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THE APPLICATION OF THE APPROXIMATE FOUR BODY EQUATIONS TO DESCRIPTION OF THE ELASTIC SCATTERING AND CHARGE EXCHANGE OF PIONS BY ^{3}He NUCLEUS

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INTRODUCTION

At present there exists a great deal of experimental data on elastic scattering and charge-exchange of $\pi^{\pm}$ mesons on nuclei ^3He and ^3H /1a,b/. Theoretical treatment of these processes is rather important because the systems $\pi\,^3He$ and $\pi\,^3H$, after πd, are the only systems for which exact dynamical equations may be formulated within the potential theory /2/ and the properties of these systems may be analysed within a consistent theory. We think that one should find solutions as exact as possible for the 4-particle equations since only then one can discuss the role of the meson degrees of freedom and the contribution of inelastic processes in a model-independent way. The significance of such processes is clear even from the data on the level shift and width in $\pi^-\,^3He$ - mesoatom (in contrast to the case of $\pi^- d$ in $\pi^-\,^3He$ $\Delta E \sim \Gamma$).

Unfortunately, until now the exact 4-body equations have not been applied to the systems $\pi 3N$. Among attemps of "a nonoptical analysis"* of these systems one should mention those based on the Glauber approximation /4/ and fixed-scatterer approximation /5/.

As is known, the Glauber approximation, at least for three particles, can be derived from the exact 3-body equations. It is to be expected that for four particles the same is valid, therefore solutions obtained in ref. /4/ on the basis of the Glauber model can be treated as an approximation to the exact equations.

The scattering within the fixed-scatterer model can also be considered as an approximation to the exact solution. However, in our opinion, the model developed in paper /5/ cannot be treated as an approximation to the exact equations. The point is that these equations can be rewritten in a form in which the kernel and inhomogeneous term are defined by a two-body operator obeying the corresponding Lippmann-Schwinger equation. The two-body operator used in /5/ does not satisfy this requirement, therefore it is not clear in what sense the equations to be solved approximate the exact equations. And what is more, the nucleon recoil is taken into account by a procedure which in no way follows from the initial 4-body equations.

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*A detailed description of these processes in the framework of the optical model with a first-order potential can be found in papers /3/ .

In what follows we shall apply approximate 4-body equations of ref. [6] to the $\pi^{\pm}\,{}^3He$ interaction in the energy region 68 MeV $\le E_\pi \le$ 208 MeV. These equations differ from the most popular description within the optical model with the first-order optical potential in two aspects: 1) The equations are derived without the impulse approximation; 2) The rescattering of pions on nucleons is taken into account in all orders.

In § 1 we derive the basic equations, in § 2 results of numerical calculations are presented and in § 3 the obtained results are discussed and compared with experimental data.

1. The mainpoint of the approximation used is as follows. Let the total Hamiltonian H of a system $\pi 3N$ be of the form

$$H = h_0 + V_{\pi c} + H_c \qquad /1/$$

with H_c , the Hamiltonian of a 3N subsystem.
The equation for the four-particle transition operator may be rewritten as:

$$T = T^\circ + T^\circ [G_0(E) - G_c(E)], \qquad /2/$$

where operator T° obeys the equation

$$T^\circ = V_{\pi c} - V_{\pi c} G_0(E) T^\circ . \qquad /3/$$

In eqs. /2/ and /3/ the following notations are used: $G_0(E) = (h_0 - E)^{-1}$, $G_c(E) = (H_c - E)^{-1}$,

$V_{\pi c} = \sum_{i=1}^{3} V_{\pi N_i}$ is the sum of two-body πN potentials; h_0 is the kinetic energy of relative motion of a pion and a nucleus. All the quantities are in the c.m.s. of four bodies.

The approximation to be made changes the nucleus Hamiltonian H_c by the first-rank operator, i.e.,

$$H_c \cong H_c^{(1)} = \epsilon_0 |\chi_0\rangle\langle\chi_0| . \qquad /4/$$

where ϵ_0 and $|\chi_0\rangle$ are the eigenvalue and eigenfunction of

the exact Hamiltonian H_c . The approximation /4/ allows us to integrate the many-dimensional equation /2/ over nuclear variables and to obtain the one-dimensional integral equation for the π^3He elastic scattering amplitude

$$\langle \vec{k} | T | \vec{k}' \rangle = \langle \vec{k} | T^\circ | \vec{k}' \rangle + \int d\vec{q} \langle \vec{k} | T^\circ | \vec{q} \rangle [G_0(q,E) - G_0(q, E-\epsilon_0)] \times$$
$$\times \langle \vec{q} | T | \vec{k}' \rangle , \qquad /5/$$

where $\langle \vec{k} | T | \vec{k}' \rangle \equiv \langle \vec{k}\chi_0 | T | \vec{k}'\chi_0 \rangle$ is the amplitude of elastic scattering of the pion on the nucleus ${}^3He({}^3H)$.

