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PARTIAL WIDTHS OF NONMESONIC WEAK DECAYS OF Λ-HYPERNUCLEI

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1 Introduction

Recent papers [1] consider the possibility of studying the spin-isospin dependence of the matrix elements of non-leptonic weak interactions $\Lambda N \to NN$ registering two correlated α-particles created in the reactions

$$\begin{array}{lclcl} {}^{10}_{\Lambda}\mathrm{Be} & \to & n+p & + & ({}^{8}\mathrm{Li} \xrightarrow{\beta^-} {}^{8}\mathrm{Be} \to \alpha\alpha) \\ & \to & n+n & + & \boxed{{}^{8}\mathrm{Be}^* \to \alpha\alpha} \\ {}^{10}_{\Lambda}\mathrm{B} & \to & n+p & + & \boxed{{}^{8}\mathrm{Be}^* \to \alpha\alpha} \\ & \to & n+n & + & ({}^{8}\mathrm{B} \xrightarrow{\beta^+} {}^{8}\mathrm{Be} \to \alpha\alpha) \end{array} \tag{1}$$

by the decay of excited states of product nucleus ^{8}Be. It was stated, that the knowledge of experimental $\alpha\alpha$-decay widths (4 for protons and 4 for neutrons) is sufficient for complete determination of all four (eight) matrix elements of effective weak interaction

$$w^{SJ}_{l=1,\tau} = \left|\sum_{L'S'} \langle NN : L'S'T' : J|V_{\mathrm{w}}|\tau p s_{\Lambda} : L = 1SJ\rangle\right|^2, \tau = n, p. \tag{2}$$

In this short note we will concentrate on two important questions:
i) Is it possible to extract 4 matrix elements $w^{SJ}_{l\,\tau}$ from the experimental data?
ii) How do the obtained matrix elements depend on the nuclear "residual" interaction employed in calculations of the wave functions for $A = 9$ and $A = 8$ nuclei?

To answer these questions, we will use standard approaches of the many-particle nuclear shell model: the spectra and wave functions of nuclear excited states are obtained by the diagonalization of the Hamiltonian matrix. The parameters of the used residual two-body interaction were determined by the fit of a large amount of experimental spectroscopic data on a wide range of nuclei.

2 Partial width of nonmesonic decays

The main observable in weak nonmesonic decays of a hypernucleus is the total decay width

$$\Gamma_{\mathrm{nm}} = \sum_{\tau=n,p} \Gamma^{\tau}_{\mathrm{nm}} = \sum_{\tau=n,p} \sum_{E_i,J_i,T_i} \left|\langle\left[\Psi^{(A-2)}(\{i\}) \cdot \Psi^{(NN)}(JT)\right]^{\mathcal{J}}|V_{\mathrm{w}}|\left[\Psi^{(A-1)}(\{c\}) \cdot \psi^{\Lambda}(\tfrac{1}{2})\right]^{\mathcal{J}}\rangle\right|^2, \tag{3}$$

here $\{c\} \equiv \{E_c, J_c, T_c, \tau_c\}$ and $\{i\} \equiv \{E_i, J_i, T_i, \tau_i\}$ are sets of quantum numbers describing the states of initial and final nuclei. The hypernucleus decays from its ground state in which the nucleons form a core nucleus in its ground state, $\Psi^{(A-1)}(E_c, J_c, T_c, \tau_c)$, and Λ-hyperon has a minimal energy. In the shell model representation:

$$|I\rangle = |l^k, E_c = 0, J_c, T_c, \tau_c; s_\Lambda : \mathcal{J}\rangle.$$

We consider two decay channels: $n\Lambda \to nn$ and $p\Lambda \to pn$, in which the Λ-hyperon picks-up one $1p$-nucleon of the core-nucleus. Employing the technique of fractional parentage coefficients (FPC), the ground state wave function of the initial hypernucleus is decomposed [2] into a complete set of wave functions of excited states in the residual $(A-2)$ nucleus $\Psi^{(A-2)}(E_i, J_i, T_i, \tau_i)$ coupled to the complete set of wave function of states of an $N\Lambda$ pair, $|\tau l, s_\Lambda : L = lSJ\rangle$. So, the wave function of the initial state of the hypernucleus is

