei. Numerical calculations have shown that the microscopic theory with pairing plus quadrupole interactions gives a satisfactory description of lowest 2^+ vibrational states. But the properties of 0^+ vibrational states, predicted by the theory are in the poor agreement with experimental data. It is not, for example, able to desribe well the energies of 0^+ states in rare-earth region, using the single coupling parameter for all nuclei.

New low lying 0^+ states have been recently discovered in a number of deformed nuclei. In samarium isotopes the new 0^+ states were studied by means of (p,t) and (t,p) reactions [17] and in Yb^{168} and Hf^{174} nuclei - by means of (p,2n) and (d,d) reactions [6]. Information about the 0^+ states in Er^{164} and Hf^{178} was obtained from beta decay studies[18,19] Second excited 0^+ states also occur in the transuranium region (U^{234},U^{238}) [20].

These new states do not appear to be beta-vibrations. On the other hand the microscopic theory with pairing plus quadrupole interactions predicts the appearance of the second 0^+ states at energies of the order of 2 MeV. It seems the pairing vibrations are not responsible for the appearance of these states too [21].

Another approach was proposed by Belyaev [22]. He derives two branches of 0^+ excitations from the pairing interaction gauge invariance requirement. The E0-transition probabilites for these states are to be quite different. But no any numerical calculations were performed and it is difficult to say if the theory can feet the experimental data.

The aim of our calculations is to clarify the efficiency of the spin-quadrupole interactions as compared to the quadrupole interactions. It is necessary to investigate how the spin-quadrupole interactions affect the properties of low energy 0^+ states. We will try to answer the question if the spin-quadrupole interactions could be responsible for the appearance of the second 0^+ states below the two-quasiparticle energy.

The numerical calculations have been carried out for nuclei in the region $150 \leq A \leq 174$, assuming the deformation parameter to be approximately constant [23]. An improved version of the Nilsson scheme [24] was used in calculating the collective states, as in ref.[25,26]. The pairing interaction parameters were determined from the pairing energy calculations [27]. In all the calculations the blocking effect, the anharmonicity and other corrections were not inserted because we have no method to take them into account consistently.

We consider at first particular cases of the eq. (26):

$$\begin{aligned} 1/\varkappa_q &= F(\omega_i) \quad , \varkappa_t = 0 \\ 1/\varkappa_t &= S(\omega_i) \quad , \varkappa_q = 0 \end{aligned} \qquad (40)$$

The empirical energies of the first 0^+ states are used to determine the variation of coupling parameters $\varkappa_q$ and $\varkappa_t$ from nucleus to nucleus. The results are given in fig.1. It appeared that $\varkappa_q$ vary much more from nucleus to nucleus than $\varkappa_t$. Furthermore the

$\varkappa_t$ values are of the same order of magnitude as $\varkappa_q$, i.e. the spin-quadrupole interaotions appear to be much more efficient in deformed nuclei than in spherical nuclei.

Neglecting the mixing term $X(\omega)$ in eq. (26), we obtain two branches of collective 0^+ excitations below the two-quasiparticle energy$^{x)}$. They seem to possess quite different decay properties. For example, they may be distinguished judging from B(E2) values. Actually we have no theoretical justification, assuming the quadrupole and spin-quadrupole interactions to be exclusive. Numerical calculations have shown that the mixing term plays an important role and is to be taken into account. We put some requirement on the choice of the spin-quadrupole coupling parameter $\varkappa_t$:

a) The first pole of $P(\omega, \varkappa_t)$ appears at higher energy than the lowest empirical 0^+ level.

b) In some cases the solutions of eqs. (40) are very sensitive to the choice of coupling parameters. The introduction of the spin-quadrupole interactions have to reduce the sensitivity of the solution of eq. (26$'$) to the choice of $\varkappa_q$. In this point we have in mind, that the quadrupole interactions dominate in the formation of collective 0^+ states.

c) We have to explain the lowering of the second 0^+ states in a number of nuclei.

d) The introduction of the spin-quadrupole interactions is to be consistent with the empirical B(E2) values for 0^+ states.

