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FLUX FLATTENING BY MEANS OF NON-UNIFORM FUEL DISTRIBUTION IN A SLAB REACTOR WITH FINITE REFLECTOR

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1. Introduction

A flattened flux of thermal neutrons in a reactor leads to a better delivery of thermal output and also to a more uniform burn-up of fuel.

One way how to flatten the flux in a reactor is the non-uniform distribution of fuel. This problem was already investigated mostly in connection with the minimum of critical mass of a reactor [1], [2].

In paper [4], where flattening of thermal flux in the system natural uranium D_2O with infinite reflector is investigated in a mathematically different way, it appears that the required concentrations of fuel are substantially lower than concentrations commonly used in power-engineering reactors and thus also the influence of the unequal p is small.

In this paper we shall study the influence of the size of a reflector on the critical dimension and concentration of fuel in a slab reactor with flattened flux. It appears that concentration of fuel increasses with the decreasing thickness of reflector. Thus the resonance escape probability falls the criticality of the reactor is becoming worse. It can be supposed that there exists a certain minimum thickness of reflector for which the reactor with flattened flux can still be realized and that the concentration of fuel for reactors with thickness of reflector near to minimum is already in the sphere of concentrations usual in power-engineering reactors.

Solving of the Problem

The reactor is described by means of two-group equations of the type

$$-D\Delta\Phi(\vec{r})+[\sigma_a^U m(r)+\Sigma_a^m]\Phi(\vec{r})=q(\vec{r}) \tag{2.1}$$

$$-\tau\Delta q(\vec{r})+q(\vec{r})=\eta p[m(\vec{r})]\cdot m(\vec{r})\cdot\sigma_a^U\cdot\Phi(\vec{r}), \tag{2.2}$$

where D - diffusion coefficient,

τ - age of neutrons down to thermal energies,

Σ_a^m - macroscopic cross section for absorption in moderator,

σ_a^U - microscopic cross section for absorption in fuel (in barns),

p - resonance escape probability,

η - regenerating factor,

$\Phi(\vec{r})$ - thermal flux,

$q(\vec{r})$ - slowing down density,

$m(\vec{r})$ - fuel concentration (10^{24} atoms/cm^3).

By means of derivation from Boltzmann's equation, with the help of the method mentioned in /8/ considering the space dependence of the coefficients it is possible to prove that this type of two-group equations is the most accurate approximation of this equation for problems with space variable fuel concentration. Other usual types of two-group equations are equivalent (2,1) and (2,2) for constant coefficients. D, τ and Σ_a^m are supposed to be space independent so we do not consider the pressing-out of the moderator by fuel.

The requirement of the flattening of thermal flux $\Phi(\vec{r})=\Phi_0$ = const. simplifies the system of equations (2,1) and (2,2):

$$[\sigma_a^U \cdot m(\vec{r}) + \Sigma_a^m]\,\Phi_0 = q(\vec{r}) \qquad (2.1)$$

$$-\tau\Lambda q(\vec{r}) + q(\vec{r}) = \eta\, p[m(\vec{r})] \cdot m(\vec{r}) \cdot \sigma_a^U \cdot \Phi_0 \,. \qquad (2.2)$$

In a reflector which is considered to be the same as moderator it holds:

$$-D\Lambda\Phi_r(\vec{r}) + \Sigma_a^m \Phi_r(\vec{r}) = q_r(\vec{r}) \qquad (2,1'')$$

$$-\tau\Lambda q_r(\vec{r}) + q_r(\vec{r}) = 0. \qquad (2,2'')$$

At the core-reflector interface the usual conditions of continuity of thermal and fast fluxes and currents hold ; Φ_r and q_r at the and of the reflector equal zero.