$$|I\rangle = \sum_{\tau,j,l} \sum_{E_i,J_i,T_i} \sum_{J,S} \sqrt{k}\,(T_i\tau_i \tfrac{1}{2}\tau|T_c\tau_c)\, U(J_i j \mathcal{J} \tfrac{1}{2} : J_c J) \times U(l\tfrac{1}{2}J\tfrac{1}{2} : jS)\, g^{E_c J_c T_c}_{E_r J_r T_r} \left[|k^{-1} E_r J_r T_r \tau_r\rangle \otimes |\tau l\, s_\Lambda : L = l, SJ\rangle\right]^{\mathcal{J}} \quad (4)$$

and

$$\Gamma_{\rm nm} = \sum_\tau \sum_i \Gamma_i^\tau$$

where

$$\Gamma_i^\tau = \sum_{SJ} G^2_{\mathcal{J}}(\{c\},\{i\},\tau lSJ)\, w^{SJ}_{l\tau} \quad (5)$$

Here

$$w^{SJ}_{l\tau} = \left|\sum_{L',S'} \langle l_1 l_2 : L'S'JT|V_{\rm w}|\tau l,\, s_\Lambda : L = lSJ\rangle\right|^2 \quad (6)$$

are unknown matrix elements of "weak interaction" to be extracted from partial transition widths. The factor $G_{\mathcal{J}}$ is equal to

$$G_{\mathcal{J}}(\{c\},\{i\},\tau lSJ) = \sum_j U(J_i j \mathcal{J}\tfrac{1}{2} : J_c J)\, U(l\tfrac{1}{2}J\tfrac{1}{2} : jS)\, S_i(\tau lj)\,, \quad (7)$$

where $S_i(\tau lj)$ are spectroscopic amplitudes for the separation of one nucleon from the ground state of the nucleus

$$S_i(\tau lj) = \sqrt{k}\,(T_i\tau_i\ \tfrac{1}{2}\tau\ |\ T_c\tau_c)\, g^{E_cJ_cT_c}_{E_iJ_iT_i}(lj), \quad (8)$$

and $g_i^c(lj)$ is a one-nucleon FPC in the intermediate coupling

$$g_i^c(lj) = \sum_{f_c L_c S_c} \sum_{f_i L_i S_i} a^{E_c J_c T_c}_{f_c L_c S_c}\, a^{E_i J_i T_i}_{f_i L_i S_i} \times \langle l^k[f_c]L_cS_cT_c\{|\ l^{k-1}[f_i]L_iS_iT_i\rangle \begin{pmatrix} L_i & S_i & J_i \\ l & \frac{1}{2} & j \\ L_c & S_c & J_c \end{pmatrix}. \quad (9)$$

The coefficients $a^{E_cJ_cT_c}_{f_cL_cS_c}$ and $a^{E_iJ_iT_i}_{f_iL_iS_i}$ are results of the shell-model Hamiltonian diagonalization, e.g. [3].

The partial widths of nonmesonic decay of $1p$-shell hypernuclei for transition into natural parity states are linear combinations of four matrix elements ($^1P_1$, $^3P_0$, $^3P_1$ and $^3P_2$) only. From the equation (4) one can easily see that partial widths corresponding to different J_i values are determined by quite definite (and different) combinations of matrix elements $w^{SJ}_{l\tau}$. The coefficients of these combinations for our case ($J_c = \frac{3}{2}$, $\mathcal{J} = 1$) are given below.

