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THE $T = 0$ AND $T = 1$ PAIRING AND THE FORMATION OF THE FOUR-PARTICLE CORRELATED STRUCTURES IN THE GROUND STATES OF $Z = N$ NUCLEI

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I. INTRODUCTION

Treatment of the $T = 1$ pair correlations among nucleons of the same kind in heavy nuclei is significantly simplified because a simple approximate expression for the ground-state wave function of the even-even nucleus – the BCS wave function [1,2] – can be here employed. A specific feature of the BCS wave function is that it is not an eigenfunction of the particle number operator. However, it is possible to restore a corresponding symmetry by a projection. The projected wave function is represented by an expression in which a two-particle-creation operator, whose structure is determined by the pairing interaction matrix elements and by the mean field single particle energies, is repeatedly applied to the inert core wave function to get the corresponding number of particles. Such a description forms a basis for the generalized seniority scheme [3] or the alternative Broken Pair Approximation approach [4,5]. Of course, it would be useful to find an analogous simple approximate expression for the ground-state wave function of an even-even nucleus also in the situation when the neutron-proton (np) pair correlations (either in the single $T = 1$ channel or in both the $T = 1$ and $T = 0$ channels) are important.

A generalization of the u-v Bogoliubov transformation to the case of the np pairing has been discussed many times (see e.g. [6–9]). Since the particle number and the isospin are not conserved, this procedure should be treated with care [10] particularly when both the $T = 1$ and $T = 0$ pair correlations are present.

An useful insight into the role of the np pairing can be obtained within the framework of exactly solvable algebraic models. The model with both $T = 1$ and $T = 0$ pairing channels has the SO(8) symmetry [10–13]. A simple expression for the SO(8) ground-state wave function has been constructed in such a way that the maximum possible number of fermions form correlated four-particle $T = 0$, $S = 0$ structures [10]. These correlated four-particle structures are characterized by the same quantum numbers as alpha particles and this result is in a correspondence with an alpha-cluster model applied to light and medium mass nuclei. However, it is necessary to note that the four-particle correlated structures that emerge are not real alpha particles. It is more appropriate to call them alpha-like structures.

In the present paper, we investigate the structure of the wave functions of the ground and the excited states of the system with np pairing in more details. A simple one-term expression written with a help of the creation operators of the four-particle correlated structures is discussed. The case with interplay of both $T = 0$ and $T = 1$ pairing interaction channels is discussed.

II. SO(8) ALGEBRAIC MODEL

The Hamiltonian of the SO(8) algebraic model which includes both $T = 1$ and $T = 0$ pairing terms has the form [13]

$$\hat{H} = -(1+x)\sum_{\mu}(P_{\mu}^{\dagger})_f(P_{\mu})_f - (1-x)\sum_{\mu}(D_{\mu}^{\dagger})_f(D_{\mu})_f \tag{1}$$

where

$$(P_{\mu}^{\dagger})_f = \sqrt{l+\tfrac{1}{2}} \sum_{m,\sigma,\tau,\tau'} C^{00}_{lml-m} C^{00}_{\frac{1}{2}\sigma\frac{1}{2}-\sigma} C^{1\mu}_{\frac{1}{2}\tau\frac{1}{2}\tau'} a^{\dagger}_{lm,\frac{1}{2}\sigma,\frac{1}{2}\tau} a^{\dagger}_{l-m,\frac{1}{2}-\sigma,\frac{1}{2}\tau'} \tag{2}$$

$$(D_{\mu}^{\dagger})_f = \sqrt{l+\tfrac{1}{2}} \sum_{m,\sigma,\sigma',\tau} C^{00}_{lml-m} C^{1\mu}_{\frac{1}{2}\sigma\frac{1}{2}\sigma'} C^{00}_{\frac{1}{2}\tau\frac{1}{2}-\tau} a^{\dagger}_{lm,\frac{1}{2}\sigma,\frac{1}{2}\tau} a^{\dagger}_{l-m,\frac{1}{2}\sigma',\frac{1}{2}-\tau} \, . \tag{3}$$

Above, $a^{\dagger}_{lm,\frac{1}{2}\sigma,\frac{1}{2}\tau}$ is fermion creation operator describing nucleon with orbital momentum l, spin projection σ, and isospin projection τ. The parameter x, which varies from -1 (pure isoscalar pairing) to 1 (pure isovector pairing), governs a relative importance of isoscalar and isovector pairing in the Hamiltonian (1).

Similarly to [10], we employ the boson mapping procedure [14] to obtain the solution of the fermion problem with Hamiltonian (1). Using the generalized Dyson boson representation of the fermion operators [14–16]

$$\begin{aligned} a_s^{\dagger}a_{s'}^{\dagger} &\to b_{ss'}^{\dagger} - [\hat{C}, b_{ss'}^{\dagger}] \\ a_{t'}a_t &\to b_{tt'} \\ a_s^{\dagger}a_{s'} &\to \sum_p b_{sp}^{\dagger}b_{s'p} \end{aligned} \tag{4}$$

where

$$\begin{aligned} \hat{C} &= \tfrac{1}{4}\sum b_{ss'}^{\dagger}b_{tt'}^{\dagger}b_{st}b_{s't'} \\ b_{ss'}^{\dagger} &= -b_{s's}^{\dagger} \\ [b_{ss'}, b_{tt'}^{\dagger}] &= \delta_{st}\delta_{s't'} - \delta_{st'}\delta_{s't} \\ [b_{ss'}, b_{tt'}] &= 0 \, , \end{aligned} \tag{5}$$