Equation /3/ in the momentum representation will be solved for a given value of the total isospin of the 4-particle system. We have

$$\langle \vec{k}\mu | V_{\pi c}^{TT_z} | \vec{k}'\mu' \rangle \equiv \langle \vec{k}\xi_\mu^{TT_z} | \sum_{i=1}^{3} V_{\pi N_i} | \xi_{\mu'}^{TT_z} \vec{k}' \rangle =$$

$$= \sum_i [c_1^T v_{s_1}^{\pi N}(k,k') + c_3^T (v_{s_3}^{\pi N}(k,k') + 3 v_p^{\pi N}(\vec{k},\vec{k}'))] \times \qquad /6/$$

$$\times e^{i(\vec{k}-\vec{k}')\vec{z}_i} \delta_{\mu\mu'} ,$$

where $\xi_\mu^{TT_z}$ is the spin-isospin function of π^3He

$$\vec{z}_1 = \frac{1}{2}\vec{r}_{12} + \frac{1}{3}\vec{r}_3$$

$$\vec{z}_2 = -\frac{1}{2}\vec{r}_{12} + \frac{1}{3}\vec{r}_3$$

$$\vec{z}_3 = -\frac{2}{3}\vec{r}_3$$

$\vec{r}_{12}, \vec{r}_3$ are the Jacobi variables in the 3 nucleon system

$$c_1^T = \frac{1}{3}[1 + 2(-1)^{T+\frac{1}{2}} \left\{ \begin{matrix} 1 & 1 & 1 \\ \frac{1}{2} & T & \frac{1}{2} \end{matrix} \right\}]$$

$$c_3^T = \frac{2}{3}[1 - 2(-1)^{T+\frac{1}{2}} \left\{ \begin{matrix} 1 & 1 & 1 \\ \frac{1}{2} & T & \frac{1}{2} \end{matrix} \right\}]$$

$v^{\pi N}_{s_1}$, $v^{\pi N}_{s_3}$ are the s-wave πN potentials in the states with isospin $t_{\pi N}=\frac{1}{2}$ and $t_{\pi N}=\frac{3}{2}$, resp., $v^{\pi N}_{p}$ is the p-wave πN potential in the state with $t_{\pi N}=\frac{3}{2}$ and $J_{\pi N}=\frac{3}{2}$. Since the πN phases P_{31} and P_{13} are small in the considered energy region, we take into account only the interaction in the resonant P_{33} state. The parameters of potentials v_{s1} and v_{s3} were defined from fitting the corresponding phases and scattering lengths. The parameters of the potential v_p were found from fitting the total cross section of πN scattering in the region $0\le E_\pi \le 250$ MeV.

Thus, this description of πN interaction disregards, in fact, the nucleon spin; πN potentials were taken in the separable form

$$v_\alpha(\vec{k},\vec{k}') = \frac{\lambda_\alpha}{4\pi^2\mu_{\pi N}} h_\alpha(\vec{k})h_\alpha(\vec{k}')$$

$$h_\alpha(\vec{k}) = g_\alpha(k), \quad \alpha = 1,2 \qquad /7/$$

$$= \sqrt{3}\, g_3(k)\frac{\vec{k}}{|\vec{k}|} \quad \alpha = 3,$$

where index $\alpha = 1,2,3$ labels potentials v_{s_1}, v_{s_3}, v_p . With potential /7/ eq. /3/ is solvable explicitly and the silution is

$$<\vec{k}|T^\circ(\vec{r}_{12},\vec{r}_3)|\vec{k}'> = \frac{1}{4\pi^2\mu_{\pi^3He}} \sum_{i,j=1}^{3} \sum_{\alpha,\beta=1}^{3} \qquad /8/$$

$$e^{i\vec{k}\vec{z}_i} \,|h^\alpha(\vec{k})[X^{-1}(\vec{r}_{12},\vec{r}_3)]^{\alpha\beta}_{ij} e^{-i\vec{k}'\vec{z}_j} h^\beta(\vec{k}'),$$

