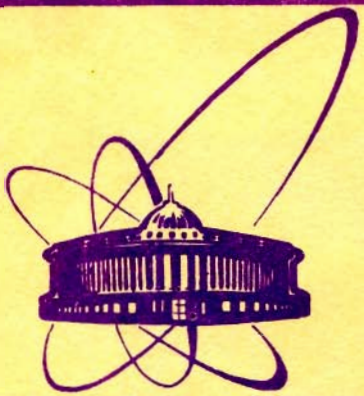

сообщения
объединенного
института
ядерных
исследований
дубна

932/2-80

3/3-80

E4 - 12893

S.N.Ershov, J.Bang, J.Cejpek, J.Dobes, A.Gareev,
B.S.Nilsson

TWO-PARTICLE TRANSFER IN HEAVY ION REACTIONS

1979

E4 - 12893

S.N.Ershov, J.Bang,* J.Cejpek, J.Dobes, A.Gareev, B.S.Nilsson *

TWO-PARTICLE TRANSFER IN HEAVY ION REACTIONS

* The Niels Bohr Institute, University of Copenhagen, DK-2100 Copenhagen Ø, Denmark.

Ершов С.Н. и др.

Е4 - 12893

Двухнуклонные передачи в реакциях с тяжелыми ионами

Вычисляются сечения двухнуклонных реакций передач между тяжелыми ионами в рамках DWBA. Ввиду важности эффектов отдачи для данных реакций, расчеты проводились в рамках метода EFR (exact-finite-range). Учитывалась только одновременная передача двух нуклонов. Двухнуклонные формфакторы передач вычислялись по методу разложения по базисным функциям Штурма-Лиувилля. Расчеты проводились для реакций $^{48}Ca(^{18}O, ^{16}O)^{50}Ca$, $^{42}Ca(^{16}O, ^{18}O)^{40}Ca$, $^{48}Ca(^{16}O, ^{14}C)^{50}Ti$. Установлено, что сечение реакции увеличивается в два-три раза с увеличением числа базисных функций, используемых для расчета двухнуклонных формфакторов. Увеличение базиса соответствует более точному учету влияния непрерывного спектра при вычислении двухнуклонных формфакторов.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1979

Ershov S.N. et al.

E4 - 12893

Two-Particle Transfer in Heavy Ion Reactions

Calculations of simultaneous two-particle transfer cross sections are performed with different overlap factors, using DWBA with the recoil included exactly. The interaction between the particles, taken into account in the overlap functions, lead to enhancements of the cross sections by factors of 2 to 3, when the overlaps are calculated in a basis with a large number of functions, corresponding to shell model continuum components.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1979

© 1979 Объединенный институт ядерных исследований Дубна

1. INTRODUCTION

Heavy-ion induced two-particle transfer cross sections have been investigated, experimentally as well as theoretically, by a number of authors [1,2]. The aim of such studies has often been the investigation of pair correlations in the wave functions of the nuclei, which take part in the transfer reaction.

These correlations are in general of a collective nature, and the cross sections are thus supposed to yield valuable information about the static or vibrating pair field of the nuclei in question [3]. However, although a number of valuable results, concerning the relative cross sections of transfer to different states of the final nuclei has been obtained, the absolute cross sections were in most cases underestimated by factors of 10 to 1000 from the theoretical values.

These deviations are seen not only in the nuclei of large collective pair correlations, but in nearly all two-particle transfers.

For one-particle transfer, the determination of spectroscopic factors by means of DWBA calculations is supported in an essential way by the successfull applications of DWBA in the cases of transfer to closed shell nuclei where the spectroscopic factor is close to unity.

In a similar way, it seems, that before any final conclusions concerning nuclei with strong collective pairing correlations can be drawn, one should be able to get agreement between theory and experiment in the simple cases, where two nucleons are transferred to a closed shell nucleus.

In heavy ion transfer reactions, this means that either projectiles or target nucleus must be of the closed shell type, and the other nucleus must have two particles outside a closed shell.

In systems of the latter type, non-collective pair correlations are important, and it has been shown, that they

influence the radial shape of the overlap functions in a way, such as to lead to large enhancements in the transfer cross sections [1].