The calculated energies of some 0^+ states and empirical data are listed in table 1. The calculations were carried out for pure quadrupde force as well as using the different $\varkappa_t$ values. The last column gives the energy of the first pole of $\Gamma(\omega)$. Numerical calculations have shown that it is impossible achieve satisfactory agreement with empirical data by the use of the single quadrupole or spin-quadrupoe- force. The second 0^+ states, predicted by the pairing plus quadrupole model, lie at or about 2 MeV. The pairing plus spin-quadrupole model predicts the second 0^+ states at higher energy.$^{xx)}$ Taking the quadrupole as well as spin-quadrupole interactions into account we obtain a better and more consistent description of the 0^+ states by the use of two coupling parameters for all the nuclei. The detailed agreement may be achieved with the fixed quadrupole coupling constant $\varkappa_q = 5.3 A^{-4/3} \hbar\omega_0$ and varying slightly the spin-quadrupole coupling parameter. It seems the more reliable $\varkappa_t$ value is to be obtained if a larger number of

$^{x)}$The mixing term may be dropped if one neglects the off-diagonal contributions in $\Gamma(\omega)$ As it was shown [11] this affects noticeably the energies of beta-vibtational states.

$^{xx)}$In this case pairing vibrations and spin-quadrupole interactions produce two different modes of collective excitations. They are the solutions of eqs.:

$$\gamma(\omega_i) = 0 \; ; \quad 1/\varkappa_t - S(\omega_i)$$

the single particle levels and transition matrix elements is involved into calculation.

The lowering of the 0^+ states is the principal qualitative result of the spin-quadrupole interactions. In particular, the tendency of the 0^+ state energy in the isotope families to increase with increasing mass number can change. But the position of the 0^+ states in some nuclei is extremely sensitive to the choice of the $\mathcal{K}_t$ value, whereas the sensitivity to the choice of the $\mathcal{K}_q$ value is decreased.

The spin-quadrupole interactions affect the decay properties of the 0^+ states. The calculated E2-transition probabilities are listed in table 2. The calculations we carried out, using the different $\mathcal{K}_q$ and $\mathcal{K}_t$ values, as well as for the empirical energies of the 0^+ states (the empirical energies are used to fix the $\mathcal{K}_q$ value for each nucleus). It was found that the spin-quadrupole interactions affect weakly the B(E2) values for nuclei in the beginning of the deformation region. All the calculated B(E2) values are consistent with the empirical data. However, in the middle of the region the spin-quadrupole interactions lead to the formation of comparatively long-lived (and low lying) 0^+ states in a number of nuclei.

The pairing plus quadrupole model predicts very small B(E2) values for the second 0^+- states. The spin-quadrupole interactions affect strongly these values. For a number of nuclei the B(E2) values for the first and for the second 0^+ states are of the same order of magnitude. Unfortunately, there are very scarce empirical data concerning the B(E2) values for the second 0^+ states.

The B(E2) values are sensitive to the choice of the $\mathcal{K}_t$ value. To show this the E2-transition probabilities for 0^+ states in ytterbium isotopes are computed by use of the different $\mathcal{K}_t$ values (the calculations are carried out for the empirical energies of the 0^+ states and for the different effective charge values). The results are given in table 3. It is seen that the spin-quadrupole effect can not be compensated by renormalizing the effective charge parameter.

The spin-quadrupole interactions affect significantly the two-quasiparticle amplitudes in the 0^+ state wave function. To illustrate this the calculated two-quasiparticle amplitudes $1/2\,(\psi_{ss'} + \omega_{ss'})$ for the first and the second 0^+ states in Er^{164} nucleus are tabulated in table 4. The introduction of the spin-quadrupole interactions leads to the decrease of the diagonal two quasiparticle amplitudes. The latter are dominant in the pairing plus quadrupole model calculations [25]. The spin-quadrupole interactions lead to the substantial increase of the off-diagonal amplitudes, that may affect noticeably the spectroscopic factor value for transfer reactions. Moreover this may hinder favoured beta decay rates to 0^+ states for a number of nuclei.

To illustrate the relation ship between the values of $F(\omega)$, $S(\omega)$, $X(\omega)$ and $P(\omega,\varkappa_t)$ these functions for Er^{164} nucleus are plotted on fig.2. The value of $X(\omega)$ is usually 5-10 per cent of $F(\omega)$ or $S(\omega)$. The plot of $P(\omega,\varkappa_t)$ is given for $\varkappa_t$ $8.8A^{-4/3}\hbar\omega_0$. The solutions of eq. (26) may be obtained by crossing this plot with a level. That corresponds to the choice of a certain $\varkappa_q$ value.

