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ON THE COMPRESSIBILITY OF NUCLEI IN THE METHOD OF HYPERSPHERICAL FUNCTIONS

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О сжимаемости ядер в методе гиперсферических функций

Изучалась природа монопольных гигантских резонансов легких ядер в методе гиперсферических функций. Расчеты проведены для ядер $A=4 \div 16$ с центральными потенциалами гауссовского типа. Показано, что в минимальном приближении метода эффект "дыхания" ядра четко проявляется при увеличении энергии возбуждения. Это иллюстрируется результатами расчетов средних квадратичных радиусов и плотностей с возбуждением "breathing mode" -состояний.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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On the Compressibility of Nuclei in the Method of Hyperspherical Functions

The nature of the monopole giant resonances of light nuclei is studied by the method of hyperspherical functions. The calculations were performed for light nuclei $A=4 \div 16$ with potentials of the Gaussian type. It is shown that already in the minimal approximation of the method the effect of nuclear "respiration" is taken into account. This effect is illustrated by the results of calculation of the mean-square radii and the densities of the ground and "breathing mode" states.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. Among various collective excitations, the monopole or "breathing mode" oscillations are distinguished by the fact that they correspond to variations in the nuclear matter density, i.e., they characterize its compressibility. The nature of the monopole giant resonances of light nuclei is studied by the method of hyperspherical functions. Such investigations are stimulated, on the one hand, by the difficulty of solving the nontrivial problem of identification of monopole giant resonances in experiments [1]; for most light nuclei it has not yet been solved unambiguously Therefore, theoretical studies, which shed light upon the nature of wave functions and various properties of monopole resonances are of interest. On the other hand, the method of hyperspherical functions [2] provides a convenient basis for a microscopic description of monopole vibrations [3,4]. The point is that in the method a collective variable (hyperradius ρ) is introduced which is associated with the mean-square nuclear radius $\rho^2 = A\langle \bar{r}^2 \rangle$, i.e., with the mean nuclear density. The excitations in this variable correspond to the monopole vibrations of the nucleus as a whole, the density being a dynamical variable.

2. In the method of hyperspherical functions, the wave function Ψ of a nucleus consisting of A nucleons should be translation-invariant. Accordingly, its arguments are usually represented by the Jacobi coordinates: $\vec{x}_1, \vec{x}_2, \dots, \vec{x}_{A-1}$

$$\begin{aligned}
\vec{x}_1 &= \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2) \\
\vec{x}_2 &= \sqrt{\frac{2}{3}}\left[\frac{1}{2}(\vec{r}_1 + \vec{r}_2) - \vec{r}_3\right] \\
&\dots\dots\dots\dots\dots\dots \\
\vec{x}_{A-1} &= \sqrt{\frac{A-1}{A}}\left[\frac{1}{A-1}\sum_{i=1}^{A-1}\vec{r}_i - \vec{r}_A\right]
\end{aligned} \tag{1}$$

The method of hyperspherical functions is specified by that in the $3(A-1)$-dimensional space of Jacobian coordinates the spherical coordinates are introduced: the hyperradius ρ and the hyperspherical angles $(3A-4)$. These angles $\theta_1, \theta_2, \dots \theta_{n-1}$ can be chosen so that the relation between the rectangular and hyperspherical coordinates has the form:

$$
\begin{aligned}
x_1 &= \rho \sin\theta_{n-1} \;\cdots\cdots\; \sin\theta_2 \sin\theta_1 \\
x_2 &= \rho \sin\theta_{n-1} \;\cdots\cdots\; \sin\theta_2 \cos\theta_1 \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
x_{n-1} &= \rho \sin\theta_{n-1} \cos\theta_{n-2} \\
x_n &= \rho \cos\theta_{n-1} \\
\rho^2 &= \sum_{i=1}^{n} x_i^2 \qquad 0 \le \rho \le \infty \\
& \qquad\qquad\qquad 0 \le \theta_1 < 2\pi \\
n &= 3(A-1)
\end{aligned}
\tag{2}
$$