Eq. (2.1) and (2.2) will be transferred to the equation for fuel concentration:

$$\Lambda m(\vec{r}) + \frac{\eta p[m(\vec{r})] - 1}{\tau} \quad m(\vec{r}) = \frac{\Sigma_a^m}{\tau\,\sigma_a^U} \,. \qquad (2.3)$$

Because of the factor $p(m)$ this equation is nonlinear. For the slab geometry, however, where

$$\frac{d^2}{dz^2} m(z) + \frac{\eta p[m(z)] - 1}{\tau}\, m(\vec{r}) = \frac{\Sigma_a^m}{\tau\sigma_a^U} \qquad (2.3')$$

holds, it has an analytical solution. We introduce a coordinate system in such a way that the origin will be situated in the plane of reactor symmetry. The end of the core be denoted as h , the end of the reflector H and the thickness of the reflector $\Lambda H = H - h$. Since in the Eq. (2.3') the independently variable does not appear explicitly, we can use the substitution /5/: $\frac{dm}{df} = f(m)$ and find the analytical solution in the implicit form:

$$z = h - \int_{m(h)}^{m(z)} \frac{dm'}{\sqrt{\int_{m'}^{m(0)} \kappa(m'')\, dm''}} \,, \qquad (2.4)$$

where $\kappa(m) = \frac{2}{\tau}\{[\eta p(m) - 1] \cdot m - \frac{\Sigma_a^m}{\sigma_a^U}\}$. Curve $\kappa(m)$ for the system natural uranium – D_2O is plotted in graph 1. Thus for the critical halfthickness of the core it holds:

$$h = \int_{m(h)}^{m(0)} \frac{dm'}{\sqrt{\int_{m'}^{m(0)} \kappa(m'')\, dm''}} \,. \qquad (2.4')$$

Concentrations in the centre $m(0)$ and on the boundary $m(h)$ have the meaning of integration constants and must be determined from the boundary conditions. For the symmetry reasons it will be sufficient to consider conditions in points 0 , h and H .

Solution in a reflector which satisfies the boundary conditions is evident; it holds:

$$\Phi_r(z)=\frac{a_1}{D}\,\frac{L^2_r}{r-L^2}\,\mathrm{sh}\,\frac{z-H}{\sqrt{r}}+a_2\,\mathrm{sh}\,\frac{z-H}{L}, \tag{2.5}$$

$$q_r(z)=a_1\,\mathrm{sh}\,\frac{z-H}{\sqrt{r}}. \tag{2.6}$$

In the core it holds $\Phi(z)=\Phi_0$; $q(z)=[m(z)\sigma_a^U+\Sigma_a^m]\,\Phi_0$. From boundary conditions $\Phi(h)=\Phi_r(h)$; $\frac{d\Phi}{dz}/_h=\frac{d\Phi_r}{dz}/_h$; $q(h)=q_r(h)$, $\frac{dq}{dz}/_h=\frac{dq_r}{dz}/_h$, both constants a_1, a_2 can be eliminated and we get equations determining $m(h)$ and $m(0)$:

$$m(h)=\frac{\Sigma_a^m}{\sigma_a^U}\left\{\frac{r-L^2}{\sqrt{r}}\cdot\frac{\mathrm{tg\,h}\,\frac{\Delta H}{\sqrt{r}}}{r\,\mathrm{tg\,h}\frac{\Delta H}{\sqrt{r}}-L\,\mathrm{tg\,h}\frac{\Delta H}{L}}-1\right\}, \tag{2.7}$$

$$\int_{m(h)}^{m(0)}\kappa(m')\,dm'=\frac{\left[m(h)+\frac{\Sigma_a^m}{\sigma_a^U}\right]^2}{r\left(\mathrm{tg\,h}\,\frac{\Delta H}{\sqrt{r}}\right)^2}. \tag{2.8}$$

Curves $m(0)$ and $m(h)$ in dependence on ΔH are plotted in graph 2.