The appearance of the low-lying spin-pole depends strongly on the choice of $\varkappa_t$ value, because $S(\omega)$ is, in general, a considerably slowly varying function than $F(\omega)$.

We are very limited in choosing the $\varkappa_q$ and $\varkappa_t$ values. We obtain the imagine solution for Er and Yb isotopes increasing the $\varkappa_t$ value. On the other hand, the spin pole shifts to the two-quasiparticle energy with decreasing the $\varkappa_t$ value and the main features became almost insensitive to the introduction of the spin-quadrupole interactions. We obtain the imagine solution for Nd and Sm isotopes increasing the $\varkappa_q$ value. The B(E2) values for these nuclei increase strongly too. On the contrary, the decrease of the $\varkappa_q$ value leads to the poor agreement with empirical energies for Er and Yb isotopes,while the empirical energies in Nd and Sm isotopes are fitted a little better.

IY. Conclusion

The calculations performed showed that the introduction of the spin-quadrupole interactions leads to qualitative changes in the picture of the collective excitations in deformed nuclei, The spin quadrupole interactions may be responsible for the lowering of the second 0^+ states below the two-quasiparticle energy. They affect strongly the B(E2) values in a number of nuclei. The introduction of the spin-quadrupole interactions gives a better and a more consistent description of the 0^+ states than hitherto possible.

The spin-quadrupole interaction effects may be revealed in transfer reactions and beta- decay studies. Further theoretical investigations along this line are necessary.

Unfortunately, at present there is not enough experimental data to adjust the most reliable $\varkappa_t$ value. The main features obtained depend on the choice of this parameter.

The numerical calculations have shown the importance of the mixing term $X(\omega)$ especially on analysis the structure of the phonon wave function. The mixing term weakens the coherence of all the processes associated with collective state. The additional study is necessary to adjust the accuracy of our approximations.

At present it is difficult to offer the physical experiment which could reveal pure effects concerning the spin-quadrupole interactions. Measurement of the life-times of 0^+ and and 2^+ states, beta-decay study and transfer reactions will be of importance.

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Table I.

Low-energy 0^+ states calculated by use of the different values for coupling parameters $\varkappa_q$ and $\varkappa_t$ (in units of $A^{4/3}\hbar\omega_o$). Experimental data are taken from ref. / I5-I8,28-35/.

Nuclei	Energy (MeV)						First root of eq.
	experim.	$\varkappa_q$ = 5,3 $\varkappa_t$ = 0	5,3 8,3	5,3 8,8	5,3 9,0	0 9,0	$f(\omega)=0$
Nd I50	0,69 –	0 I,93	0 I,65	0 I,36	0 I,I6	I,I0	I,9I
Sm I52	0,685 I,I0	0,70 I,98	0,68 I,79	0,67 I,45	0,65 I,27	I,2I	I,95
Sm I54	I,I04 I,2I8	I,02 I,98	I,00 I,78	0,96 I,44	0,90 I,28	I,I3	I,94
Gd I54	0,68 –	0,94 I,98	0,93 I,87	0,9I I,59	0,90 I,43	I,36	I,97
Gd I56	I,04 –	I,20 I,98	I,I7 I,88	I,I3 I,65	I,09 I,44	I,34	I,97
Gd I58		I,57 I,98	I,53 I,93	I,39 I,68	I,2I I,64	I,30	I,96
Gd I60		I,77 I,94	I,68 I,82	I,28 I,78	I,03 I,77	I,06	I,84
Dy I58	0,99I –	I,29 2,08	I,27 I,92	I,23 I,63	I,I8 I,50	I,39	2,06
Dy I60	I,263 –	I,65 2,00	I,6I I,97	I,45 I,75	I,28 I,7I	I,34	I,98
Dy I62		I,78 I,99	I,73 I,83	I,34 I,80	I,09 I,79	I,I2	I,84
Dy I64		I,55 I,90	I,45 I,55	0,97 I,55	0,64 I,55	0,68	I,56

Table 1 (continued).