	$^3P_0$	$^1P_1$	$^3P_1$	$^3P_2$
$J_i = 0$		$\sqrt{\frac{2}{3}}\, g_{\frac{3}{2}}$	$\sqrt{\frac{1}{3}}\, g_{\frac{3}{2}}$	
$J_i = 1$	$\sqrt{\frac{2}{3}}\, g_{\frac{1}{2}}$	$-\sqrt{\frac{1}{9}}\, g_{\frac{1}{2}} + \sqrt{\frac{5}{9}}\, g_{\frac{3}{2}}$	$\sqrt{\frac{2}{9}}\, g_{\frac{1}{2}} + \sqrt{\frac{5}{18}}\, g_{\frac{3}{2}}$	$\sqrt{\frac{1}{6}}\, g_{\frac{3}{2}}$
$J_i = 2$		$-\sqrt{\frac{1}{3}}\, g_{\frac{1}{2}} + \sqrt{\frac{1}{3}}\, g_{\frac{3}{2}}$	$\sqrt{\frac{2}{3}}\, g_{\frac{1}{2}} + \sqrt{\frac{1}{6}}\, g_{\frac{3}{2}}$	$\sqrt{\frac{1}{2}}\, g_{\frac{3}{2}}$
$J_i = 3$				$g_{\frac{3}{2}}$

As a result, in an ideal case when the transitions to final states with $J_i = 0, 1, 2$ and 3 are observed, one can unambiguously determine all four matrix elements $w^{SJ}_{l\tau}$. The nuclear residual interaction accounted by many-particle shell model influences the relative $g_{\frac{1}{2}}$ and $g_{\frac{3}{2}}$ quantities.

It is supposed that from the measurements of correlated α-particles emitted in decay of ^{8}Be* (either direct or delayed, after $\beta^\pm$-decay of ^{8}B and ^{8}Li) one can extract the partial widths related to the following states of ^{8}Be: $(0^+0)_{\rm g.s.}$, $(2^+0)_1$, $(2^+1)_1$, $(2^+0)_2$, and $(1^+1)_1$ (see Fig. 1). The latter transition goes through β-decay of ^{8}Li. There is no observed transition to the 3^+ state, but the 2^+ states with excitation energies near to 3 MeV and 16 MeV relate with **different** configurations: $^1D_2$ and $^3P_2$ respectively. Therefore, **we have a necessary number** of linear independent equations for determination of **all four (eight) matrix elements** $w^{SJ}_{l\tau}$.

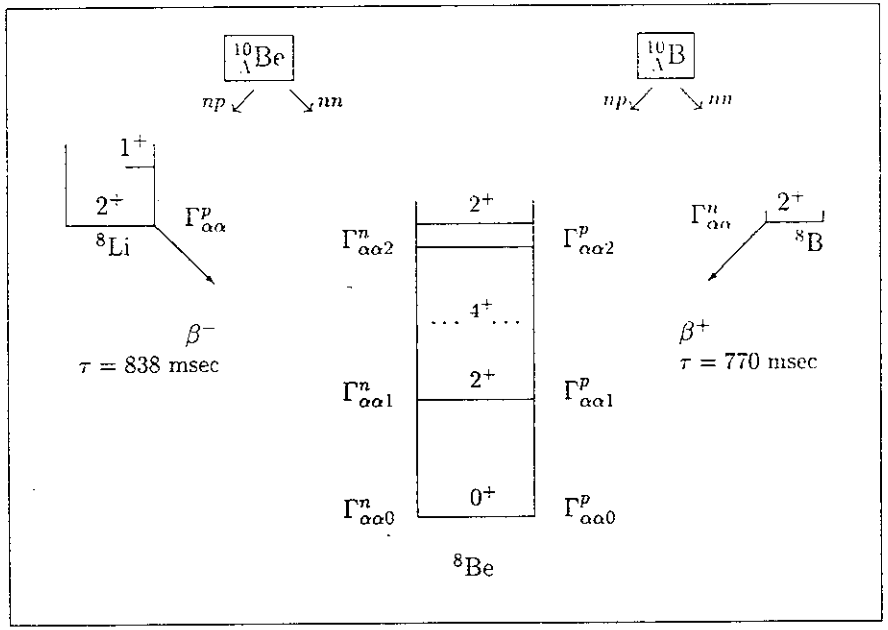

Fig. 1: The notation of the partial widths $\Gamma^{\tau}_{\alpha\alpha i}$.

