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P-VIOLATING EFFECTS IN LOW-ENERGY COMPTON SCATTERING

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The effect of Compton scattering in SM was considered [1, 2] for the range of the very high energies of LEP energy range. It was shown that radiative corrections to cross-section are essential and can results about 10% of contribution. In this paper was mentioned that the effect of RC is negligible for the case of the low energies. We too considered the last case for which analytical analysis are presented in the literature. The effect absent in the frames of QED. Nevertheless of smallness of mentioned process, one can use the powerful laser beam to experimentally test the process. The main effect comes from the W axial–vector current. The reason why we don't use Z boson is clear. In the case of W boson the ratio of vector and axial coupling constants is 1. On the other hand in the case of Z boson is order of 10^{-2}.

Due to renormalizability of SM there is no dependence of the choice of the propagator of gauge bosons and ghosts and besides of gauge invariance and Bose symmetry present in amplitude. Besides there is no dependence of renormalization scheme of the theory. Below we use the on mass–shell regularization scheme for fermion self-energy and vertex function. We use Feynman gauge W propagator. In both cases of polarized photon and electron the interference of vector and axial–vector couplings of W boson with fermion is essential. We start from consideration of self energy and vertex function of fermion.

W contribution to the self-energy of fermion has the form:

$$\Sigma^u(p) = \frac{-ig^2}{2^6\pi^2}(1+\gamma_5)\int\frac{d^4k}{i\pi^2}\gamma_\lambda(\hat{p}-\hat{k})\gamma_\lambda\frac{1}{(k^2-M^2)(p-k)^2}\,, \tag{1}$$

and after the standard four dimension integration by loop momenta we obtained for the unrenormalized self–energy operator:

$$\Sigma^u(p) = \frac{-ig^2}{2^5\pi^2}(1+\gamma_5)\hat{p}\left[\frac{1}{2}L+\frac{1}{6}\frac{p^2}{M^2}\right]\,. \tag{2}$$

Where $L = \ln\frac{\Lambda^2}{M^2}$, Λ is the ultraviolet cut–off and M is W boson mass. Renormalization implies two subtractions: left and right (here we have some deviation from QED scheme) and results

$$\Sigma^r(p) = \frac{ig^2}{6\times 2^5\pi^2M^2}(\hat{p}-m)\hat{p}\gamma_5(\hat{p}-m)\,. \tag{3}$$

Here and below we pay attention for contributions that containing γ_5. Consider next the vertex function. It has a form

$$V^\mu(p_1,p_1+k_1) = \frac{-ieg^2}{2^6\pi^2}\int\frac{d^4k}{i\pi^2}\frac{V_{\mu\lambda\sigma}\gamma_\sigma(p_1-k)\gamma_\lambda(1-\gamma_5)}{(012)}\,, \tag{4}$$

where $V_{\mu\lambda\sigma}$ is defined as

$$V_{\mu\lambda\sigma} = g_{\mu\lambda}(k-k_1)_\sigma + g_{\lambda\sigma}(-2k-k_1)_\mu + g_{\sigma\mu}(2k_1+k)_\lambda\,, \tag{5}$$

$$(0) = k^2-M^2\,,\quad (1) = (p_1-k)^2\,,\quad (2) = (k+k_1)^2-M^2$$

Performing the loop momentum integration and imposing the condition $V_\mu(p, p+k_1)|_{k_1=0} = 0$ we obtain

$$V^r_\mu(p_1, p_1+k_1) = \frac{ieg^2}{2^6\pi^2M^2}\left[-\frac{3}{2}\hat{k}_1\gamma_\mu m - \frac{17}{12}\chi_1\gamma_\mu + \frac{7}{6}p_{1\mu}\hat{k}_1\right], \chi_{1,2} = 2p_1k_{1,2} \quad (6)$$

The remaining part of diagram contribution is ultraviolet finite and may be calculated by standard way. As a result the radiative correction to the matrix element may be written in the form:

$$M_1 = i\frac{N}{2}\bar{u}(p_2)\gamma_5 O^{(1)}_{\mu\nu}u(p_1)e_{1\mu}e^*_{2\nu}, N_1 = \frac{e^4}{2^5\pi^2M^2\sin^2\theta_W}, sin^2\theta_W = 0.23. \quad (7)$$

where

$$O^{(1)}_{\mu\nu}e^*_{1\mu} = V_{1\nu} + V_{2\nu} + V_{3\nu} + V_{4\nu} + \Sigma_\nu + D_\nu + G_\nu + B_\nu, \quad (8)$$

where V_i represent the vertex contributions, Σ is the self energy of fermion, D correspond to four boson vertex diagram contribution, G takes the account the ghost contribution and finally B correspond to the box contribution.