Considering the physically meaningful cases (the requirement of non-negativity of the expression under radical in Eq. (2.4) we get a condition for maximum admissible concentration of fuel in the centre of reactor M_0 :

$$\kappa(M_0)=0 \tag{2.9}$$

which determines ΔH_{min} by means of (2.8) and (2.7).

By means of a slow mathematical analysis of conditions (2.4), (2.7), and (2.8) we can show that the consequence of exponential character of dependence $p(m)$ is $\lim\limits_{\Delta H\to\Delta H_{min}} h(\Delta H)=\infty$.

Integrands in (2.4) and in the relation for calculation of mean concentration of fuel

$$\bar{m}=\frac{1}{h}\int_{m(h)}^{m(0)}\frac{m'\,dm'}{\sqrt{\int_{m'}^{m(0)}\kappa(m^*)\,dm^*}} \tag{2.10}$$

have divergence of the type $1/\sqrt{x}$; this fact, however, does not matter in numerical calculation. But ΔH being near to ΔH_{min} and thus $m(0)$ near to M_0 the numerical integration in (2,4) and (2.10) is impossible and in the vicinity of these points the asymptotic expressions hold:

$$h \approx \frac{1}{\sqrt{|\kappa'(M_0)|}} \left| \lg \frac{M_0 - m(0)}{2M_0} \right| \tag{2.11}$$

$$\bar{m} \approx M_0 - \frac{M_0 - m(h)}{\left| \lg \frac{M_0 - m(0)}{2[M_0 - m(h)]} \right|} \tag{2.12}$$

which we get by substituting the $\kappa(m)$ by expansion $\kappa(m) = \kappa'(M_0)(m - M_0)$.

For h equalling about one triple of the critical size of the reactor with infinite reflector, the error in critical size according to (2.11) is now only about 3% and in the mean concentration according to (2.12) about 2%.

3. Numerical Results and Conclusions

Natural uranium as the fuel technical heavy water as the moderator were considered for numerical calculations. Solution was performed for homogeneous mixture of fuel and moderator; the dependence $p(m) = \exp\left[-\frac{m}{\xi \Sigma_s^m} \cdot 3,9 \left(\frac{\Sigma_s^m}{m}\right)^{0,415}\right]$ was taken from paper /3/. For a heterogeneous arrangement of fuel in the form of infinite slab elements having the constant thickness 2δ , the modified relation /6/ was used. ($\frac{2\delta}{a} = \frac{m}{\mu} 10^{24}$; a is the pitch of the elemehts; μ is Loschmidt's number, $\mu_{uranium} = 0,0472 \cdot 10^{24}$):

$$p(m) = \exp\left[-\frac{b}{\mu} 10^{24} \left(\frac{0,435}{2\sqrt{\delta}} + \frac{A}{\pi}\right) m\right],$$

where A and b are constants mentioned in /6/. With respect to the fact that the method is based on homogeneous approximation , the blocking effect is not considered in the thermal region of energies. This effect is studied individually in paper /9/ on principle of the heterogeneous method. In the solution we did not consider the influence of pressing-out the moderator by fuel. It can be awaited that this influence will be very small because even for the maximum concentration of the fuel M_0 the volume ratio of fuel and moderator is about 1 : 30.

The calculation was effected according to the following scheme :

1)for the chosen ΔH we calculate $m(h)$ according to (2.7) and $m(0)$ according to (2,8),

2) the course of $m(z)$ and the critical halfthickness h will be gradually calculated from (2.4) and (2.4') and $\bar{m}$ will be calculated from (2.10).

The calculated dependences of critical sizes on the thickness of reflector are plotted in graph 3.

Calculating the critical size for great thickness of reflector we find an apparently paradox result. The thickness of reflector being decreased from an infinite value up to a certain value ΔH, the critical dimension falls, too. This fact is possible to be exactly shown by expansion of relations where ΔH appears in the neighbourhood of the point $1/\Delta H = 0$. This effect, however, is compensated by the increase of the total critical mass. As it is evident from graph 3 this effect is very small. The space distributions of fuel concentration are demonstrated in graph 4.