In this way the collective variable ρ is introduced, the Laplace operator being then given by

$$\Delta_n = \sum_n \frac{\partial^2}{\partial x_n^2} = \frac{1}{\rho^{n-1}} \frac{\partial}{\partial \rho}\left(\rho^{n-1} \frac{\partial}{\partial \rho}\right) + \frac{1}{\rho^2} \Delta_{\Omega_n} . \tag{3}$$

The angular part of the Laplacian is written as follows:

$$\Delta_{\Omega_n} = \frac{1}{\sin^{n-2}\theta_{n-1}} \frac{\partial}{\partial \theta_n}\left(\sin^{n-2}\theta_{n-1} \frac{\partial}{\partial \theta_{n-1}}\right) + \frac{1}{\sin^2\theta_{n-1}} \Delta_{\Omega_{n-1}} . \tag{4}$$

The hyperspherical harmonics are the eigenfunctions of the angular part of the Laplace operator

$$\Delta_{\Omega_n} Y_{k\gamma}(\theta_i) = -k(k+n-2) Y_{k\gamma}(\theta_i). \tag{5}$$

Here k is the analog of the angular momentum with $n=3$ and is called the global momentum. γ- denotes all quantum numbers necessary to express various degenerate states in the equation. For example, $\gamma = [f]\,(r)\,LM$.

In the method of k-harmonics, the wave function for the nucleus A is sought as an expansion in the standard hyperspherical functions

$$\Psi = \rho^{-\frac{3A-4}{2}} \sum_{k,\gamma} \chi_{k,\gamma}(\rho) Y_{k,\gamma}(\theta_i), \tag{6}$$

where

$$\int \chi_{k\gamma}^2(\rho)\, d\rho = 1, \qquad \gamma = [f] \in LST.$$

The nuclear Hamiltonian has the form

$$H = -\frac{h^2}{2m} \frac{1}{\rho^{3A-4}} \frac{\partial}{\partial \rho}\left(\rho^{3A-4} \frac{\partial}{\partial \rho}\right) - \frac{h^2}{2m\rho^2} \Delta_\Omega + V(\rho) \tag{7}$$

and the set of equations for finding the radial eigenfunctions and eigenvalues is written as:

$$
\begin{aligned}
\left\{ \frac{d^2}{d\rho^2} - \frac{\mathcal{L}_k(\mathcal{L}_k+1)}{\rho^2} - \frac{2m}{h^2}\left(E + W_{k\gamma}^{k\gamma}(\rho)\right)\right\} \chi_{k\gamma}(\rho) &= \\
= \frac{2m}{h^2} \sum_{k'\gamma' \ne k\gamma} W_{k\gamma}^{k'\gamma'}(\rho)\, \chi_{k'\gamma'}(\rho), &
\end{aligned}
\tag{8}
$$

where $\mathcal{L}_k = K + \frac{3A-6}{2}$, $W_{k\gamma}^{k'\gamma'}(\rho)$ are the matrix elements of the potential energy of the nucleon-nucleon interation

$$V = \sum_{i<j} V(r_{ij}), \qquad V(r_{ij}) = f(r_{ij}) W_{\sigma\tau} \tag{9}$$

which may be expressed in terms of the two-particle fractional parentage coefficients in the form:

$$W^{\bar k \bar\gamma}_{k\gamma}(\rho) =$$

$$= \langle AK[f]\epsilon\, LSTM_L M_S M_T \,|\, V \,|\, A\bar K[\bar f]\bar\epsilon\, \bar L \bar S \bar T M_L M_S M_T \rangle = \tag{10}$$

$$= \frac{A(A-1)}{2} \sum_{K'[f']\epsilon'\Lambda' S' T' \lambda S_0 T_0} \langle AK[f]\epsilon LST \,|\, A-2K'[f']\epsilon' L'S'T';\, \lambda S_0 T_0 \rangle \times$$

$$\times \langle A\bar K[\bar f]\bar\epsilon\, \bar L \bar S \bar T \,|\, A-2K'[f']\epsilon' L'S'T';\, \lambda S_0 T_0 \rangle \times$$

$$\times \langle S_0 T_0 \,|\, W_{\sigma\tau} \,|\, S_0 T_0 \rangle \cdot R^{k\bar k}_{k'\lambda}(\rho).$$