Er I64	I,245 I,698 I,765	I,75 I,84 2,00	I,7I I,84 2,00	I,34 I,77 I,84	I,I0 I,77 I,85	I,I2	I,83
Er I66	I,46 –	I,54 I,8I	I,46 I,54	0,98 I,54	0,65 I,49	0,65	I,55
Er I68		I,56 I,78	I,56 I,62	I,23 I,57	I,02 I,57	I,05	I,58
Er I70		I,46 I,79	I,45 I,74	I,38 I,48	I,2I I,47	I,23	I,54
Yb I68	I,I56 I,I96 I,543	I,53 I,74 I,85	I,52 I,57 I,76	I,I6 I,53 I,75	0,92 I,48 I,75	0,94	I,55
Yb I70	I,065 – –	I,56 I,7I I,83	I,55 I,67 I,75	I,36 I,56 I,73	I,I9 I,56 I,72	I,20	I,57
Yb I72	I,045 – –	I,44 I,73 2,06	I,44 I,74 I,83	I,42 I,75 I,80	I,34 I,46 I,75	I,38	I,53
Yb I74	I,32 – –	I,43 I,73 I,94	I,43 I,73 I,80	I,43 I,49 I,74	I,34 I,44 I,74	I,39	I,45

Table 2

Calculated and experimental /I5,I6/ B(E2) values (in single particle units) for the first and second 0^+ states.

I. $\varkappa_t = 0$; $\varkappa_q = 5{,}3\ A^{-4/3}\hbar\omega_o$

2. $\varkappa_t = 0$; $\varkappa_q = \varkappa_q$ (ω_{0^+} experim.)

3. $\varkappa_t = 8{,}8\ A^{-4/3}\hbar\omega_o$; $\varkappa_q = \varkappa_q$ (ω_{0^+} experim.)

4. $\varkappa_t = 8{,}8\ A^{-4/3}\hbar\omega_o$; $\varkappa_q = 5{,}3\ A^{-4/3}\hbar\omega_o$.

The corresponding energies of 0^+ states are listed in table I.

Nuclei	B (E2, $0^+ \to 2^+$), $e_{eff} = 0{,}3$				B(E2) exper.
	I	2	3	4	
Nd ^{150}	– 0,0I	5,3 –	4,7 –	– 0,08	5 ± I,3 –
Sm ^{152}	4,3 0,08	4,4 2,3	4,0 I,85	4,I 0,05	2,5 ± 0,6 –
Sm ^{154}	2,85 0,03	2,4 –	I,8 –	2,5 0,2	I,2 ± 0,3 –
Gd ^{154}	2,6 $\sim 10^{-3}$	3,9 –	3,6 –	2,5 0,04	4,8 ± I,2 –
Gd ^{156}	I,8 $\sim 10^{-3}$	2,4 –	2,I –	I,7 0,I0	2,8 ± I,2 –
Gd ^{158}	I,I $\sim 10^{-3}$	– –	– –	0,6 0,5	–
Gd ^{160}	0,3 0,I	– –	– –	0,I 0,2	– –
Dy ^{158}	I,4 $\sim 10^{-3}$	2,3 –	2,0 –	I,3 0,I	> 0,3 –
Dy ^{160}	0,8 0,0I	I,8 –	I,3 –	0,5 0,3	– –

Table 2 (continued).

Dy I62	0,I4	–	–	0,I	–
	0,09	–	–	0,I	–
Dy I64	0,0I	–	–	0,04	–
	0,02	–	–	0,0I	–
Er I64	0,24	2,3	0,6	0,I2	$>$ 0,05
	$\sim$ IO^{-3}	0,5	0,4	0,2	–
Er I66	0,03	0,5	0,4	0,04	–
	0,I	–	–	0,03	–
Er I68	0,02	–	–	0,05	–
	0,I5	–	–	0,02	–
Er I70	0,I	–	–	0,I	–
	0,03	–	–	0,04	–
Yb I68	0,04	3,0	0,04	0,04	–
	0,3	2,7	$\sim$ IO^{-3}	0,03	
Yb I70	0,03	3,0	I,9	0,06	–
	0,3	–	–	0,02	–
Yb I72	0,2	I,7	I,5	0,2	–
	0,2	–	–	0,I	–
Yb I74	0,05	0,8	0,6	0,06	$\leq$ 0,5
	0,2	–	–	0,04	–

Table 3.