Table 1 illustrates the influence of a nuclear model on the calculated one-body spectroscopic amplitudes. The used models employ the full $1p$-shell space and differ mainly in the nuclear residual interactions. We have selected several models used usually in the calculations of characteristics of $1p$-shell nuclei. The calculations were performed with the many-particle shell model code OXBASH [4].

The main attention in this paper is paid to the theoretical analysis, nevertheless, a few words should be said about the experimental situation. It is quite possible that $\Gamma_{\alpha\alpha 1}$ and $\Gamma_{\alpha\alpha 2}$ widths could be measured at the Dubna Nuclotron [5]. The fact that

$$\Gamma_{\alpha\alpha 2} = \Gamma_{\alpha\alpha}\left((2^+1)_1\right) + \Gamma_{\alpha\alpha}\left((2^+0)_2\right),$$

does not complicate the analysis. We would like to stress that the measurement of $\Gamma^p_{\alpha\alpha}$ ($^{10}_{\Lambda}$Be) is very important, because only this partial width contains the information about w^{10}_{1p}.

It should be noted that $\Gamma^{\tau}_{\alpha\alpha 0}$ does not practically depend on details of the ^{8}Be structure.

Table 1: Neutron spectroscopic factors in ^{9}Be

		States of ^{8}Be				
Model	Quantity	$(0^+0)_{g.s.}$	$(2^+0)_1$	$(2^+0)_2$	$(2^+1)_1$	$(1^+1)_1$
Experi-ment	E_x (MeV) [6]	0.0	3.04	16.92	16.63	17.64
	$S^2_{1/2}+S^2_{3/2}$ [7]	0.67(14)	1.49(23)	1.14(19)		0.27(5)
	$S^2_{1/2}+S^2_{3/2}$ [8]	0.60(17)	1.20(21)	0.65(15)		0.10(7)
CK$_{pot}$ [9]	E_x	0.0	3.82	12.87	16.19	17.10
	$S_{1/2}$		−0.2045	0.3186	0.1936	0.2068
	$S_{3/2}$	0.7534	0.8578	0.5973	0.6463	−0.1664
	$S^2_{1/2}+S^2_{3/2}$	0.5676	0.7775	0.4582	0.5129	0.0705
CK I [9]	E_x	0.00	3.41	14.43	15.80	16.88
	$S_{1/2}$		−0.2424	0.2881	0.2257	−0.3144
	$S_{3/2}$	0.7616	0.8166	0.5886	0.6698	0.3210
	$S^2_{1/2}+S^2_{3/2}$	0.5800	0.7256	0.4295	0.4996	0.2018
CK II [9]	E_x	0.00	3.55	13.36	16.19	16.93
	$S_{1/2}$		−0.2428	0.3107	0.2025	0.2369
	$S_{3/2}$	0.7620	0.8180	0.6112	0.6776	−0.1758
	$S^2_{1/2}+S^2_{3/2}$	0.5806	0.7281	0.4700	0.5002	0.0870
PBA I [10]	E_x	0.00	3.39	14.33	15.73	16.77
	$S_{1/2}$		−0.2715	0.2937	0.2160	−0.3180
	$S_{3/2}$	0.7689	0.7942	0.5899	0.6549	0.3263
	$S^2_{1/2}+S^2_{3/2}$	0.5912	0.7045	0.4342	0.4388	0.2076
PBA III [10]	E_x	0.00	2.86	16.39	15.95	16.95
	$S_{1/2}$		−0.2827	0.2512	0.2308	−0.3329
	$S_{3/2}$	0.7703	0.7766	0.5611	0.6525	0.3336
	$S^2_{1/2}+S^2_{3/2}$	0.5934	0.6830	0.3779	0.4791	0.2221
WB(5−16) [11]	E_x	0.00	3.57	15.44	15.85	17.39
	$S_{1/2}$		−0.1676	0.2699	0.2260	0.2836
	$S_{3/2}$	0.7409	0.8649	0.5904	0.6812	−0.2999
	$S^2_{1/2}+S^2_{3/2}$	0.5489	0.7761	0.4215	0.5151	0.1704