Contribution of vertex type Feynman diagram has a form (we put here only the contributions providing the parity odd effect in the cross section):

$$V_{1\nu} = \frac{2p_{2\nu} + \gamma_\nu\hat{k}_2}{\chi_1}\left[-\frac{3}{2}\hat{k}_1\hat{e}_1 m - \frac{17}{12}\chi_1\hat{e}_1 + \frac{7}{6}(p_1e_1)\hat{k}_1\right], \quad (9)$$

$$V_{2\nu} = \frac{2p_2e_1 - \hat{e}_1\hat{k}_1}{-\chi_2}\left[\frac{3}{2}\hat{k}_2\gamma_\nu m + \frac{17}{12}\chi_2\gamma_\nu - \frac{7}{6}p_{1\nu}\hat{k}_2\right], \quad (10)$$

$$V_{3\nu} = \left[\frac{3}{2}\gamma_\nu\hat{k}_2 m - \frac{17}{12}\gamma_\nu\chi_1 + \frac{7}{6}\hat{k}_2p_{2\nu}\right]\frac{(2p_1e_1) + \hat{k}_1\hat{e}_1}{\chi_1}, \quad (11)$$

$$V_{4\nu} = \left[-\frac{3}{2}\hat{e}_1\hat{k}_1 m + \frac{17}{12}\chi_2\hat{e}_1 - \frac{7}{6}\hat{k}_1(p_2e_1)\right]\frac{2p_{1\nu} - \hat{k}_2\gamma_\nu}{-\chi_2}. \quad (12)$$

Fermion self energy diagram contribution is

$$\Sigma_\nu = \frac{1}{3}\left[\gamma_\nu(\hat{p}_1 + \hat{k}_1)\hat{e}_1 + \hat{e}_1(\hat{p}_2 - \hat{k}_1)\gamma_\nu\right]. \quad (13)$$

The $\gamma\gamma WW$ vertex containing Feynman diagram contribution is

$$D_\nu = e_{1\nu}\left(m + \frac{1}{2}\hat{q}\right) + \gamma_\nu\left((p_1e_1) + \frac{1}{2}(qe_1)\right) + \hat{e}_1\left(p_{1\nu} + \frac{1}{2}q_\nu\right), \quad (14)$$

where $q = \hat{k}_1 - \hat{k}_2$.

Only two Feynman diagrams containing the ghost particle contribution are relevant (we may neglect interaction ghost with fermion, which contribution is suppressed by additional factor $\frac{m}{M}$, m is electron mass):

$$G_\nu = \frac{1}{2}\hat{e}_1p_{1\nu} + \frac{1}{2}\gamma_\nu(p_1e_1) - \frac{1}{2}e_{1\nu}m + \gamma_\nu\left(2\hat{k}_1 - \hat{k}_2\right)\hat{e}_1 + \hat{e}_1\left(\hat{k}_1 - 2\hat{k}_2\right)\gamma_\nu. \quad (15)$$

At least the contribution of Feynman diagrams containing three boson vertices has a form:

$$B_\nu = \frac{1}{24}\Big[e_{1\nu}(-32m - 28\hat{q}) + \gamma_\nu\left(35\hat{k}_2 + 23\hat{k}_1\right)\hat{e}_1 + \\ + \hat{e}_1\left(-35\hat{k}_1 - 23\hat{k}_2\right)\gamma_\nu + 28\hat{e}_1 p_{1\nu} + 26\hat{e}_1 k_{1\nu} + 28\gamma_\nu(p_1 e_1) - 26\gamma_\nu(k_2 e_1)\Big]. \quad (16)$$

Parity odd contributions to cross–section arise from interference of the Born matrix element $M^{(0)} = ie^2\bar{u}(p_2)O^{(0)}_{\mu\nu}u(p_1)e_{1\mu}e^*_{2\nu}$ with the one loop corrected matrix element given above. This summed on polarization states of electrons and photons Born matrix element squared is known proportional to:

$$\begin{aligned} \sum |M^{(0)}|^2 &\sim \frac{1}{4}Sp(\hat{p}_2 + m)O^{(0)}(\hat{p}_1 + m)\tilde{O}^{(0)} = S = \\ &= 2\left(\frac{\chi_1}{\chi_2} + \frac{\chi_2}{\chi_1}\right) + 8m^2\left(\frac{1}{\chi_1} - \frac{1}{\chi_2}\right) + 8m^4\left(\frac{1}{\chi_1} - \frac{1}{\chi_2}\right)^2. \end{aligned} \quad (17)$$