Here

$$R^{k\bar k}_{k'\lambda_0}(\rho) = \int d\theta_1 (\sin\theta_1)^{3A-7} (\cos\theta_1)^2 \times$$

$$\times f(\rho\cos\theta_1) N_{kk'\lambda_0} N_{\bar k k'\lambda_0} (\sin\theta_1)^{2k'} (\cos\theta_1)^{2\lambda_0} \times$$

$$\times P_{k-k'-\lambda_0}^{k'+\frac{3A-6}{2}-1,\, \lambda_0+\frac{1}{2}}(\cos 2\theta_1) P_{\bar k-k'-\lambda_0}^{k'+\frac{3A-6}{2}-1,\, \lambda_0+\frac{1}{2}}(\cos 2\theta_1). \tag{11}$$

The matrix elements for an effective interaction are calculated by the method proposed by A.I.Baz' in ref./3/. According to this method a matrix element, which depends on ρ, is expressed in the hyperspherical function technique in terms of a matrix element in the translation-invatiant shell model:

$$W^{k'\gamma'}_{k\gamma}(\rho) = \frac{\Gamma(k+\frac{3A-6}{2})}{2\pi i\, \rho^{2k+3A-3}} \int_c \frac{dS\, e^{S\rho^2}}{S^{k+\frac{3A-1}{2}}} W^{k'\gamma'}_{k\gamma}(S^{-1/2}). \tag{12}$$

The oscillator parameter r_0 is related to the integration variable S as follows $r_0 = S^{-1/2}$.

Having found the effective-interaction matrix element $W^{k'\gamma'}_{k\gamma}(\rho)$, we substitute it into eq. (8) and find its eigenvalues E and eigenfunctions $\chi_{k\gamma}(\rho)$. The equation is first solved for the ground state of the nucleus and then for the first, second, etc., monopole excitations of the nucleus as a whole. Let us give an expression for some matrix elements of the physical operators in the hyperspherical function technique, which we shall use in what follows.

In the hyperspherical function technique, the 1p-shell nuclear density has the form

$$n_{ij}(r) = \frac{16}{\sqrt{\pi}} \frac{\Gamma(\frac{5A-11}{2})}{\Gamma(\frac{5A-14}{2})} \int_r^\infty \frac{(\rho^2-r^2)^{\frac{5A-16}{2}}}{\rho^{5A-13}} \chi_i(\rho)\chi_j(\rho)d\rho +$$

$$+ \frac{8}{3}\frac{(A-4)}{\sqrt{\pi}} \frac{\Gamma(\frac{5A-11}{2})}{\Gamma(\frac{5A-16}{2})} \int_r^\infty \frac{r^2(\rho^2-r^2)^{\frac{5A-15}{2}}}{\rho^{5A-13}} \chi_i(\rho)\chi_j(\rho)d\rho \tag{13}$$

and the mean-square radius is written as:

$$\bar R^2_{ii} = \langle r^2_{ii} \rangle = \frac{\int n_{ii}(r) r^2 dV}{\int n_{ii}(r) dV} = \frac{\int n_{ii}(r) r^4 dr}{\int n_{ii}(r) r^2 dr}, \tag{14}$$

where the density is normalized as follows:

$$4\pi \int n(r) r^2 dr = A.$$

The monopole isoscalar sum rule can be written as:

$$\sum_n (E_n - E_0)|M_{0n}|^2 = \frac{h^2 Z}{m} \langle 0 | r^2 | 0 \rangle, \tag{15}$$

where

$$M_{0_n} = \langle 0 | \frac{1}{2} \sum_{i=1}^{A} r_i^2 | 0_n \rangle$$

3. Let us discuss more thoroughly the method for solving eq. (8). Let us formulate the boundary-value problem

$$\chi_{k\gamma}(0)=\chi_{k\gamma}(R)=0, \qquad R \gg 0$$

and designate the effective potential including the centrifugal term as

$$U(\rho)=\chi_{k\gamma}(\rho) \tag{16}$$

$$W(\rho) = -\frac{h^2}{2m}\frac{L_k(L_k+1)}{\rho^2} - W_{k\gamma}(\rho) - \sum_{k'\gamma'\neq k\gamma} W_{k\gamma}^{k'\gamma'}(\rho) \tag{17}$$