we can get a boson image of the Hamiltonian (1). In our case $s = lm, \frac{1}{2}\sigma, \frac{1}{2}\tau$. Introducing boson operators

$$\begin{aligned} P_{\nu}^{\dagger} &= \tfrac{1}{\sqrt{2}} \sum_{m,\sigma,\tau,\tau'} C^{00}_{lml-m} C^{00}_{\frac{1}{2}\sigma\frac{1}{2}-\sigma} C^{1\nu}_{\frac{1}{2}\tau\frac{1}{2}\tau'} b^{\dagger}_{lm,\frac{1}{2}\sigma,\frac{1}{2}\tau,l-m,\frac{1}{2}-\sigma,\frac{1}{2}\tau'} \\ D_{\mu}^{\dagger} &= \tfrac{1}{\sqrt{2}} \sum_{m,\sigma,\sigma',\tau} C^{00}_{lml-m} C^{1\mu}_{\frac{1}{2}\sigma\frac{1}{2}\sigma'} C^{00}_{\frac{1}{2}\tau\frac{1}{2}-\tau} b^{\dagger}_{lm,\frac{1}{2}\sigma,\frac{1}{2}\tau,l-m,\frac{1}{2}\sigma',\frac{1}{2}-\tau} \, , \end{aligned} \tag{6}$$

we obtain the boson representations of the fermion operators $(P_{\mu}^{\dagger})_f$, $(D_{\mu}^{\dagger})_f$, $(P_{\mu})_f$, and $(D_{\mu})_f$

$$
\begin{aligned}
(P_\mu^\dagger)_f &\to \sqrt{2l+1}(P_\mu^\dagger - [\hat{C}, P_\mu^\dagger]) \\
(P_\mu)_f &\to \sqrt{2l+1} P_\mu \\
(D_\mu^\dagger)_f &\to \sqrt{2l+1}(D_\mu^\dagger - [\hat{C}, D_\mu^\dagger]) \\
(D_\mu)_f &\to \sqrt{2l+1} D_\mu \ ,
\end{aligned} \tag{7}
$$

where for $\hat{C}$ we get from (5)

$$
\begin{aligned}
\hat{C} &= \tfrac{1}{2l+1}[\tfrac{1}{2}(\hat{n}_p + \hat{n}_d)^2 - \tfrac{1}{2}(\hat{n}_p + \hat{n}_d) \\
&+ \tfrac{1}{4}((P^\dagger \cdot P^\dagger)(D \cdot D) + (D^\dagger \cdot D^\dagger)(P \cdot P) \\
&- (P^\dagger \cdot P^\dagger)(P \cdot P) - (D^\dagger \cdot D^\dagger)(D \cdot D))] \ .
\end{aligned} \tag{8}
$$

Above

$$
\begin{aligned}
\hat{n}_p &\equiv \sum_\mu P_\mu^\dagger P_\mu \\
\hat{n}_d &\equiv \sum_\mu D_\mu^\dagger D_\mu \\
P^\dagger \cdot P^\dagger &= \sum_\mu (-1)^\mu P_\mu^\dagger P_{-\mu}^\dagger \\
D^\dagger \cdot D^\dagger &= \sum_\mu (-1)^\mu D_\mu^\dagger D_{-\mu}^\dagger \ .
\end{aligned}
$$

Using (7) and (8) we obtain the boson representation of the Hamiltonian (1)

$$
\begin{aligned}
\hat{H} &= -(1+x)(2l+1)\hat{n}_p - (1-x)(2l+1)\hat{n}_d \\
&+ (1+x)\hat{n}_p(\hat{n}_p + \hat{n}_d - 1) \\
&+ (1-x)\hat{n}_d(\hat{n}_p + \hat{n}_d - 1) \\
&- \tfrac{1}{2}(1+x)(P^\dagger \cdot P^\dagger)(P \cdot P) - \tfrac{1}{2}(1-x)(D^\dagger \cdot D^\dagger)(D \cdot D) \\
&+ \tfrac{1}{2}(1+x)(D^\dagger \cdot D^\dagger)(P \cdot P) + \tfrac{1}{2}(1-x)(P^\dagger \cdot P^\dagger)(D \cdot D) \ .
\end{aligned} \tag{9}
$$

The above boson Hamiltonian is equivalent to the the boson Hamiltonian from [10] where a slightly different form has been used.