In the case of polarized electron we use it's density matrix: $u(p,a)\bar{u}(p,a) = (\hat{p} + m)(1 - \gamma_5\hat{a})$. Corresponding interference matrix has a form:

$$\begin{aligned} S_1 &= \frac{1}{4}Sp(\hat{p}_2 + m)\gamma_5 O^{(1)}(\hat{p}_1 + m)(-\gamma_5\hat{a})\tilde{O}^{(0)} = \\ &= (k_1 a)m\left[-6\frac{\chi_2}{\chi_1} - \frac{7m^2}{3\chi_1} + 7\frac{m^2}{\chi_2} - 1\right] + \\ &+ (k_2 a)m\left[6\frac{\chi_1}{\chi_2} + 7\frac{m^2}{\chi_1} - \frac{7m^2}{3\chi_2} + 1\right]. \end{aligned} \quad (18)$$

In the laboratory frame $\chi_1 = 2m\omega_1$, $\chi_2 = 2m\omega_2$, and the energy of scattering photon is $\omega_2 = \omega_1/\left[1 + (\omega_1/m)(1 - c)\right]$, $c = \cos\theta$ and θ is the angle between the initial and scattered photon 3–momenta. Scalar products entering S_1 are: $(k_1 a) = -\omega_1|a|\cos\theta_0$, $(k_2 a) = -\omega_2|a|(\cos\theta\cos\theta_0 - \sin\theta\sin\theta_0\cos\phi)$, ϕ is the azimuthal angle between the planes containing initial photon and electron spin and the plane containing initial and scattered photons momenta.

For the case unpolarized electron and circularly polarized initial photon (it's spin density matrix has a form $e_{1,i}e^*_{1,j} = \frac{1}{2}(1 + \xi_2\sigma_2)_{ij}$, $i,j = x,y$) the corresponding interference has a form

$$\begin{aligned} S_2 &= \frac{1}{8m^2}\chi_1\chi_2{}^2(1 - c^2)\xi_2(A_2 - A_1) - \frac{1}{2}\xi_2\chi_1 A_3 - \\ &- \frac{1}{2}\xi_2\chi_2 A_4 + \frac{1}{4m^2}\xi_2(1 - c)\chi_1\chi_2 A_5 \end{aligned} \quad (19)$$

with

$$A_1 = -\frac{5m^2}{2\chi_1\chi_2} - \frac{31}{12\chi_1} + \frac{23}{4\chi_2} - \frac{4m^2}{\chi_2^2}, \quad (20)$$

$$
\begin{aligned}
A_2 &= -\frac{m^2}{6\chi_1\chi_2}+\frac{119}{12\chi_1}+\frac{21}{4\chi_2}+\frac{4m^2}{3\chi_2^2},\\
A_3 &= \frac{29}{12}-\frac{4\chi_1}{3\chi_2}+\frac{5\chi_1 m^2}{4\chi_2^2}+\frac{25\chi_2}{8\chi_1}+\frac{25m^2}{6\chi_1}-\frac{11\chi_2 m^2}{6\chi_1^2}-\frac{29m^2}{12\chi_2},\\
A_4 &= \frac{5}{12}+\frac{\chi_1}{3\chi_2}-\frac{7\chi_1 m^2}{12\chi_2^2}-\frac{31\chi_2}{24\chi_1}-\frac{7m^2}{12\chi_1},\\
A_5 &= -\frac{1}{6}-\frac{4\chi_1}{3\chi_2}+\frac{5\chi_1 m^2}{4\chi_2^2}-\frac{31\chi_2}{24\chi_1}+\frac{7m^4}{6\chi_1\chi_2}-\frac{5m^2}{12\chi_1}+\frac{14m^2}{\chi_2}+\frac{7m^4}{6\chi_2^2}.
\end{aligned}
$$

The corresponding asymmetries are:

$$
\begin{aligned}
\mathcal{A}_1 &= N\frac{S_1}{Sm^2}=\frac{d\sigma(\vec{a},\theta,y)-d\sigma(-\vec{a},\theta,y)}{d\sigma(\vec{a},\theta,y)+d\sigma(-\vec{a},\theta,y)},\\
\mathcal{A}_2 &= N\frac{S_2}{Sm^2}=\frac{d\sigma_R(\theta,y)-d\sigma_L(\theta,y)}{d\sigma_R(\theta,y)+d\sigma_L(\theta,y)},\\
N &= \frac{g^2m^2}{32\pi^2M^2}\approx\frac{\alpha m^2}{2\pi M^2}=4\times10^{-13},\ y=\omega_1/m.
\end{aligned}
\tag{21}
$$

In Thomson limit $y\to 0$ we have:

$$
\frac{S_1}{Sm^2}=\frac{7}{6}\frac{(\vec{n}_1+\vec{n}_2)\vec{a}}{(1+c^2)},
\tag{22}
$$

$$
\frac{S_2}{Sm^2}=\frac{7\xi_2(1-c)}{24(1+c^2)},
\tag{23}
$$

$\vec{n}_1$, $\vec{n}_2$ are the orts along initial and the scattered photons momenta, $\vec{a}$ is the polarization vector of the initial electron, ξ_2 is degree of circular polarization of initial photon.

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