These enhancements are nevertheless not by themselves large enough to remove the above-mentioned discrepancies. Only when the previously neglected two-step transfer processes are introduced alongside with the simultaneous transfer [2,4] one now seems to be able to obtain agreement between theory and experiment. However, detailed investigations of these problems are necessary, before the use of two particle transfer in nuclear structure research can be brought on a solid basis.

We shall in this article concentrate on the first problem, the two particle overlap functions, and consequently, we shall consider one step transfer as the mechanism. Since recoil seems important in these processes, we shall take it into account exactly.

2. THE TWO-PARTICLE OVERLAP FUNCTION

Since the formalism of calculating two-particle overlaps has been given several times, see, e.g., ref. [1], we shall here not derive the relevant equations, but just give the final results.

Let us consider the special case of a two particle transfer to a closed shell nucleus. The core excitations can be neglected, and the overlap function satisfies the equation

$$(-\frac{\hbar^2}{2m}(\Delta_1+\Delta_2)+V(r_1)+V(r_2)+\gamma V_{12}(\bar{r}_1-\bar{r}_2)-E)\Phi=0. \qquad (2.1)$$

We shall here consider only the case that the closed shell nucleus is so heavy (mass= $A\cdot M$, where M is the nucleon mass) that the single particle kinetic energy can be written as in (2.1) with $m=M\frac{A}{A+1}$ and r_1, r_2 being the distances between the center of mass A and the two extra nucleons.

In the general case, where A is not a closed shell nucleus, the two-particle overlaps will satisfy a set of coupled equations, where, however the coupling terms are in many cases smaller than the terms already present in (2.1) and always of finite range. Therefore, some main features of the solution of (2.1) will be found in the general case, too, particularly as concerns the asymptotic behaviour of the solutions. This, in a more exact way shows the necessity of finding precise solutions of the problem

of a closed shell plus two nucleons, before more complicated problems can be attacked.

The main difficulty of solving equation (2.1) consists in the fact, that the core-particle Hamiltonian has both a continuous spectrum and a discrete one, and that when V_{12} is taken into account, the overlap function Φ will contain components from state of both the discrete and the continuous spectrum. The neglect of the continuum admixtures being ever so justified in many appreciations of the shell model, is disastrous in transfer calculations. This is clear from the fact that the amplitudes of these reactions get their main contributions from the regions of space, where the bound state components are falling off exponentially.

Different ways of including the continuum components in the solution of (2.1) have been suggested /5/. However, only the two methods mentioned below have proven themselves efficient in including such components where both particles are in the continuum, and in two particle transfer where such components play an essential role.

One method, introduced by Ibarra, Vallieres and Feng /6/ consists of expanding in terms of harmonic oscillator eigenfunctions.

The main disadvantage of this method is that each of these functions falls off very fast, as $\exp(-cr_i^2)$, where r_i is the core-particle distance, and in general a very large number of components is needed in order to reproduce the correct behaviour of Φ for large r_i-values.

The other method, suggested by Bang and Gareev /7/ consists in expanding Φ in a set of products of Sturmian wave functions, i.e., in the antisymmetrized, angular-momentum coupled products of solutions of

$$(-\frac{\hbar^2}{2m}\Delta + \lambda_i V_s(r) - E_s)\psi_i = 0, \qquad (2.2)$$

where E_s is a prescribed energy, whereas the eigenvalue λ_i corresponds to different well depths.

This basis, which, like the one above, contains only a discrete set of functions, has the advantage, that the value of E_s can be chosen so, that all components approximate the correct asymptotic shape. This will in general lead to a particularly fast convergence of this type of expansions, as was shown in /8/.

So we write

$$\Phi = \Sigma\, c_{mn}\psi_n(r_1)\psi_m(r_2). \qquad (2.3)$$

When we insert (2.3) in (2.1), multiply from the left with $\Psi_\ell^*(r_1)\Psi_k^*(r_2)$ and integrate over r_1 and r_2, we get, using (2.2) and the weight-orthogonality of the Sturm-Liouville functions, the matrix equation /1/

$$\sum c_{mn}\,((E_{s_1}+E_{s_2}-E)\langle k\ell\,|\,mn\rangle \; + \; + \gamma\langle mn\,|\,V_{12}\,|\,k\ell\rangle \; + \; + (\lambda_1-1)\langle\ell\,|\,n\rangle_2\,\delta_{km} + (\lambda_2-1)\langle k\,|\,m\rangle_1\,\delta_{\ell n}\;) \;=\; 0, \tag{2.4}$$

where we have used $V = V_s$.