From the comparison of the mean concentration of fuel and dimensions of the core with analogous quantities in real heavy water reactors /7/, mentioned in the following table we can judge that even the reguirements of constructional and technique-economical character can be met.

Reactor	ηf	$B^2[\bar{m}^2]^+$	$\bar{m} \cdot 10^3$
NPD	1,15	2,74	1,12
CVTR (enriched by 1,8% of U 235)	1,61	10	1,55
R - 3/ ADAM	1,28	2,81	1,29
Homog.Slab Reactor with flattened therm. flux h = 90 cm	1,34	2,78+	1,2

+ fictive parameter

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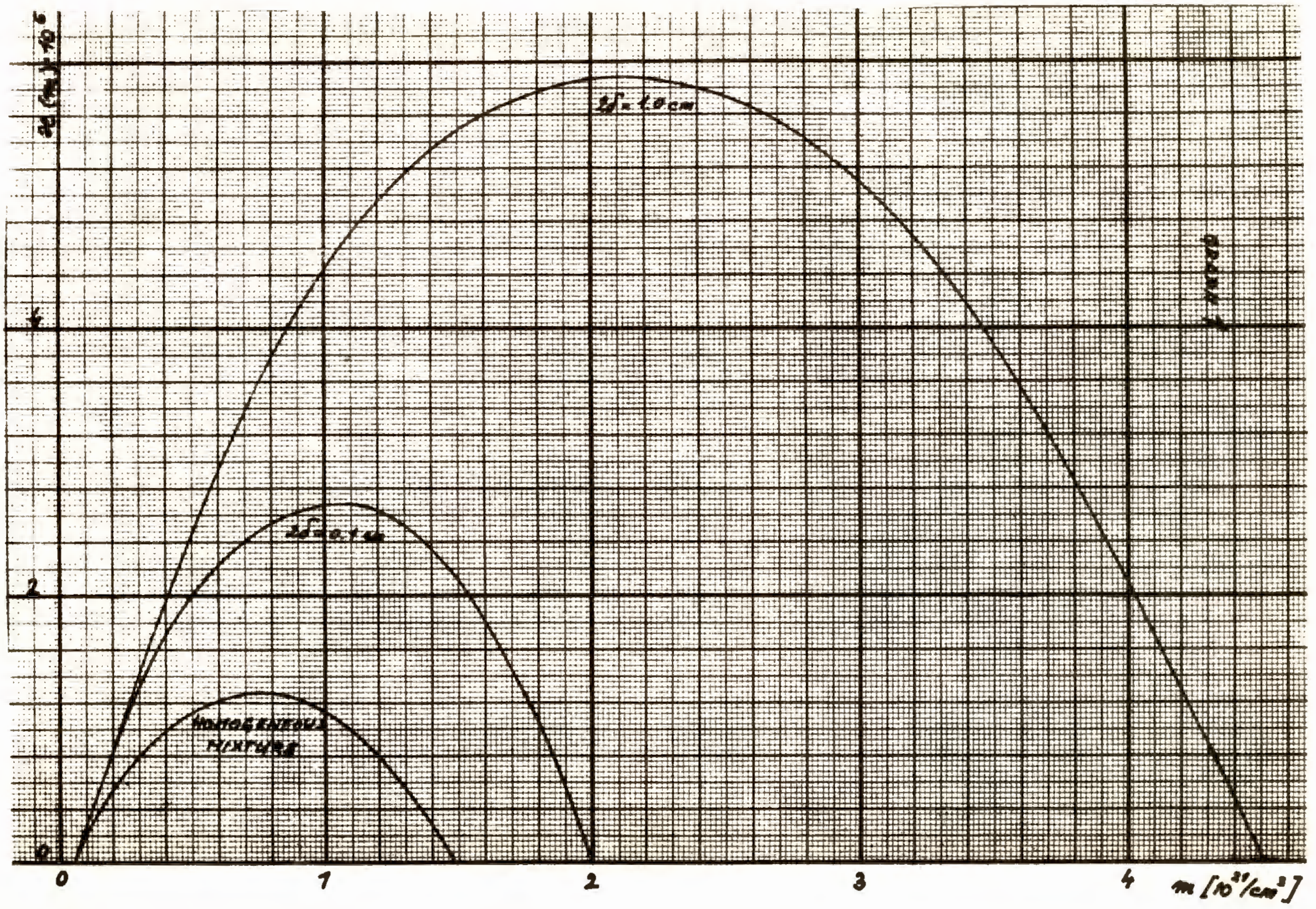
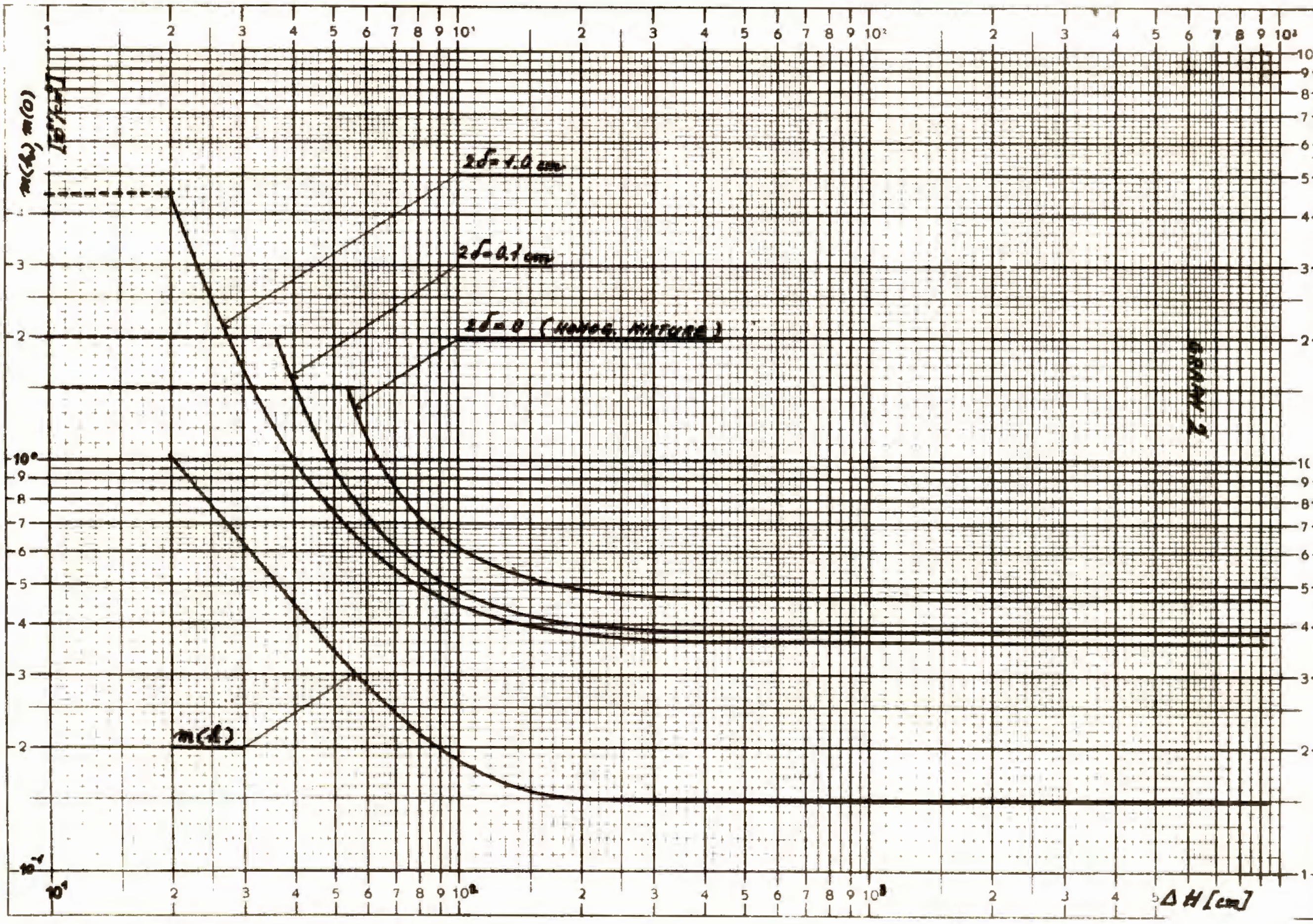
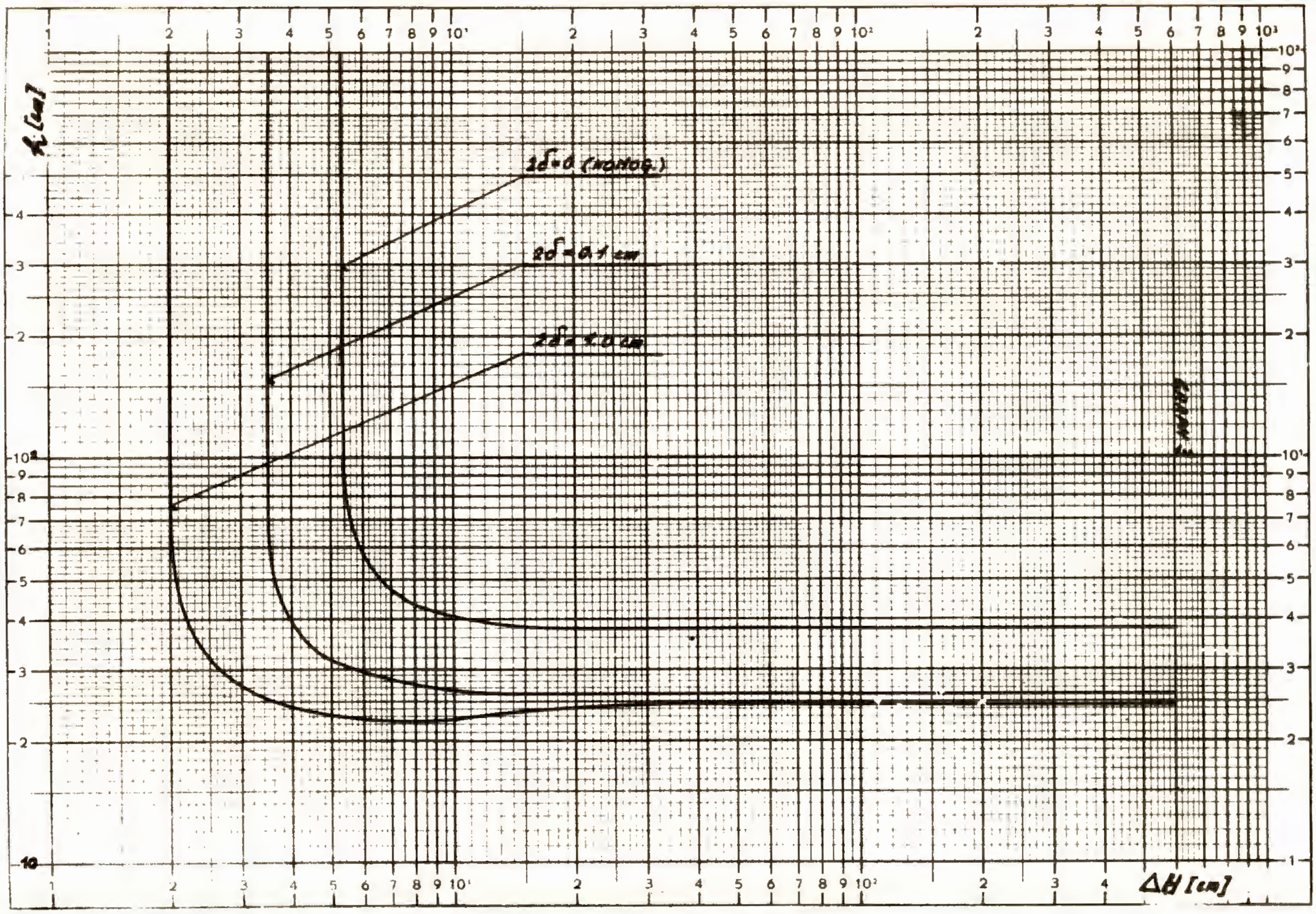
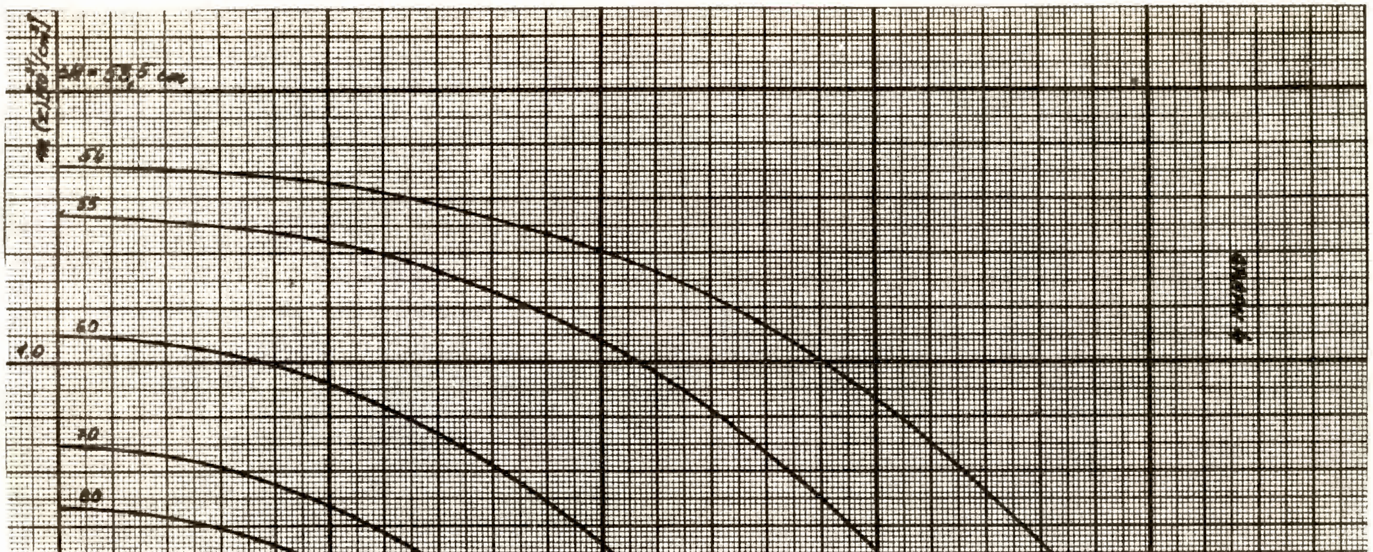

2δ = 1.0 cm
2δ = 0.1 cm
HOMOGENEOUS MIXTURE
0
1
2
3
4
m [10²¹/cm³]
GRAPH 1


2δ = 1.0 cm
2δ = 0.1 cm
2δ = 0 (HOMOG. MIXTURE)
m(h)
ΔH [cm]
GRAPH 2


2δ=0.1 cm
ΔH [cm]


ΔH = 53,5 cm
56
55
60
70
80
100
150
0
20
40
60
80
z [cm]