where matrix $\hat{X}$ has the structure

$$\hat{X} = \frac{\mu_{\pi N}}{\mu_{\pi^3He}} \hat{\Lambda}^{-1} + \xi$$

$$[\hat{\Lambda}]^{\alpha\beta}_{ij} = \Lambda^\alpha \delta_{\alpha\beta}\delta_{ij}, \; \Lambda^1 = c_1^T\lambda_{s_1}, \Lambda^2 = c_3^T\lambda_{s_3}, \; \Lambda^3 = c_3^T\lambda_p.$$

Matrix ξ is given in Appedix A. So if we restrict ourselves to the inhomogeneous term T°, for the amplitude of

elastic scattering of the pion on ^{3}He we obtain:

$$f(\vec{k},\vec{k}') = -4\pi^2\mu_{\pi^3He}\int d\vec{r}_{12}d\vec{r}_3\Psi^2_{^3He}(\vec{r}_{12},\vec{r}_3)\times \qquad /9/$$

$$\times<\vec{k}|T^\circ(\vec{r}_{12},\vec{r}_3)|\vec{k}'>.$$

2. We used two sets of elementary πN potentials I and II with the different form factors in the p-wave potential

$$g_\sigma(k) = \frac{1}{\alpha_\sigma^2+k^2} + \frac{S_\sigma}{\beta_\sigma^2+k^2}; \quad \sigma = 1,2$$

$$g_3(k) = \frac{k}{(\alpha_3^2+k^2)^2} + S_3\frac{k^3}{(\beta_3^2+k^2)^2} \quad [I]$$

$$= \frac{k}{(\tilde{\alpha}_3^2+k^2)^2} + \tilde{S}_3\frac{k^2}{(\tilde{\beta}_3^2+k^2)^2} \quad [II].$$

If in eqs. /3/, /5/ and the equation for the two-body πN t-matrix (in p-state) the free Green function is taken in the relativistic form with respect to pion and nucleon momenta, then the arising integrals in the case of parametrization /1/ diverge and a cutoff is necessary. We used the nonrelativistic form of the nucleon kinetic energy, in this case the integrals converge though slowly and we retain the cutoff. The parameters used are listed in the Table.

Table

Parameters of πN potentials

	α (fm)	β (fm)	S	λ	K_{max}
S_{31}	3.382	1.107	-0.02733	71.3585 (fm^{-3})	
S_{11}	3.188	0.823	0.05015	-6.137 (fm^{-3})	
P_{33}(I)	1.497	2.467	1.3803	-1.383 (fm^{-1})	15 fm^{-1}
P_{33}(II)	1.623	4.467	12.407	-2.149 (fm^{-1})	20 fm^{-1}

K_{max} is the cutoff momentum.

The spatial part of the wave function contained only the symmetric S - component and was chosen in the Irving form[7].

We have calculated here the angular distributions for the following processes

$$\pi^{+}\,{}^{3}He \rightarrow \pi^{+}\,{}^{3}He$$

$$\pi^{-}\,{}^{3}He \rightarrow \pi^{-}\,{}^{3}He$$

$$\pi^{-}\,{}^{3}He \rightarrow \pi^{0}\,{}^{3}H .$$

Equation /5/ was solved in the two different approximations:

1) $T \approx T^{\circ}$,

2) $T \approx T^{\circ} + T^{\circ}[G_0(E) - G_0(E-\epsilon_0)]\,T^{\circ}$.

As is seen from Figs.1 to 5, the theoretical elastics cross sections are in qualitative agreement with experimental curves to all the approximations and in both the fitting variants I and II. The best agreement is observed at E_π =132 MeV and E_π =145 MeV. All the theoretical curves posses a deep minimum about 90° which corresponds to the dominating P -wave contribution (in the system $\pi 3N$). All the experimental cross sections also have min though rather less pronounced. This disagreement between theory and experiment should, in our opinion, be related both to the neglect of spin terms in the elementary amplitude used in the calculation and "small" components of the ^3He wave-function (S′- and D - states).

From the results for differential cross section of elastic scattering it is clear that the solutions to eq. /5/ obtained in different approximations are similar. So, for instance, the first iteration of eq. /5/ gives the cross section whose maximum difference from the cross section calculated in the zeroth approximation is ≈ 30% at E_π =145 MeV and θ =140°. At other energies and angles the difference is still smaller. Thus, the application of approximation /4/ to the system $\pi 3N$ allows us to be restricted only to the fixed-scatterer approximation (i.e., the inhomogeneous term in eq. /5/).