The lack of a Moshinsky-type transform for the Sturmian functions is not a serious drawback, since many interactions directly have simple expressions in terms of r_1 and r_2.

The asymptotic form of Φ, i.e., the form when $|r_1|$, $|r_2|$, or $|r|$ is large, was investigated in /8/. In general, it is determined by the binding energies, i.e., by the binding of one particle, and of the two interacting particles to the core. In order to get realistic overlap functions, it is therefore necessary to adjust the interactions in (2.1) in such a way that these binding energies are identical to the exact ones. With identical particles, this means, that V must be chosen so as to reproduce the one-particle energy, whereas the interaction between the particles is adjusted to get the correct two-particle binding energy. In practice, this is in both methods done by the choice of the coupling constant, γ in (2.1).

Still, with the Sturmian method, E_s and V_s must be chosen. In principle, any $E_s<0$ can be used, but the fastness of the convergence is strongly dependent on this choice. It was shown in /8/, that in the region r_1, r_2 which is the most important for simultaneous transfer,

$$\Phi \sim \exp(-\kappa\frac{r_1+r_2}{\sqrt{2}}) \qquad \frac{h^2\kappa^2}{2m} = E \tag{2.5}$$

for a class of overlap functions, which include those considered in this paper.

This corresponds to the choice

$$E_s = \frac{E}{2} \tag{2.6}$$

a choice, which also is useful in the sense, that only one sort of Sturmian functions appear, and that antisymmetric Φ's are easy to construct.

For calculational reasons, it is most convenient to choose the V_s of equation (2.2) identical to the V of equation (2.1). It should be noted, that in that case, the convergence of the Sturmian expansion is not so fast in regions where $|r_1| \gg |r_2|$ or $|r_2| \gg |r_1|$ /8,9/.

This may play some role for sequential transfer, but in the one step process, treated here, it seems to be of minor importance, as is also seen from the fast convergence of the cross sections with enlargement of the basis, shown below.

The Pauli principle is in these calculations taken into account in an approximation which is consistent with the assumption of an inert closed shell core. The occupied eigenstates of the core are calculated, and the Hamiltonian of (2.1) is diagonalized in the space, orthogonal to these states. This was done by adding a term $g \sum_c |u_c><u_c|$, $g \to \infty$ to the Hamiltonian, where $|u_c>$ are the forbidden core states /7/. With the oscillator expansion /2/, the forbidden states were approximated by oscillator states with the same quantum numbers.

With the form factors calculated in the way just indicated, the cross sections were calculated in the distorted wave Born approximation, using the form factors

$$f = \int dr[\Phi^*_{final}(r_1, r_2)\{V(r_1) + V(r_2)\}\Phi_{initial}(r_1, r_2)]. \qquad (2.7)$$

$$r = r_2 - r_1$$

In this expression, V may be chosen as the interaction between the particles and any of the two cores (post-prior symmetry).

In this connection, the following should be noted. In many calculations, the basis functions (2.2) are used in the calculation of the form factor (2.7) ("well depth procedure") with given expansion coefficients, often found by some method, different from equation (2.4). One may, e.g., use coefficients calculated in the ordinary shell model, multiplied with such Sturmian components which have the same quantum numbers (i.e., limiting the components to such q.n. which correspond to bound states of V). Such methods may be thought of as good or bad approximations to the real Sturmian calculation of equation (2.4).

It would, however, be vary dangerous to interpret them as if $\lambda_i V$ was the core interaction of a particle with quan-

tum numbers n_i, ℓ_i, j_i and then use $\lambda_i V$ with such components in (2.7) (and thus in the transfer matrix elements) instead of V. For some small components one may easily have $\lambda_i \gtrsim 2$, and if these components play a role in the transfer, they get large, spurious enhancements. A similar effect, but may be even more dangerous, will be seen in the sequential transfer amplitude.

This warning seems appropriate, since DWBA codes often have overlap factor calculations built into them, and at least one such code in common use /10/ combines the overlap and transfer calculations in the above-mentioned erroneous way.

3. CROSS SECTIONS

The overlap factors for a number of nuclei were calculated, using the equations (2.4).