The hermicity of Hamiltonian (9) can be restored by the following transformation which conserves commutation relations

$$
\begin{aligned}
P_\mu^\dagger &\to (1+x)^{\frac{1}{4}} P_\mu^\dagger, \quad P_\mu \to (1+x)^{-\frac{1}{4}} P_\mu \\
D_\mu^\dagger &\to (1-x)^{\frac{1}{4}} D_\mu^\dagger, \ D_\mu \to (1-x)^{-\frac{1}{4}} D_\mu \ .
\end{aligned} \tag{10}
$$

Applying transformation (10) to the Hamiltonian (9), we get

$$
\begin{aligned}
\hat{H} &= -(1+x)(2l+1)\hat{n}_p - (1-x)(2l+1)\hat{n}_d \\
&+ (1+x)\hat{n}_p(\hat{n}_p+\hat{n}_d-1) + (1-x)\hat{n}_d(\hat{n}_p+\hat{n}_d-1) \\
&+ \tfrac{1}{2}(1+x)(P^\dagger\cdot P^\dagger)(P\cdot P) + \tfrac{1}{2}(1-x)(D^\dagger\cdot D^\dagger)(D\cdot D) \\
&+ \tfrac{1}{2}\sqrt{1-x^2}((D^\dagger\cdot D^\dagger)(P\cdot P) + (P^\dagger\cdot P^\dagger)(D\cdot D)) \,.
\end{aligned}
\tag{11}
$$

The Hamiltonian (11) has been diagonalized and the eigenvectors have been constructed using the basis

$$|NJTk\rangle = (P^\dagger\cdot P^\dagger)^k (D^\dagger\cdot D^\dagger)^{\frac{N-J-T}{2}-k} (P_1^\dagger)^T (D_1^\dagger)^J |0\rangle \,. \tag{12}$$

Here N is a total number of bosons, J is angular momentum, and T is isospin. For simplicity we have considered as a basis the state vectors with the maximum values of the spin and isospin projections.

Having obtained exact eigenstates, we investigate a possibility to represent them within terms of alpha-like two-boson $T=0$, $J=0$ structures. Those are obtained as the linear combinations of the operators $(P^\dagger\cdot P^\dagger)$ and $(D^\dagger\cdot D^\dagger)$:

$$A^\dagger = (P^\dagger\cdot P^\dagger)\cos\theta - (D^\dagger\cdot D^\dagger)\sin\theta \tag{13}$$

and the orthogonal one

$$A'^\dagger = (P^\dagger\cdot P^\dagger)\sin\theta + (D^\dagger\cdot D^\dagger)\cos\theta \,. \tag{14}$$

First, let us consider eigenstates with $J=0$ and $T=0$. The lowest state is approximated by

$$A^{\dagger\frac{N}{2}}|0> \,, \tag{15}$$

The parameter θ in (13) is determined so as to get a maximum overlap of the state vector (15) with the lowest $J=0$, $T=0$ exact eigenstate. The next excited eigenstate with $J=0$, $T=0$ is approximated by the form

$$A^{\dagger\frac{N}{2}-1}A'^\dagger|0> \,, \tag{16}$$

from which a projection on the vector (15) is subtracted. The higher eigenstates are constructed similarly by increasing a degree of the $A'^\dagger$ operator in the expression for the state vector and orthogonalizing this approximate state to the previously obtained lower-lying alpha-correlated expressions. For every eigenstate we determine the value of θ, which gives a maximum overlap of the corresponding approximate state vector with the exact one. Thus, the values of θ are state–dependent.

The calculations have been done for the total number of bosons $N=6$ and the eigenstates with isospin $T=0$ and angular momentum $J=0$.

The results of calculations are shown in Fig. 1. All overlaps of the exact and the corresponding approximate state vectors are larger then 0.975 for all values of x