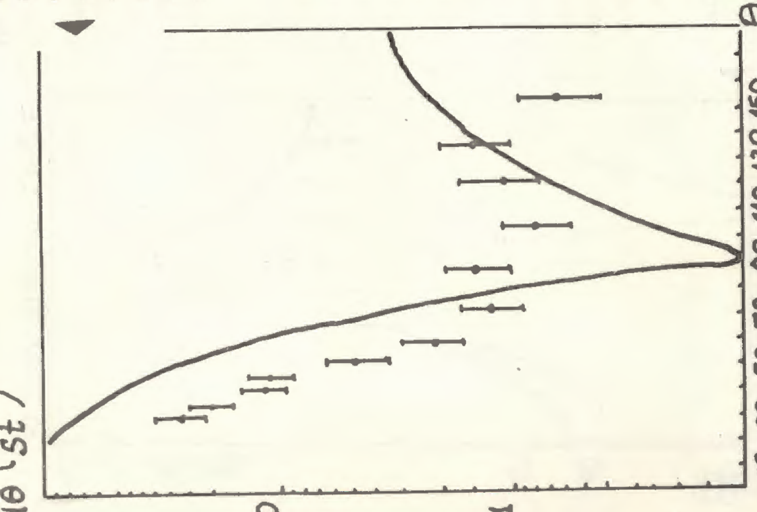


Fig.1. The full line represents the calculations of the differential cross section for $\pi^{+3}He$ elastic scattering at energy E_π =132 MeV with the parameters of fit (II)*.

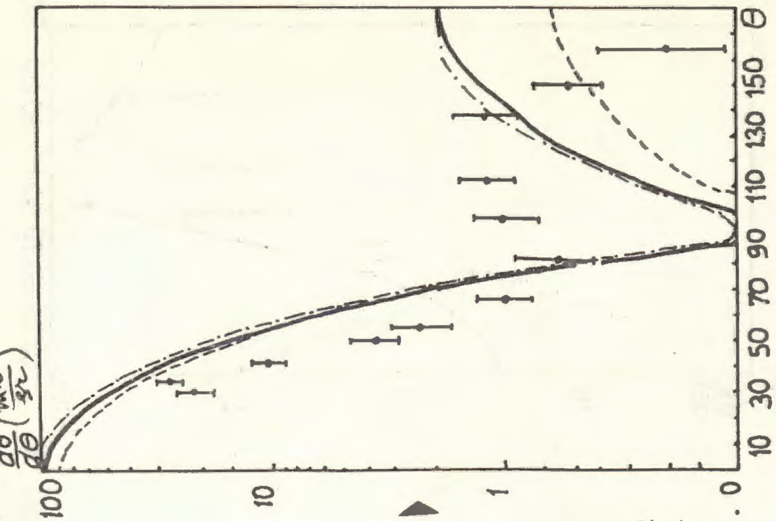


Fig.2. ——— the differential cross section of $\pi^{+3}He$ el. scattering at energy E_π =145 MeV calculated with fit (II) (zero iteration of eq. /5/). —·— the same calculation with the first iteration of eq. /5/. — — — the same curve calculated in the zero iteration of eq. /5/ with the parameters of fit (I).

* Experimental data on all figures were taken from ref. [1a,b].

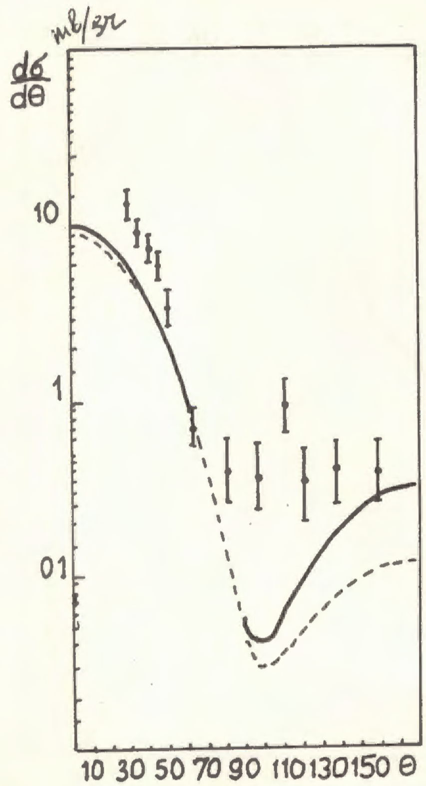


Fig.3. ——— The differential cross section of the $\pi^-\ {}^3He$ el. scattering at energy 145 MeV with the parameters of fit (II) calculated in the zero iteration of eq. /5/. — — — the same curve calculated with the parameters of fit (I).