The problem involving eqs. (8)-(16) will be solved by the finite difference method. On a segment $[0,R]$ we introduce the difference set $\bar{\omega}=\{\rho_i = ih_\rho,\ i=0,1,2...N,\ h_\rho = R/N\}$. Let $U_i = U(\rho_i)$, $W_i = W(\rho_i)$. We shall write the difference scheme, which approximates eq. (8) and the boundary conditions (16)

$$\frac{h^2}{2m}\frac{U_{i+1}-2U_i+U_{i-1}}{h_\rho^2} + W_iU_i = E^{h_\rho}U_i, \quad i=1,2,...N-1 \tag{18}$$

$$U_0=U_N=0$$

The accuracy of the scheme (18) is a function of $O(h_\rho^2)$. The set of linear algebraic equations (18) shall be written in the form:

$$AU=E^hBU, \tag{19}$$

where A is the tridiagonal symmetrical matrix of an order of N-1 and B is the unitary matrix. The eigenvalue matrix problem (19) has been solved by the inverse iteration technique [10]. This method enables one to calculate all values, i.e., eigenvalues, and the wave functions in the problem (19), which correspond to the bound states in eq. (8).

4. Using the method described above, we have performed the calculations for light nuclei A =4-16 with potentials of the Gaussian type [5].

Figure 1 shows the energies of the first and second monopole resonances as a function of A for nuclei from A=4

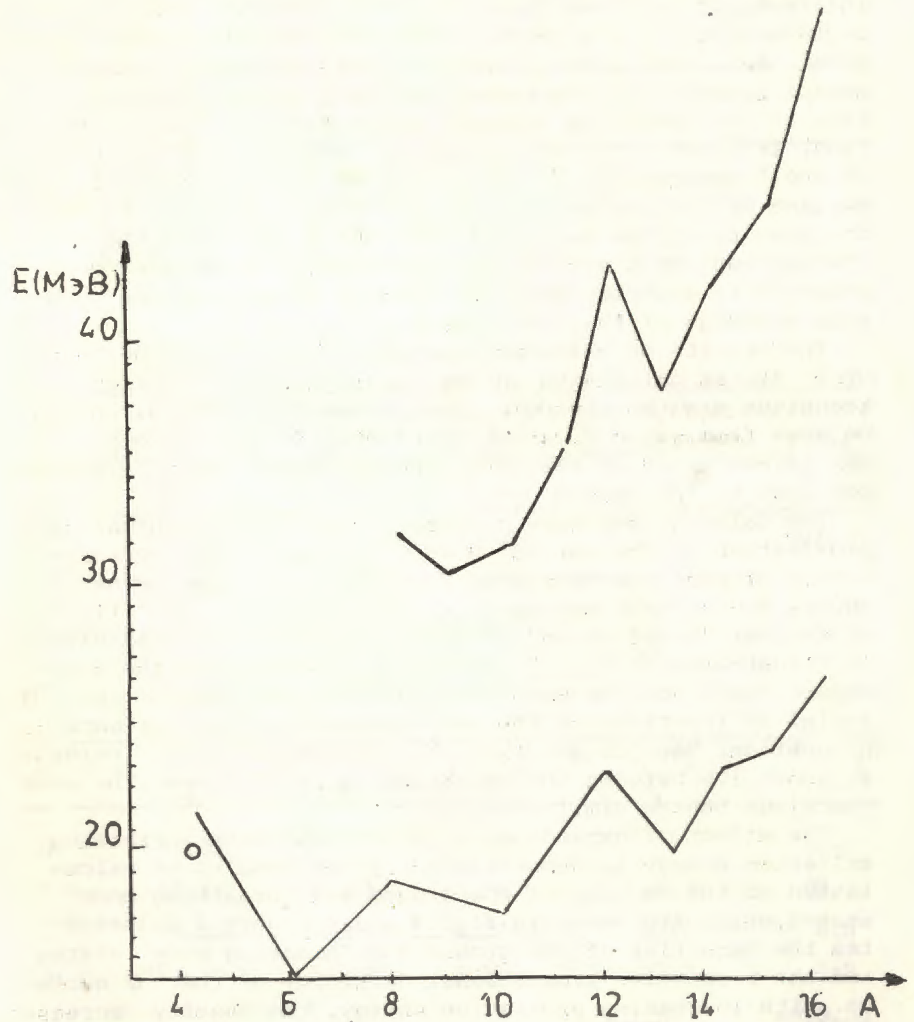

Fig.1. Positions of the first and second monopole resonances as a function of A.