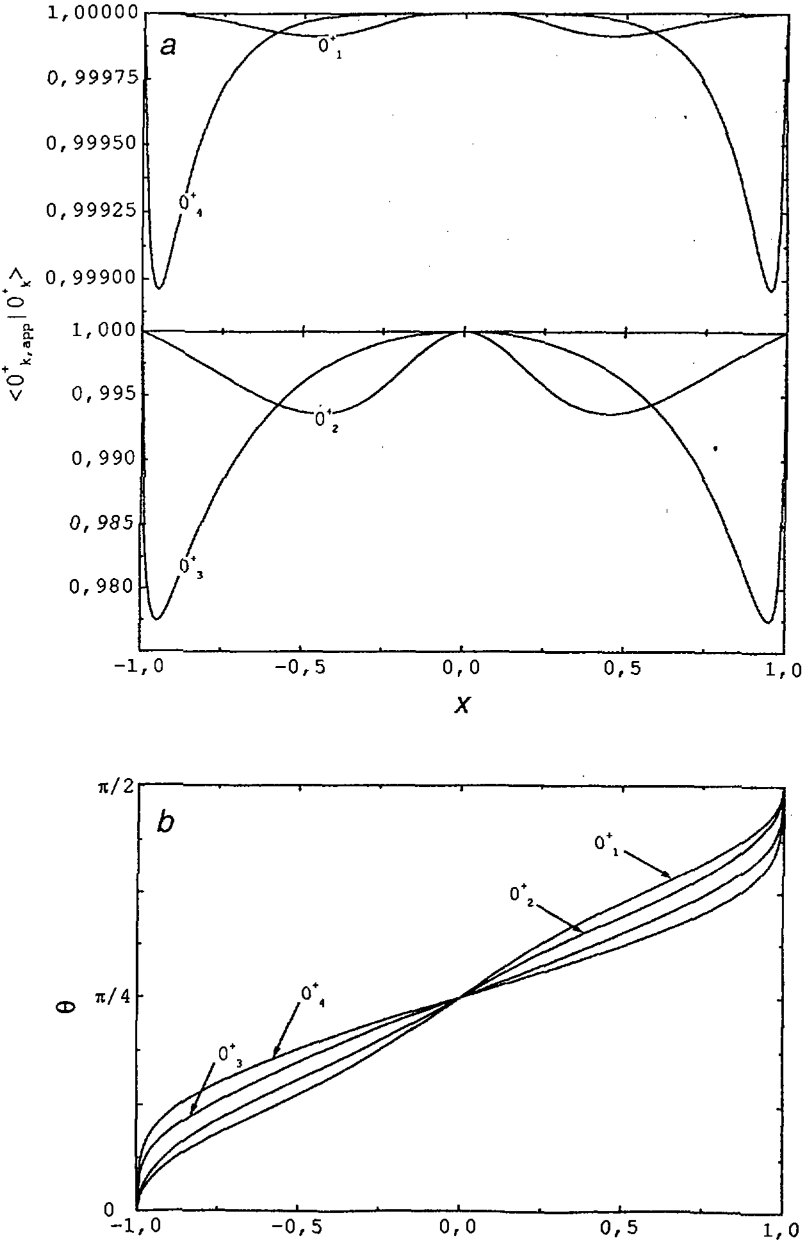


FIG. 1. a) Dependence of an overlap of the exact and approximate state vectors of the 0^+ states in the SO(8) algebraic model on the parameter x.
b) Dependence of the angle θ introduced in the Eqs. (13 , 14) on x.

between -1 and 1^*. The values of θ do not depend on the state in the dynamical symmetry limits at $x = 0$ (SU(4)), $x = 1$ ($SO^T(5)$), and $x = -1$ ($SO^S(5)$). Between the dynamical symmetry limits, the values of θ are state dependent but this dependence is not strong. The calculations show that if we take for all eigenstates the same (average) value of θ, we get for the overlaps the values equal or larger than 0.97 for all values of x.

These results mean that the alpha-like bi-bosons, which are the boson analogs of the four-fermion T=0, J=0 correlated structures, are very important structure units. A representation of the eigenstates is simplified significantly with the help of them. It stresses the important role of the four-nucleon alpha-like correlations in the regime of np pairing.

The lowest eigenvectors with a nonzero T and J can be described approximately by the expression

$$(A^\dagger)^{\frac{N-J-T}{2}}(P_1^\dagger)^T(D_1^\dagger)^J|0\rangle \,. \tag{17}$$

Again, the calculations confirm almost perfect overlap of this construction with the exact solution.

Similar discussion of the SO(8) wave functions in terms of alpha-like structures has been performed in [10]. There, the ground-state wave functions have been analysed. The variational principle has been employed and the genuine fermionic wave functions (both the exact and approximate) have been considered in [10].

In the present study, we search the approximate wave function by the principle of maximal overlap. In view of the almost perfect agreement of the approximate and exact solutions, we have found a little difference between the maximal overlap and variational procedures.

The another aspect which makes a difference between present calculations and those of Ref. [10] is our employment of the bosonized and hermitized SO(8) Hamiltonian and the comparison of the exact solutions of this Hamiltonian with the bosonic analogues of the alpha-like correlated wave functions. This approach does nor agree completely with the genuine fermionic procedure but again the differences are not large and are of the order up to $1/(l+\frac{1}{2})$.

III. SINGLE j-LEVEL WITH SURFACE DELTA INTERACTION

The SO(8) model comprising the fermion pairs with the values of the angular momentum $J = 0$ and $J = 1$ only is, of course, an idealization of a real situation. In fact, we should take into account also the fermion pairs with other values of an angular momentum J. As a rule, fermion pairs with $T = 0$ and angular momentum

*For $x = 0$ (SU(4) symmetry limit), an overlap is 100% !

J equal to the maximum angular momentum allowed by a corresponding shell model configuration have low energy and play an important role. One should clarify whether the conclusions of the preceding section about a possibility to approximate the exact wave functions by the four-particle $T = 0$, $J = 0$ correlated structures are also valid in the more general case.

We consider a model Hamiltonian with nucleons of both kinds interacting by the Surface-Delta Interaction (SDI) $(1 + y(\vec{\tau_1} \cdot \vec{\tau_2}))\delta(|\vec{r_1} - \vec{r_2}|)\delta(r_1 - R_0)$ and occupying isolated single particle level with angular momentum j

$$\hat{H} = -(1-x)\sum_J G_{0,J}\sum_M A^\dagger_{JM,00}A_{JM,00} - (1+x)\sum_J G_{1,J}\sum_{M,M_T} A^\dagger_{JM,1M_T}A_{JM,1M_T} \,. \quad (18)$$

Here,

$$A^\dagger_{JM,TM_T} = \sum_{m,m'} C^{JM}_{jm,jm'}\sum_{\tau,\tau'} C^{TM_T}_{\frac{1}{2}\tau,\frac{1}{2}\tau'} a^\dagger_{jm,\frac{1}{2}\tau} a^\dagger_{jm',\frac{1}{2}\tau'} \,, \quad (19)$$

where $a^\dagger_{jm,\frac{1}{2}\tau}$ is a creation operator of a nucleon with angular momentum j, its projection m, and isospin projection τ. Other notations are

$$\begin{aligned} x &= -\tfrac{1}{3} + \tfrac{2}{3}y, \\ G_{0,J} &= (1-(-1)^J)(f^2_{j,J} + g^2_{j,J}), \\ G_{1,J} &= (1+(-1)^J)\tfrac{1}{3}f^2_{j,J} \end{aligned} \quad (20)$$

and

$$\begin{aligned} f_{j,J} &= \tfrac{2j+1}{\sqrt{2J+1}} C^{J0}_{j\frac{1}{2},j-\frac{1}{2}}, \\ g_{j,J} &= \tfrac{2j+1}{\sqrt{2J+1}} C^{J1}_{j\frac{1}{2},j\frac{1}{2}}. \end{aligned} \quad (21)$$

Parameter x regulates a relative role of the $T = 1$ and $T = 0$ pairing in the Hamiltonian (18).