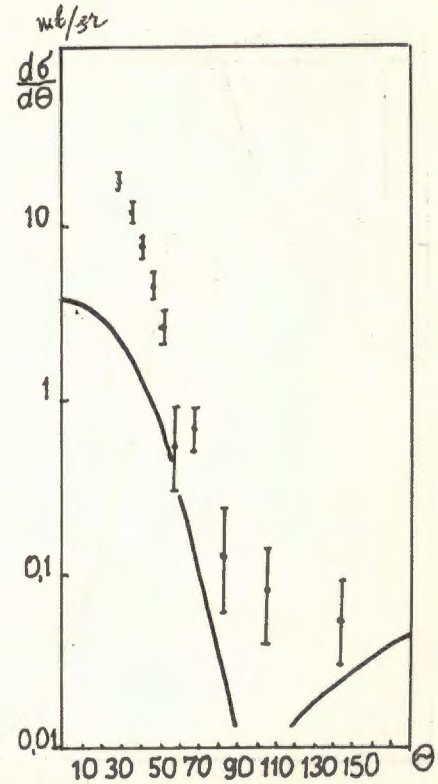


Fig.4. The same curve as in Fig.3 calculated at energy E_π=208 MeV.

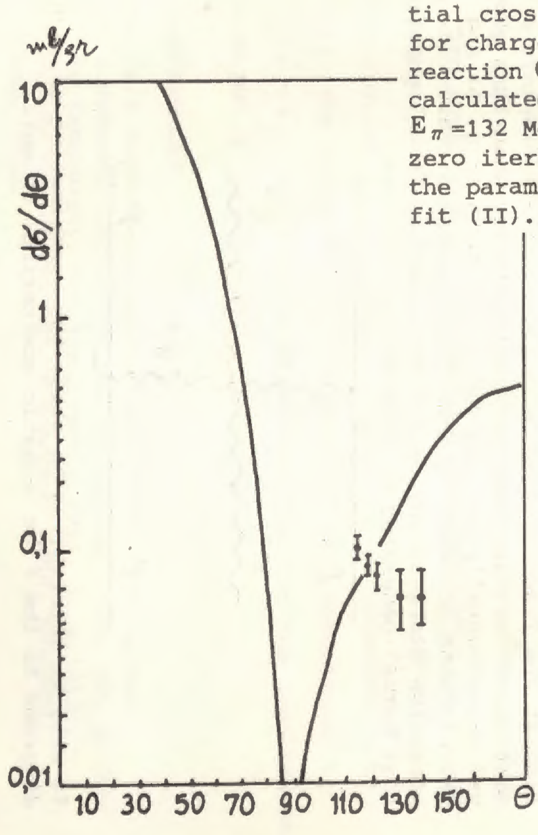


Fig.5. The differential cross section for charge exchange reaction ($\pi^-\ {}^3He \to \pi^0\ {}^3H$) calculated at energy E_π=132 MeV in the zero iteration with the parameters of fit (II).

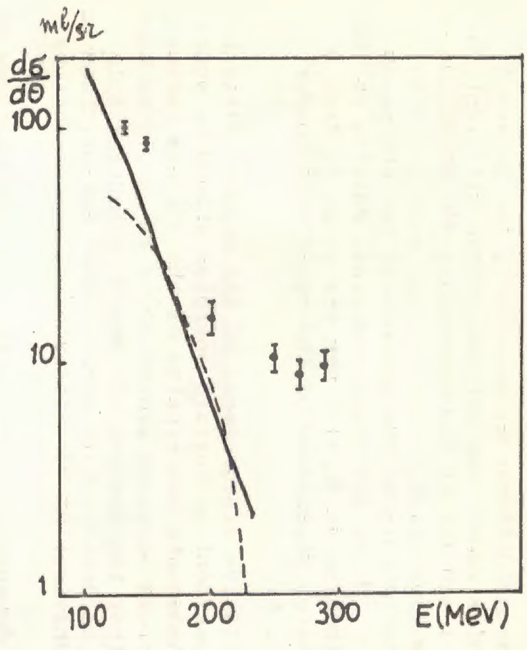


Fig. 6. -- -- -- the energy dependence of the differential cross section calculated at the scattering angle 115° (zero iteration, fit II); straight line [4].