The single particle potentials used in these calculations have the form

$$V(r) = V_C(r) + V_{S.O.}(r), \tag{3.1}$$

where

$$V_C(r) = V_C\,(1 + \exp(r - r_0 A^{1/3})/a)^{-1} + V_{coul}\ (r), \tag{3.2}$$

$$V_{S.O.}(r) = V_{S.O.}\ (\frac{h}{m_\pi C})^2 \frac{1}{r} \frac{d}{dr}[1 + \exp(r - r_0 A^{1/3})/a]^{-1}. \tag{3.3}$$

The parameters of these potentials and particularly the well depths were, as mentioned above, chosen so as to yield the correct binding energies of the corresponding odd systems. These parameters are given in table 1.

The two particle interaction was chosen as

$$V_{12}(r) = \gamma(a + b\sigma_1\sigma_2)\frac{e^{-\mu r}}{\mu r}. \tag{3.4}$$

The parameters a, b and μ were chosen in accordance with earlier published work /1/, γ was determined by fitting the experimental two-particle binding energy by the above mentioned procedure. Here $\mu = 0.71$ fm^{-1}, $a = 0.25$, $b = 0.75$ for ^{18}O, $a = 1.0$, $b = 0$ for the other nuclei.

The two-particle overlaps were used to calculate two-particle transfer cross sections for the processes

$^{48}Ca(^{18}O, ^{16}O)\,^{50}Ca$, $^{42}Ca(^{16}O, ^{18}O)\,^{40}Ca$ and $^{48}Ca(^{16}O, ^{14}C)\,^{50}Ti$ (ground states in all cases).

The optical potentials used in the DWBA calculations were of the usual type

$$V_{optical} = V_0\left(1 + \exp\left[\frac{r - r_R(A_1^{1/3} + A_2^{1/3})}{a_R}\right]\right)^{-1} + \quad (3.5)$$

$$+ W\frac{d}{dr}\left(1 + \exp\left[\frac{r - r_I(A_1^{1/3} + A_2^{1/3})}{a_I}\right]\right)^{-1}.$$

The parameters of these potentials were chosen so as to reproduce the corresponding elastic cross sections in entrance and exit channels /11,12/. These parameters are given in table 2.

Table 1

Parameters for single-particle potentials

	V_0 (MeV)	$V_{s.o.}$ (MeV)	r_0 (fm)	a (fm)	r_c (fm)
^{15}N	- 55.76	5.0	1.24	0.75	1.24
^{17}O	- 53.41	5.87	1.24	0.65	
^{41}Ca	- 54.63	6.01	1.24	0.65	
^{49}Ca	- 48.26	8.2	1.24	0.65	
^{49}Sc	- 65.05	5.5	1.20	0.65	1.20

Table 2

Reactions analyzed and corresponding optical parameters used

	E_{Lab} (MeV)	V_0 (MeV)	r_R (fm)	a_R (fm)	W (MeV)	r_I (fm)	a_I (fm)	Ref.
$^{42}Ca(^{16}O, ^{18}O)\,^{40}Ca$	56	35.9	1.35	0.432 (0.960)	101.5	1.272	0.286	11
$^{48}Ca(^{18}O, ^{16}O)\,^{50}Ca$	50	97.0	1.210	0.497	59.7	1.140	0.442	12
$^{48}Ca(^{16}O, ^{14}O)\,^{50}Ti$	56	33.9	1.344	0.424 (0.850)	110.2	1.274	0.280	11

When there is double entry, those in parenthesis are used for exit channel.

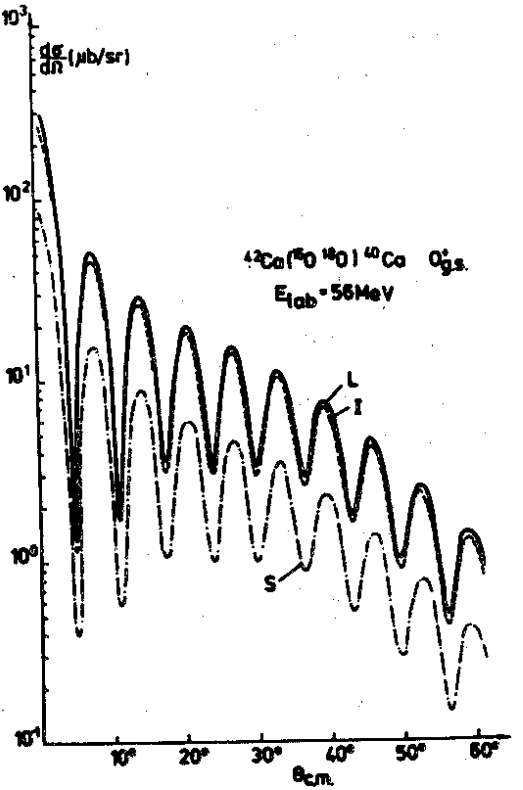

Fig. 1. The cross section of ^{42}Ca(^{16}O, ^{18}O) ^{40}Ca calculated with different numbers of Sturmian components in the form factor basis: S - small, I - intermediate, L - large basis (cfr. Table 3) L = 2.