The calculations have been done for the single particle angular momentum $j = \frac{7}{2}$, total number of nucleons equal to 8, and the eigenstates with isospin $T = 0$ and angular momentum $J = 0$. The following fermionic basis have been used for exact diagonalization of the Hamiltonian (18)

$$|I, \alpha, \beta\rangle = [\nu^\alpha_I \times \pi^\beta_I]_0|0\rangle, \quad (22)$$

where

$$\begin{aligned} \nu^\alpha_{I,M} &= (a^\dagger_n a^\dagger_n a^\dagger_n a^\dagger_n)^\alpha_{I,M}, \\ \pi^\alpha_{I,M} &= (a^\dagger_p a^\dagger_p a^\dagger_p a^\dagger_p)^\alpha_{I,M} \end{aligned} \quad (23)$$

are orthonormal fully antisymmetric basis vectors of neutron (n) and proton (p) subsystems, respectively. These vectors are constructed using CFP's with definite angular momentum I and have additional quantum number α to distinguish orthonormal states with the same I.

We investigate the possibility to approximate exact ground states of the Hamiltonian (18) by using the alpha-like correlated structures. The creation operator of the four-particle $T = 0$, $J = 0$ correlated structure is introduced as

$$A^\dagger = \frac{1}{2}\sum_\lambda c_\lambda[[a_{\rm n}^\dagger \times a_{\rm n}^\dagger]_\lambda \times [a_{\rm p}^\dagger \times a_{\rm p}^\dagger]_\lambda]_0 \,. \tag{24}$$

In (24) the coefficients c_λ should satisfy to the following relation

$$4c_0 = \sum_{\lambda=\rm even} \sqrt{2\lambda+1}c_\lambda \tag{25}$$

in order to get a $T = 0$ operator. Moreover, the coefficients c_λ in (24) are normalized so that

$$\langle 0|AA^\dagger|0\rangle = 1.$$

Some of the c_λ coefficients are negative.

The lowest state with $J = 0$ and $T = 0$ is approximated by

$$|J = 0, T = 0\rangle_{\rm app} = \mathcal{N}^{-\frac{1}{2}}(A^\dagger)^2|0\rangle, \tag{26}$$

and the coefficients c_λ are determined so as to get a maximum overlap of (26) with the corresponding exact eigenvector. In our case of $j = \frac{7}{2}$, there are only two free parameters in the expression (26).

As it is shown in Fig. 2a, the overlap of exact and approximate ground-state vectors is larger than 0.93 for all values of the Hamiltonian parameter x. The dependence of the coefficients c_λ on the parameter x is displayed in Fig. 2c. For the $x = 1$ case with the pure $T = 1$ pair interaction, the coefficient c_λ with $\lambda = 0$ is much larger than the other ones. This finding reflects the prevailing role of the $T = 1$, $J = 0$ pair and usefulness of the seniority classification in that case.

Thus for the model with SDI, we obtain a similar picture as in the section II where the SO(8) algebraic model has been considered. However, the model with SDI is more realistic than the SO(8) model. What is especially important, it includes fermion pairs with the angular momentum $J \neq 0, 1$.

IV. INFLUENCE OF THE np-PAIR CORRELATIONS ON SOME PHYSICAL QUANTITIES

It was discussed above that the ground-state wave functions of the even-even $Z = N$ nuclei can quite well be approximated by the expressions in which the maximum possible number of fermions form the correlated four-particle $T = 0$, $J = 0$ structures. All information about the ground state is thus contained in the structure of the four-particle creation operators (13) and (24). The important