Calculated B(E2, $0^+ \to 2^+$)values (in single particle units) for the first 0^+ states in Ytterbium isotopes. The calculations are made by use of the different $\varkappa_t$ and e_{eff} values and experimental energies of 0^+states/I5,I6,34/

Nucleus and energy of 0^+ state	$\varkappa_t A^{4/3}$ / $\hbar\omega_o$	e_{eff}			$B(E2)_{s.p.u.}$ experim.
		0.2	0.3	0.4	
Yb^{I68} I,I56 MeV	0.0	2.34	3.03	3.8I	>3.6·I0^{-3} (~ I.0)
	8.3	I.77	2.29	2.88	
	8.8	0.03	0.04	0.06	
	9.0	I.02	I.3I	I.63	
Yb^{I70} I,065 MeV	0.0	2.30	2.98	3.76	
	8.3	2.02	2.62	3.3I	
	8.8	I.47	I.92	2.42	
	9.0	0.66	0.86	I.08	
Yb^{I72} I,045 MeV	0.0	I.3I	I.73	2.20	
	8.3	I.25	I.65	2.II	
	8.8	I.I6	I.53	I.96	
	9.0	I.03	I.35	I.73	
Yb^{I74} I,32 MeV	0.0	0.58	0.75	0.95	≤ 0.5
	8.3	0.56	0.73	0.93	
	8.8	0.48	0.6I	0.77	
	9.0	0.I2	0.I6	0.20	

Table 4.

Two-quasiparticle amplitudes $\frac{1}{2}\,(\,g_{ss'} + w_{ss'})$ in 0^+ state phonon wave function in nucleus Er^{164}. The calculations are made using experimental energies of 0^+ states /18/.

Configuration		$\omega_1 = 0,167\hbar\omega_0$ (1,25 MeV)		$\omega_2 = 0,227\hbar\omega_0$ (1,7 MeV)	
		$\varkappa_t = 0$	$8,8A^{-4/3}\hbar\omega_0$	0	$8,8\,A^{-4/3}\hbar\omega_0$
		neutron two-quasiparticle states			
505+	505+	-.988	-.429	-.433	-.417
651+	651+	.145	.063	.119	.115
521+	521+	-.358	-.156	-.099	-.095
642+	642+	.499	.217	.840	.808
523-	523-	-.641	-.279	-.712	-.685
633+	633+	-.290	-.126	-.132	-.127
521-	521-	-.278	-.121	-.118	-.114
512+	512+	-.337	-.114	-.136	-.131
541-	510+	.045	.461	.019	-.164
532-	521+	.044	.164	.021	-.049
521+	512-	.028	.320	.011	-.110
523-	512+	.041	.355	.019	-.142
532+	523-	.027	.143	.010	-.039
		proton two-quasiparticle states			
411-	411-	-.075	-.033	-.327	-.314
404-	404-	-.430	-.187	-.324	-.312
532+	532+	.255	.111	.152	.146
523+	523+	.425	.185	.486	.468
523+	514-	.019	.149	.007	-.040
411+	402-	-.025	-.272	-.010	.094
420+	411-	-.030	-.273	-.012	.093
541+	532-	.069	.279	.026	-.067
514+	514+	-.214	-.093	-.195	-.187