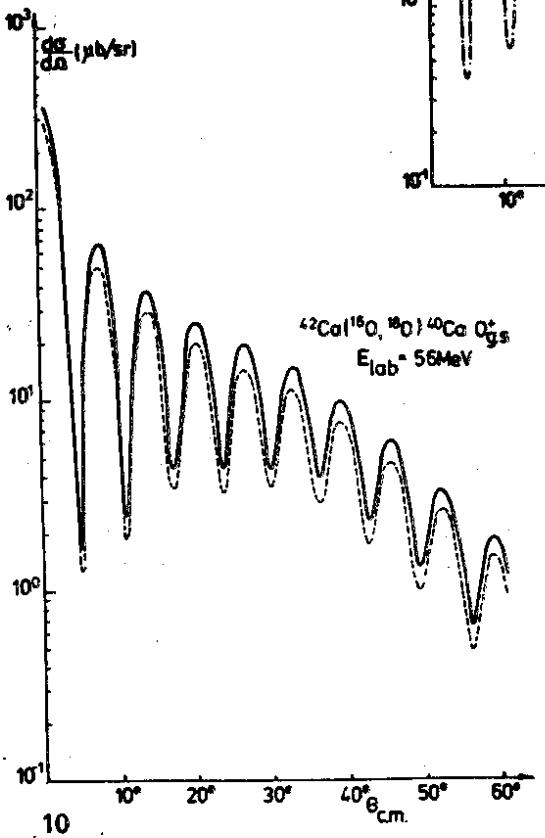

Fig. 2. The cross section of ^{42}Ca(^{16}O, ^{18}O) ^{40}Ca; the full curve corresponds to L = 4, the dashed one to L = 2 (cfr. text). Basis L.

The angular cross sections, obtained in these calculations are shown in figs. 1-7.

The dependence of the cross sections on the number of components in the basis is shown in figs. 1,4 and 6, the meaning of which is further elucidated in table 3.

In the simultaneous transfer, it is practical to transform the two particle overlaps $\Phi(r_1, r_2)$ to functions of the relative coordinate $r_1 - r_2$ and of the centre of mass of the two particles, relative to the core. In the latter coordinate, only comparatively small values of the angular momentum will contribute appreciably to the cross sections, as shown in figs. 2 and 5, where curves for different values for L are given, where L is the maximal value of the sum of orbital angular momenta (1) in the initial and final channels.

It is seen, that the change of the cross section obtained by enlarging L further than $L = 2$ is small further than $L = 4$ negligible.

Figure 3 illustrates a point mentioned in section 2. When the correct V in the transfer matrix element is replaced by $\lambda_i V$, some components get large, spurious, en-

Table 3

Enhancement of cross sections due to extension of basis

Cross Sections	E (MeV)	Dimension	Peak (μb/sr)	Enhancement
$^{48}Ca(^{18}O, ^{16}O)^{50}Ca$	50	3 - 3 (S)	100.	1
		23 -20 (I)	165.	1.65
		30 -28 (L)	181.	1.81
$^{42}Ca(^{16}O, ^{18}O)^{40}Ca$	56	3 - 4 (S)	2.22	1.
		16 -22 (I)	6.55	2.95
		30 -28 (L)	7.26	3.27
$^{48}Ca(^{16}O, ^{14}C)^{50}Ti$	56	4 - 4 (S)	.315	1
		23 -22 (I)	.510	1.62
		28 -28 (L)	.556	1.7

The first number in column "dimension" gives the dimension of the projectile overlap, while the second is for the overlap of the heavy nucleus.