to A =16. As can be seen from the comparison with experiment (a point on the curve), these calculations reproduce fairly well the position of the monopole resonance of the ^{4}He nucleus. Unfortunately, there are no other data available on the monopole giant resonances in light nuclei. Nevertheless, one may think that this curve is thought to be of interest. It has some characteristic features, namely, it increases with A and shows the cluster structure. One should think that, upon normalizing the curve at several experimental points, one can rather reliably predict the position of the remaining monopole resonances. In this connection, it is of interest to trace in what way the position of the "respiratory" 0^+ state depends on the choice of a variant of the nucleon-nucleon potential. Table 1 shows the results of the calculation of the 0^+ state of the ^{4}He nucleus for 8 variants of potentials, which have been selected to describe well the binding energy and the mean-square radius of the ^{4}He nucleus.

The results of calculation enable one to conclude that the 0^+ states calculated in the hyperspherical function technique are the monopole giant resonance. Indeed, as can be seen from table 2, about 80 to 90% of the monopole sum is exhausted by the first excited state and only several per cent by the second one.

The calculations have been performed in the minimal approximation of the method. However, even in such approximation of the hyperspherical function technique the mean-square radius is a dynamical variable, so that the effect of nuclear "respiration" is taken into account. This effect is illustrated in fig. 2 and table 2 which show the mean-square radii and the wave functions for ^{4}He, ^{8}Be, ^{12}C and ^{16}O nuclei as functions of the excitation energy of the nucleus in question. One can see that the mean-square radii increase by about 10% between the neighbouring excitations. The wave functions behave similarly.

The effect of expansion of the nucleus with increasing exitation energy is demonstrated by the results of calculation of the density of the ground and "breathing mode" states, which are shown in figs. 3 and 4. Figure 3 illustrates the densities of the ground and "breating mode" states and the respective transitional densities of the ^{16}O nucleus. With increasing excitation energy, the density decreases inside the nucleus and increases at the boundary. Indeed, with increasing exitation energy of the ^{16}O nucleus up to 100 MeV, the nuclear density at the point r =0 at the maximum (r =1) and at the boundary of the nucleus $r = \langle R^2 \rangle^{1/2} =$

Table 1

E_B^0	-28.6	-29.4	-29.3	-29.2	-28.I	-28.8	-28.0	-28.5
E_B^1	-3.2	-4.5	-2,9	-2.2	-3.5	-3.8	-3.7	-3.8
E_{ex}^{0+}	25.4	24.9	26.4	27.0	24.6	25.0	24.4	24.7
	I	2	3	4	5	6	7	8

Table 2

A = 4

N	E_B	E_{ex}	$\langle R^2 \rangle^{1/2}_{ii}$	$(E_i - E_0)\|M_{0i}\|^2 = S_i$	S_i/S_2 %
0	-28.00	0	I.767	0	0
I	-3.68	24.32	3.487	I93.8	74.7

S_2 = 259.4 $\quad \sum_i S_i$ = I93.8

A = 8

N	E_B	E_{ex}	$\langle R^2 \rangle^{1/2}_{ii}$	$(E_i - E_0)\|M_{0i}\|^2 = S_i$	S_i/S_2 %
0	-40.I2	-	2.353	-	-
I	-2I.59	I8.63	2.940	830.95	90.4
2	-9.23	30.89	3.774	59.3	6.5
3	-I.99	38.03	5.II6	II.3	I.2
				= 90I.59	=9I9.30

Table 2 (cont.)