Table 2: Weights of $w^{SJ}_{1\,n}$ in the α-decay widths.

State in ^{8}Be	Model	$^{2S+1}L_J$: $^3P_0$	$^1P_1$	$^3P_1$	$^3P_2$	Norm factor
$(0^+0)_1$	CK_{pot}		0.6667	0.3333		0.5676
	CK I		0.6667	0.3333		0.5800
	CK II		0.6667	0.3333		0.5806
	PBA I		0.6667	0.3333		0.5912
	PBA III		0.6667	0.3333		0.5934
	WB(5–16)		0.6667	0.3333		0.5489
$(2^+0)_1$	CK_{pot}		0.4837	0.0432	0.4731	0.7775
	CK I		0.51523	0.0253	0.4595	0.7256
	CK II		0.5152	0.0253	0.4595	0.7281
	PBA I		0.5374	0.0149	0.4477	0.7045
	PBA III		0.5476	0.0109	0.4415	0.6830
	WB(5–16)		0.4579	0.0602	0.4819	0.7761
$(2^+0)_2$	CK_{pot}		0.0565	0.5542	0.3893	0.4582
	CK I		0.0701	0.5266	0.4033	0.4295
	CK II		0.0640	0.5387	0.3973	0.4700
	PBA I		0.0673	0.5321	0.4006	0.4342
	PBA III		0.0847	0.4988	0.4165	0.3779
	WB(5–16)		0.0812	0.5052	0.4136	0.4215
$(2^+1)_1$	CK_{pot}		0.1598	0.3767	0.4635	0.5129
	CK I		0.1316	0.4194	0.4490	0.4996
	CK II		0.1505	0.3905	0.4590	0.5002
	PBA I		0.1374	0.4103	0.4523	0.4888
	PBA III		0.1238	0.4319	0.4444	0.4791
	WB(5–16)		0.1341	0.4155	0.4504	0.5151
$(1^+1)_1$	CK_{pot}	0.4047	0.5285	0.0014	0.0655	0.0705
	CK I	0.3264	0.5864	0.0022	0.0851	0.2018
	CK II	0.4299	0.5068	0.0041	0.0592	0.0870
	PBA I	0.3247	0.5874	0.0023	0.0855	0.2076
	PBA III	0.3326	0.5822	0.0016	0.0835	0.2221
	WB(5–16)	0.3147	0.5938	0.0035	0.0880	0.1704

3 Conclusion

The properties of nonmesonic decays of $1p$-shell nuclei can be described in terms of few weak interaction phenomenological matrix elements $w^{SJ}_{1\,\tau}$ defined by Eq. (6) [1]. The present consideration shows that these matrix elements can be extracted from the measured values of $\Gamma^{\tau}_{\alpha\alpha_i}$, partial widths of nonmesonic decays $^{10}_{\Lambda}$Be and $^{10}_{\Lambda}$B. Also it is shown that the uncertainties related to the description of nuclear structure are not essential for this task.

The relation between $w^{SJ}_{1\,\tau}$ and "elementary" weak $\Lambda N \to NN$ interaction (exchange by one pion, exchange by one kaon, two-meson exchange, etc) will be discussed in subsequent papers together with possible reasons for the known problems in explanations of the experimental ratio Γ^n/Γ^p.

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