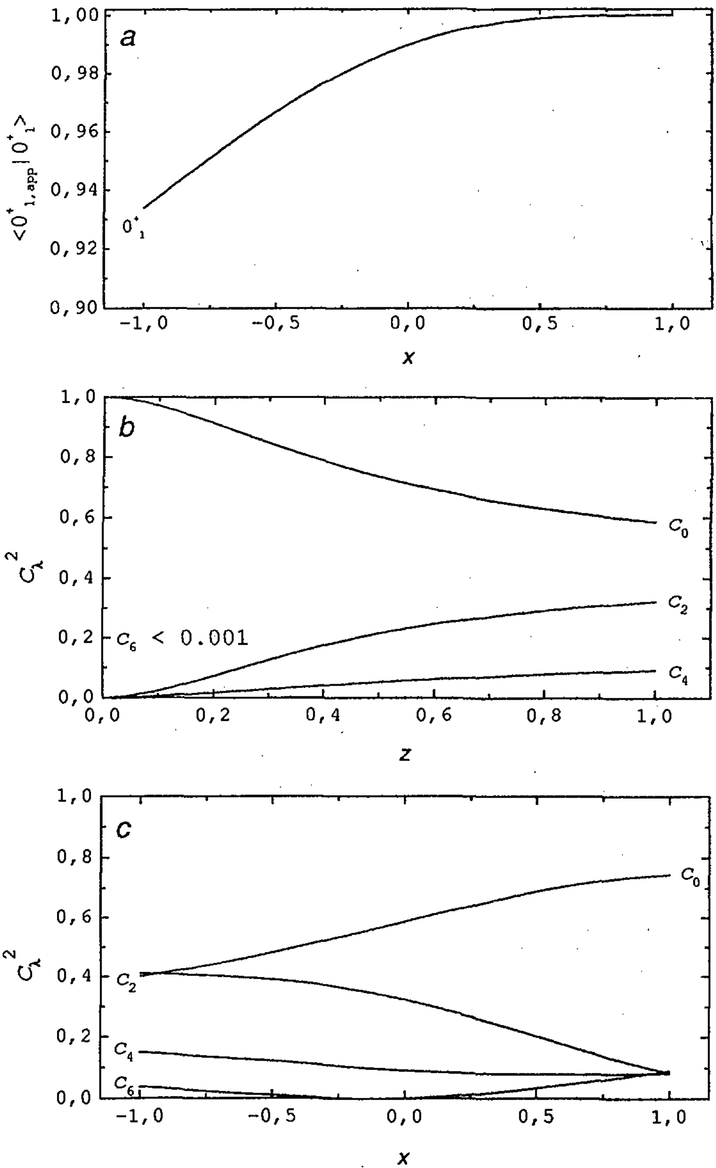


FIG. 2. a) Dependence of an overlap of the exact and approximate ground-state vectors on the parameter x in the single–j model with SDI.

b)Dependence of the squares of the coefficients of the ground-state vector multipole expansion on the parameter z regulating a strength of the np interaction. The results are shown for x=0.

c) The same as in b) but for dependence on the parameter x (z=1).

question arises whether do a relevant experimental quantities exist in which such an alpha-like structure would be revealed and which would confirm an importance of the np pairing degrees of freedom.

In the present section, we discuss the quadrupole sum rule calculated for the ground state of the even-even $Z = N$ nucleus and the ground-state magnetic moment of the odd nucleus with $Z = N \pm 1$. The former quantity mainly characterizes the E2-transition probability from the ground to the first 2^+ state. Both the E2 sum rule and magnetic moment depend on the angular momenta of the neutron and proton subsystems. As in the preceding section, we consider 8 nucleons in the $j = \frac{7}{2}$ ($f_{\frac{7}{2}}$) shell. An additional odd particle is taken to be a neutron.

To investigate an influence of the np pairing forces, we multiply np–interaction terms both in the T=1 and T=0 parts of the Hamiltonian (18) by a factor z which varies from zero (absence of the np–pairing force) to one (full presence of the np–pairing force). Of course, the isospin invariance of the Hamiltonian is broken for $z \neq 1$. Using this artificial procedure we get some insight into an effect of different type pairing correlations.

As it is seen from Fig. 2b, the values of the coefficients c_λ depend strongly on the parameter z. Of course when z is less than 1, the isospin invariance is broken and the relation (25) does not hold. For $z = 0$, i.e. for only nn- and pp-pairing forces, the coefficient $c_0 = 1$ and the neutron and proton subsystems have got zero angular momenta separately. This finding can be explained by the separation of the neutron and proton degrees of freedom and by the seniority conservation for the SDI in single-j shell. With increasing z, c_0^2 decreases and c_λ^2's with $\lambda \neq 0$ increase. Therefore with increasing z, neutron and proton parts of the four-particle correlated structures possess nonzero angular momenta and can influence the ground-state magnetic moment and the quadrupole sum rule.

In Fig. 2c, similar correlations are observed between the values of c_λ and the parameter x. However, as it follows from (25), $c_0^2 < 1$ when $z = 1$ even for $x = +1$, i.e. for only $T = 1$ pairing force being present in the Hamiltonian. The reason for this is the presence of the $T = 1$, $J = 0$ neutron-proton pair correlations which create some angular momentum in the neutron and proton subsystems separately.

The square of the quadrupole proton operator $Q_{2\mu}$

$$Q^{\rm p}_{2\mu} = \sum_{m,m'} C^{jm}_{jm',2\mu} a^\dagger_{jm,{\rm p}} a_{jm',{\rm p}}$$

averaged over the ground state

$$\langle 0^+_{\rm I} | (Q^{\rm p}_2 \cdot Q^{\rm p}_2) | 0^+_{\rm I} \rangle \tag{27}$$

give us a value of the quadrupole sum rule. The dependence of the quantity (27) on z and x is illustrated in Figs. 3a and 3b. No significant change of the sum rule with x is observed. Therefore, the quadrupole sum rule can not be used to get an information on the $T = 1$ and $T = 0$ pairing competition. The sensitivity of this quantity on z is more pronounced. The sum rule value is larger when the np-correlations are absent.