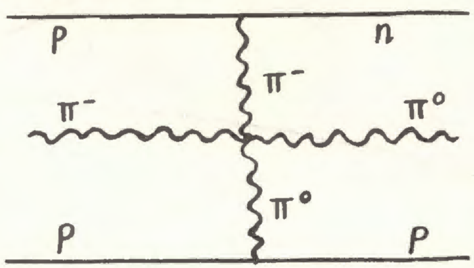
A rather larger difference, especially for backward scattering angles comes from the use of two different parametrizations (I and II) of the elementary πN amplitude in state P^{33} (see Figs. 2,3).

We have also calculated the process of the charge-exchange $\pi^{-}\,{}^{3}He \to \pi^{0}\,{}^{3}H$ at different energies. Results for the angular distribution at $E_{\pi}=132$ MeV are given in Fig. 5 and for the energy dependence of the cross section for $\theta=115^{\circ}$ in Fig. 6.

3. A quantitative disagreement of the angular distributions with experiment at certain energies should be attributed to an inadequate description of the πN scattering differential cross sections around 90°. It should be kept in mind also that the neglect of the πN potential spin structure, as is done in this work, cannot apriori lead to the correct behaviour of πN differential cross sections around 90°.

The strong dependence of the obtained cross sections of elastic scattering on the form of parametrization of the elementary πN interaction indicates the necessity of a more thorough study of π- nuclear cross sections as functions of the off-mass-shell behaviour of elementary πN amplitudes. Note, that an analogous conclusion has been made for the large-angle scattering in calculations with the first-order optical potential [8].

The energy dependence of the charge-excharge cross section at the fixed scattering angle ($\theta=115^{\circ}$) displays a more sharp decrease with increasing energy than that observed experimentally both in our calculation (Fig. 6) and calculations of other authors [4]. A possible reason for a more slow decrease may be the contribution from "nonpotential terms" generated by diagrams of the form



It will be interesting to study the contribution of such diagrams to the $\pi\,{}^{3}He$ elastic scattering, as well.

APPENDIX

We bring here the formulae for the matrix $\hat{\xi}$ (see the formula below /8/)

$$(\hat{\xi})^{\alpha\beta}_{ij} = \frac{1}{\pi\mu_{\pi\,{}^3He}} \int_0^\infty q^2 dq\, g_\alpha(q) g_\beta(q)\, j_0(q|\vec{z}_i - \vec{z}_j|) \times$$

$$\times \frac{1}{\frac{q^2}{2m_{^3He}} + \sqrt{q^2+\mu^2} - \mu - E} + \qquad /A.1/$$

$$+ i \frac{r_0(E)}{\mu_{\pi\,{}^3He}} q_1(E)\, j_0(q_1(E)|\vec{z}_i - \vec{z}_j|)\, g_\alpha[q_1(E)]\; g_\beta[q_1(E)],$$

where

$$r_0(E) = \frac{m_{^3He}[\mu^2 + q_1^2(E)]^{1/2}}{m_{^3He} + [\mu^2 + q_1^2(E)]^{1/2}}$$

$$q_1(E) = \{ [\sqrt{m^2_{^3He} + 2m_{He}(\mu + E + \frac{\mu^2}{2m_{^3He}})} - m_{^3He}]^2 - \mu^2 \}^{1/2}$$

$$\mu_{\pi^3He} = \frac{\mu m_{^3He}}{\mu + m_{^3He}},$$

μ is the π- meson mass, $m_{^3He}$ is the mass of ${}^3He$ nucleus.

The integrals ξ_{ij} for $\alpha,\beta = 3$ have been calculated neglecting the terms proportional to $Y_{2m-m'}(\hat{\vec{z}_i - \vec{z}_j})$ whose contribution to the elastic scattering is expected to be small. The integrals ξ^{13}_{ij} and ξ^{23}_{ij} are proportional to $Y_{1m}(\hat{\vec{z}_i - \vec{z}_j})$ and they have been omitted for the same reason. So we put $\xi^{13}_{ij} = \xi^{23}_{ij} = 0$.

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