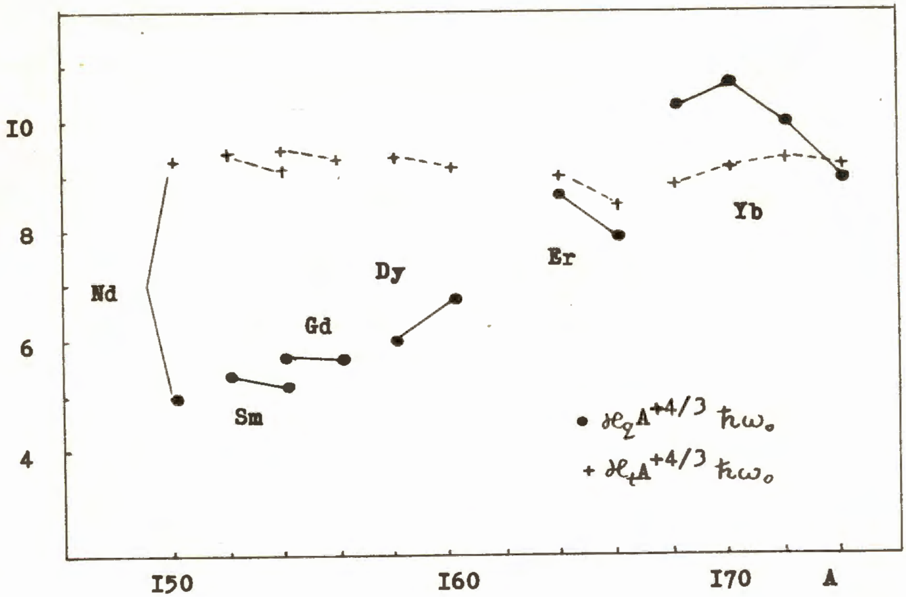

Fig.I. The quadrupole and spin-quadrupole coupling parameters, estimated from empirical energies of the first 0^+ state.

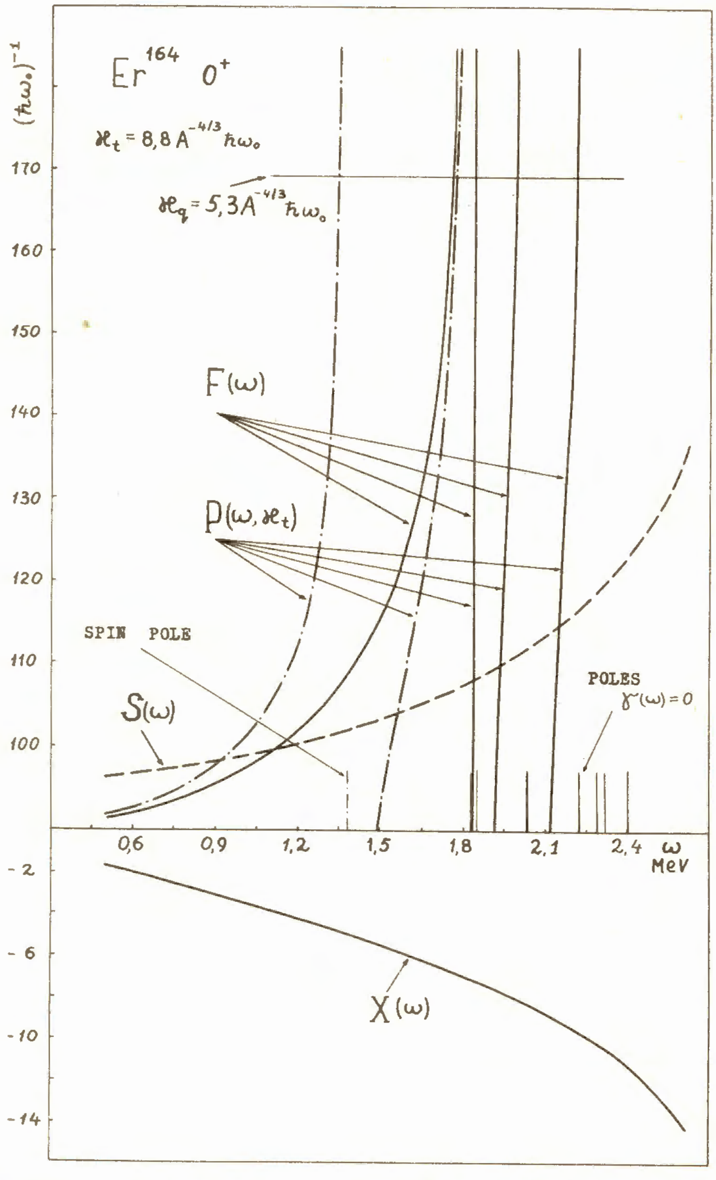

Fig.2. Typical plots of functions $F(\omega)$, $S(\omega)$ and $X(\omega)$. The plot of $P(\omega,\varkappa_t)$ is given for $\varkappa_t = 8,8A^{-4/3}\hbar\omega_0$.