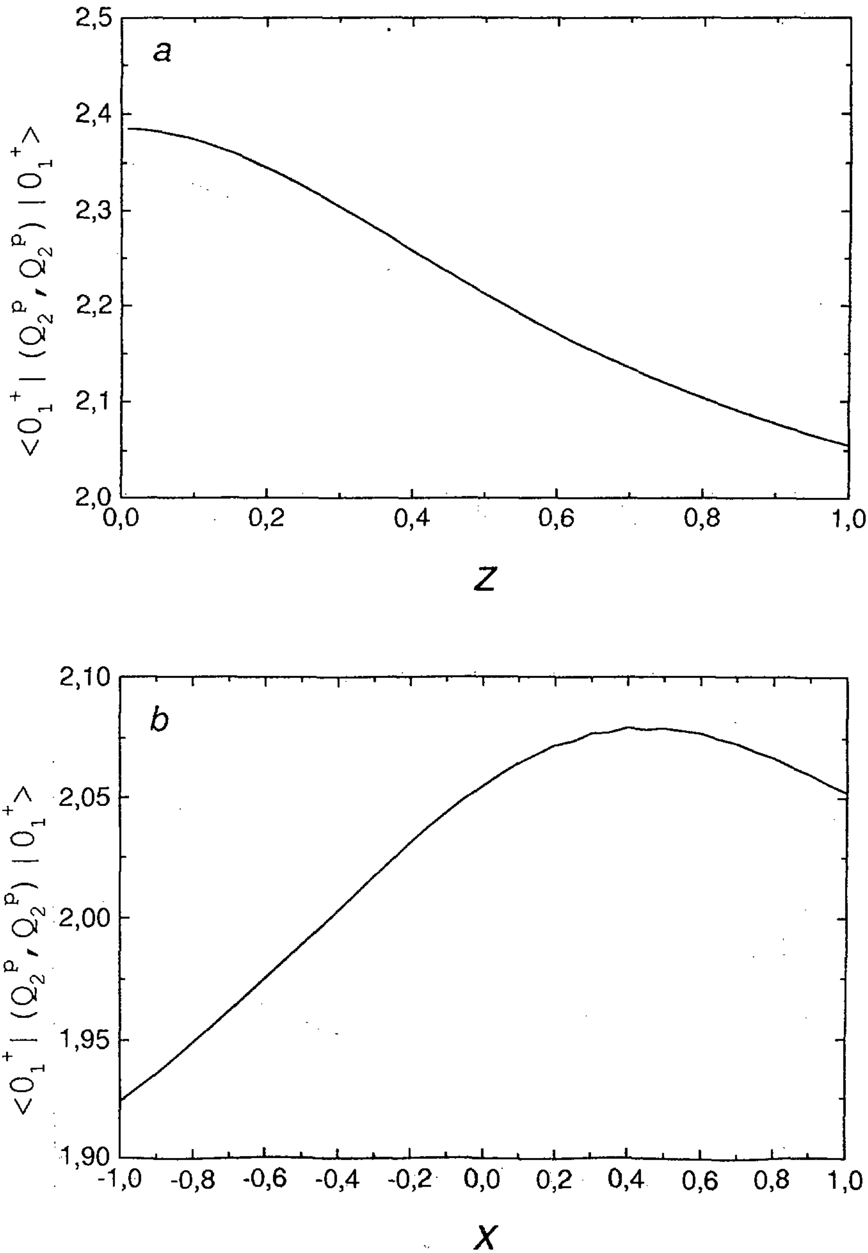
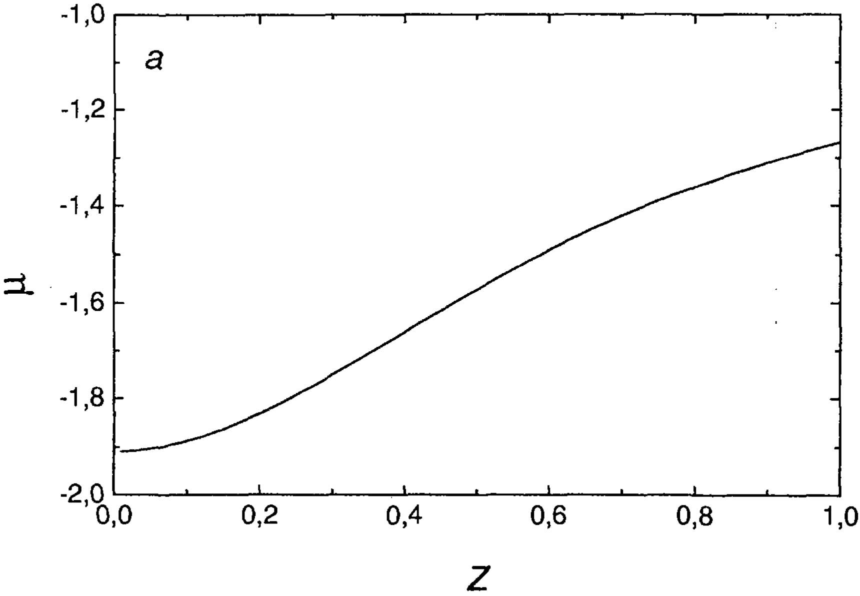


FIG. 3. Dependence of the ground-state quadrupole sum rule of the $N = Z$ nucleus on the parameters z (x=0) (a) and x (z=1) (b) in the single-j model calculations.