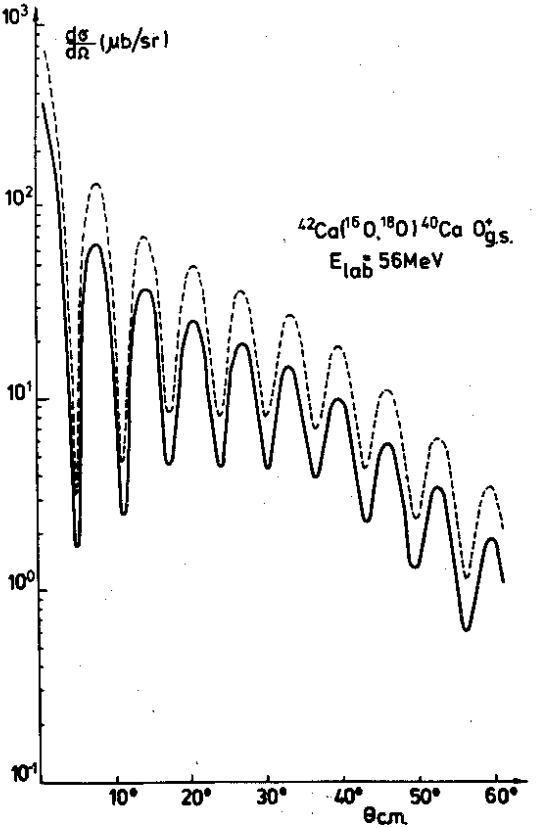

Fig. 3. The cross section of $^{42}Ca(^{16}O,^{18}O)^{40}Ca$. The full curve is calculated with the correct potential in the transfer potential, the dashed with the well depth potential.

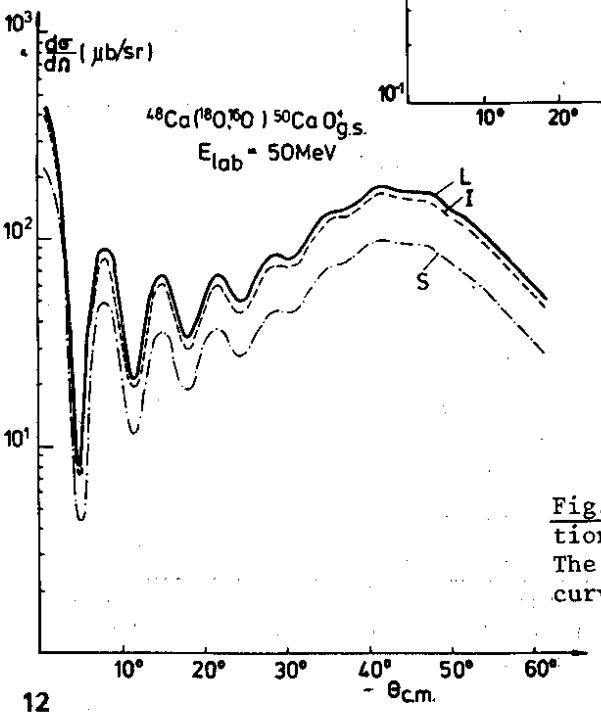

Fig. 4. The cross section of $^{48}Ca(^{18}O,^{16}O)^{50}Ca$. The meaning of the curves as in fig. 1.

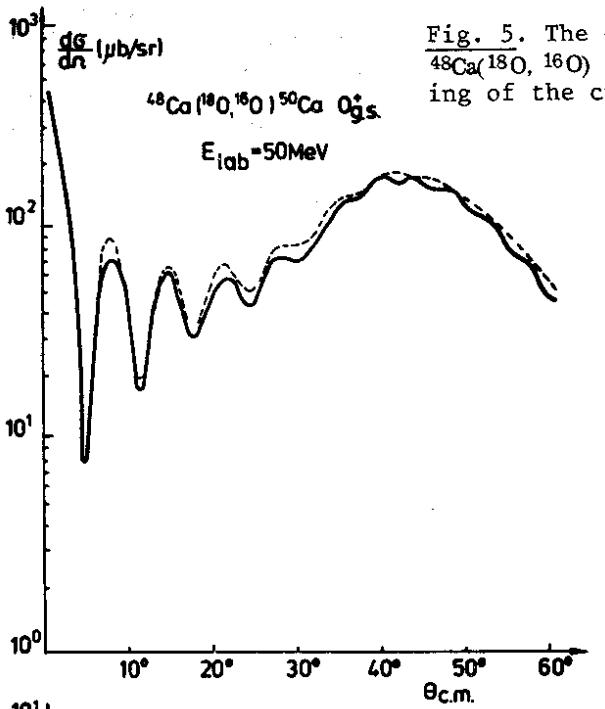

Fig. 5. The cross section of $^{48}Ca(^{18}O, ^{16}O)\ ^{50}Ca$. The meaning of the curves as in fig.2.

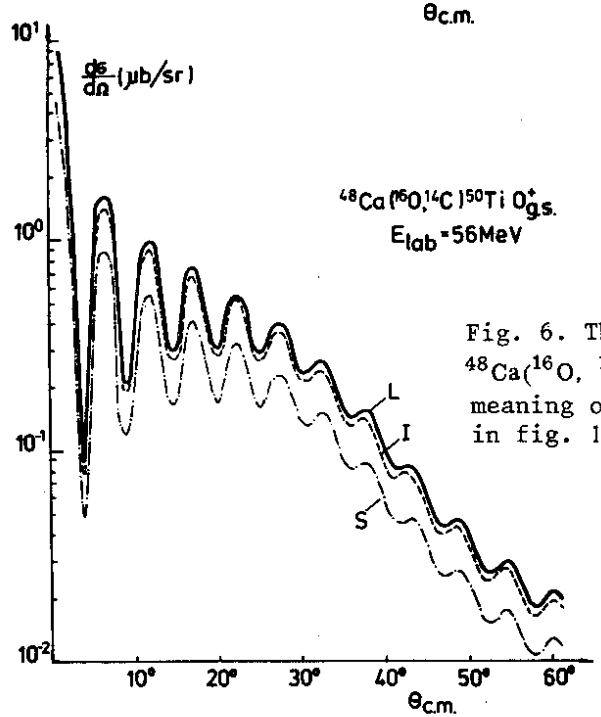

Fig. 6. The cross section of $^{48}Ca(^{16}O, ^{14}C)\ ^{50}Ti$. The meaning of the curves as in fig. 1.

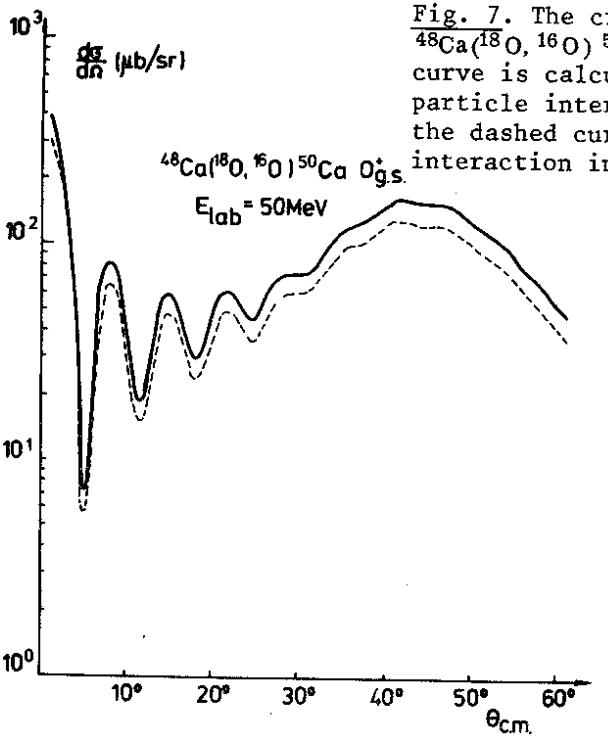

Fig. 7. The cross section of $^{48}Ca(^{18}O, ^{16}O)^{50}Ca$. The full curve is calculated with the two particle interaction of the text, the dashed curve without spin-spin interaction in ^{18}O.

hancement factors, and the cross section is enlarged by a factor of two.

The dependence of the cross section on the residual interaction is to a large extent a question of the asymptotic form of the overlap function, and the details of the interaction are of smaller importance. This is illustrated by the fact, that Feng et al. [2] in their most carefully calculated case obtain a similar enhancement as ours, using very difficult interactions. Nevertheless, this interaction does play a role, and in fig. 7 we see, that the neglect of a spin-spin term in the ^{18}O interaction leads to a cross section, which is smaller than the previous one by 15%.