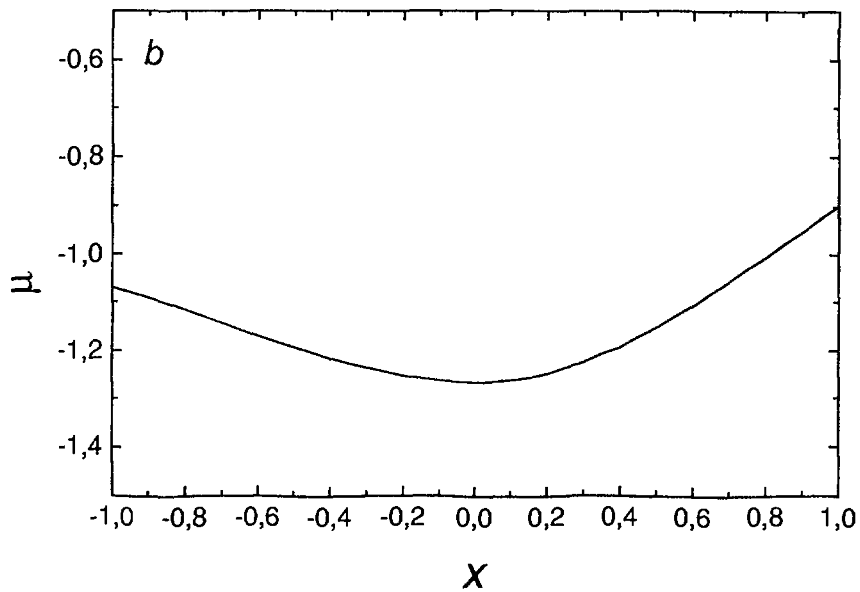


FIG. 4. The same as in Fig. 3 but for the ground-state magnetic moment of the $N = Z + 1$ odd nucleus.

For the ground-state wave function of the odd nucleus adjacent to the $T = 0$ line, we use the alpha-like correlated form

$$|jm\rangle = \mathcal{N}_{\rm odd}^{-\frac{1}{2}} a_{jm}^{\dagger} (A^{\dagger})^2 |0\rangle. \tag{28}$$

The magnetic moment operator is written as

$$\hat{\mu} = g_{j,{\rm n}} \hat{j}_{{\rm n},z} + g_{j,{\rm p}} \hat{j}_{{\rm p},z} \tag{29}$$

with $\hat{j}_{\rm n(p),z}$ being the z-component of the neutron (proton) angular momentum. For the $f_{\frac{7}{2}}$ shell, $g_{j,{\rm n}}$=-0.55 and $g_{j,{\rm p}}$=1.66.

Using the state vector (28) and its expansion in the basis (22), we obtain for the magnetic moment by a direct calculation

$$\mu \equiv \langle jj|\hat{\mu}|jj\rangle = g_{j,{\rm n}} j - (g_{j,{\rm n}} - g_{j,{\rm p}}) \frac{\mathcal{N}_{\rm even}}{\mathcal{N}_{\rm odd}} \frac{1}{(j+1)(2j+1)} \langle \hat{I}^2 \rangle_{\rm p} \, , \tag{30}$$

where $\langle \hat{I}^2 \rangle_{\rm p}$ is an average value of the squared angular momentum of the proton (the same for neutron) subsystem of the even–even core described by the state vector $\mathcal{N}_{\rm even}^{-\frac{1}{2}} (A^{\dagger})^2 |0\rangle$.

The first term in (30) is the single particle magnetic moment of an odd neutron. The second term is a contribution of the protons and neutrons forming the four-particle $T = 0$, $J = 0$ correlated structures. Of course, this $Z = N$ core contribution appears due to the nonzero angular momenta of the proton and neutron subsystems of the core. It is proportional to the core average of the squared proton (neutron) angular momentum operator. If the proton and neutron subsystems of the even-even $Z = N$ nucleus have zero angular momenta, the core term is equal to zero and the magnetic moment approaches its single particle value. This is illustrated in Fig. 4a where the dependence of the magnetic moment μ on the Hamiltonian parameter z is displayed. For z equal to one, i.e. for the np-correlations fully included, the contribution of the core nucleons becomes essential.

A sensitivity of the magnetic moment to parameter x, shown in Fig. 4b, is weak and insufficient to study $T = 1$ and $T = 0$ pairing competition. This finding is connected with the fact that both the isovector and isoscalar np-pair correlations introduce nonzero angular momentum into the neutron and proton subsystems and cause deviation of the magnetic moment from the single particle value.

V. CONCLUSIONS

We have discussed the $T = 0$ and $T = 1$ pairing correlations within the framework of two simple models: SO(8) algebraic model and the single-j model with the Surface Delta Interaction. We investigate a possibility to represent the wave vector of the ground state by the simple one-term expression obtained by using a creation operator of the four-particle $T = 0$, $J = 0$ correlated structures. In the cases studied, an accuracy better than 93% of the overlap of the exact and approximate wave functions has been obtained. Thus, a possibility opens to formulate an approximate approach, similar to the Broken Pair Approximation for nuclei with like nucleon pairing, to describe nuclei in which both like-nucleon and np pair correlations are important.

Employing this approximation, we have investigated an influence of the different kinds of the pair correlations on the ground-state magnetic moments of an odd nucleus and on the electric quadrupole sum rule. The magnetic moment appears to be quite sensitive to the presence of the np-correlations. However, both the magnetic moment and quadrupole sum rule are not sensitive enough to the competition between the $T = 1$ and $T = 0$ pair correlations.

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