In the figures, as well as in table 3, the small basis (S) contains Sturm-Liouville components with the same quantum numbers as the shell-model states, i.e., the non-occupied bound states in the potential V. In a sense, this basis therefore represents a self-consistent use of the well depth procedure, and actually, the results obtained with this basis are not very different from what one gest from the usual well depth procedure, with coefficients calcu-

lated in the shell-model (where the coupling constants are generally also chosen so that the binding energies are well approximated). This well depth procedure already represents a large step in the direction of including continuum components, in comparison with the simple shell model. It is seen from the figures, that the main effect of including further continuum components is an enhancement of the absolute cross sections, which seems to reach a saturation, when the basis, going through "I", is enlarged to that, which is called L.

The meaning of the letters S, I and L for each case is given in table 3, where also the enhancement factors are shown.

It is interesting to note, that the enhancement factors of the processes $^{42}Ca(^{16}O,^{18}O)^{40}Ca$ and $^{48}Ca(^{18}O,^{16}O)^{50}Ca$ differ by more than 100%. This contradicts the assumption of Feng et al.[2] that the enhancement factor connected with the oxygen should be dominating all transfer reactions of this type.

4. CONCLUSIONS

We have in this work considered different two-particle transfer processes in the approximation of 1 order DWBA - simultaneous transfer.

This approximation is in itself not precise enough to permit a successful comparison with experimental data.

The large enhancements, found when the continuum contributions to the overlap functions are taken into account, nevertheless surely have a physical significance, as can be seen by comparison with the work of Feng et al. [2]. These authors include sequential transfer in the calculations, but treat the enhancement in the one-step form factor in an approximate way. In a special case, though, their enhancement is nearly correct, and with that, together with the sequential transfer, they obtain a complete agreement with the experimental cross section.

This 100% agreement is to our mind not so significant, since the result is sensitive to many details of the calculations which still have to be investigated. What is significant is the fact, that the two effects, the overlap enhancement and the inclusion of sequential transfer, are of comparable order of magnitude, and, taken together, lead to cross sections of the right order of magnitude.

The neglect of the interaction-enhancement for the two-step amplitude [2] seems not fully justified. Together with

the fact, that our enhancement factors are in some cases larger than supposed by Feng et al., this may lead to theoretical cross sections, which are still larger than those of ref. /2/.

This could either give room for spectroscopical factors <1, or be compensated by other corrections.

One particular correction, stressed by Pinkston /13/ concerns the use of coordinates and masses in the form factor calculations. With a finite core mass, $A \cdot M$, the three body problem should be solved with Jacobian coordinates and corresponding reduced masses. Often, the overlap functions are calculated for an infinite A (the m in equation (2.1) will then be the nucleon mass). In the present approach, however, where m in equations (2.1) and (2.2) is $M\frac{A}{A+1}$, the neglected terms in the Hamiltonian are small. We shall in later work return to the above-mentioned questions.

ACKNOWLEDGEMENT

Two of the authors, J.B. and J.C., want to express their gratitude to JINR for paying their stay in Dubna during the period in which this work was performed, and J.B. gratifies the Danish Research Council for paying his travel expenses.

REFERENCES

1. Bang J. et al. Nucl.Phys., 1976, A264, p.157.
2. Feng D.H., Udagawa T., Tamura T. Nucl.Phys., 1976, A274, p.262 and references therein.
3. Broglia R.A., Hansen O., Riedel C. Advances in Nuclear Physics, vol. 6 (Academic Press, N.Y., 1973, p.283).
4. Bang J., Wollesen S. Phys.Lett., 1970, 33B, p.395.
5. Ibarra R.H., Bayman B.F. Phys.Rev., 1970, C1, p.1786.
6. Ibarra R.H., Vallieres M., Feng D.H. Nucl.Phys., 1975, A241, p.386.
7. Bang J., Gareev F.A. Nucl.Phys., 1974, A232, p.45.
8. Bang J., Gareev F.A. Physica Scripta, 1978, 18, p.297.
9. Vaagen J.S. et al. Nucl.Phys., 1979, A319, p.143.
10. Feng D.H. et al. CPC, 1976, 12, p.293.
11. Eisen Y. et al. Phys.Rev., 1976, C12, p.699.
12. Petersen J.F. et al. Phys.Rev.Lett., 1976, 36, p.307.
13. Pinkston W.T. Nucl.Phys., 1977, A291, p.342.

Received by Publishing Department
on October 30 1979.