N A = I2	E_B	E_{ex}	$\langle R^2 \rangle^{1/2}_{ii}$	$(E_i - E_0)\lvert M_{0i}\rvert^2 = S_i$	S_i/S_2 %
0	-83.I0	-	2.385	-	-
I	-6I.57	2I.53	2.689	I354.3	95.6
2	-43.78	39.32	3.046	49.0	3.5
3	-29.43	53.67	3.474	5.7	0.4
4	-I8.2I	64.99	4.000	I.2	0.I
5	-9.79	73.3I	4.67I	0.4	0.0
6	-3.84	79.26	5.572	0.0	

S_2 = I4I6.3 $\quad\quad \sum_i S_2$ = I4I0.5

A = I6

N	E_B	E_{ex}	$\langle R^2 \rangle^{1/2}_{ii}$	$(E_i - E_0)\lvert M_{0i}\rvert^2 = S_i$	S_i/S_2 %
0	-I46.52	0	2.252	-	-
I	-I2I.I4	25.38	2.449	I525.9	90.6
2	-98.74	47.78	2.665	38.0	2.2
3	-79.I4	67,38	2.904	3.0	0.2
4	-62.06	84.46	3.I73	0.5	0.0
5	-47.40	99.I2	3.476	0.I	
6	-34.98	II2.54	3.823	0.0	
7	-24.64	I22.88			
8					
9					

S_2 =I684.8 $\quad\quad \sum_i S_2$ =I567.6

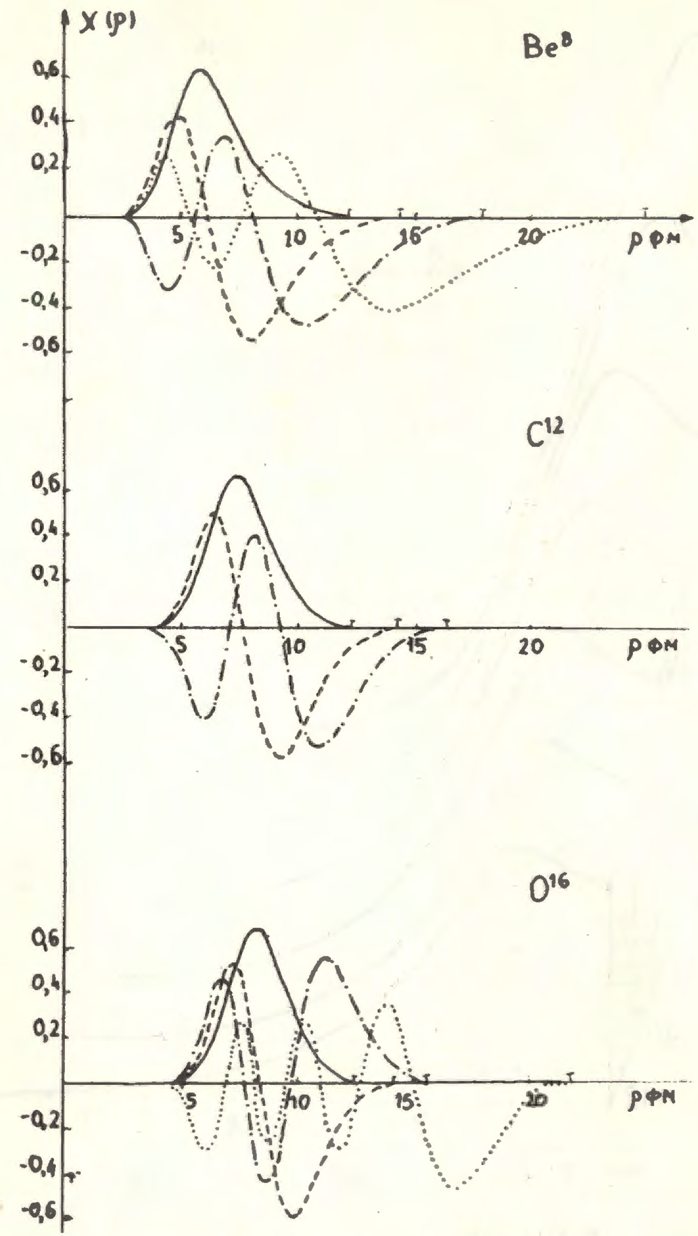

Fig.2. Radial part of the wave function $\chi(\rho)$ for ^{8}Be, ^{12}C and ^{16}O nuclei.

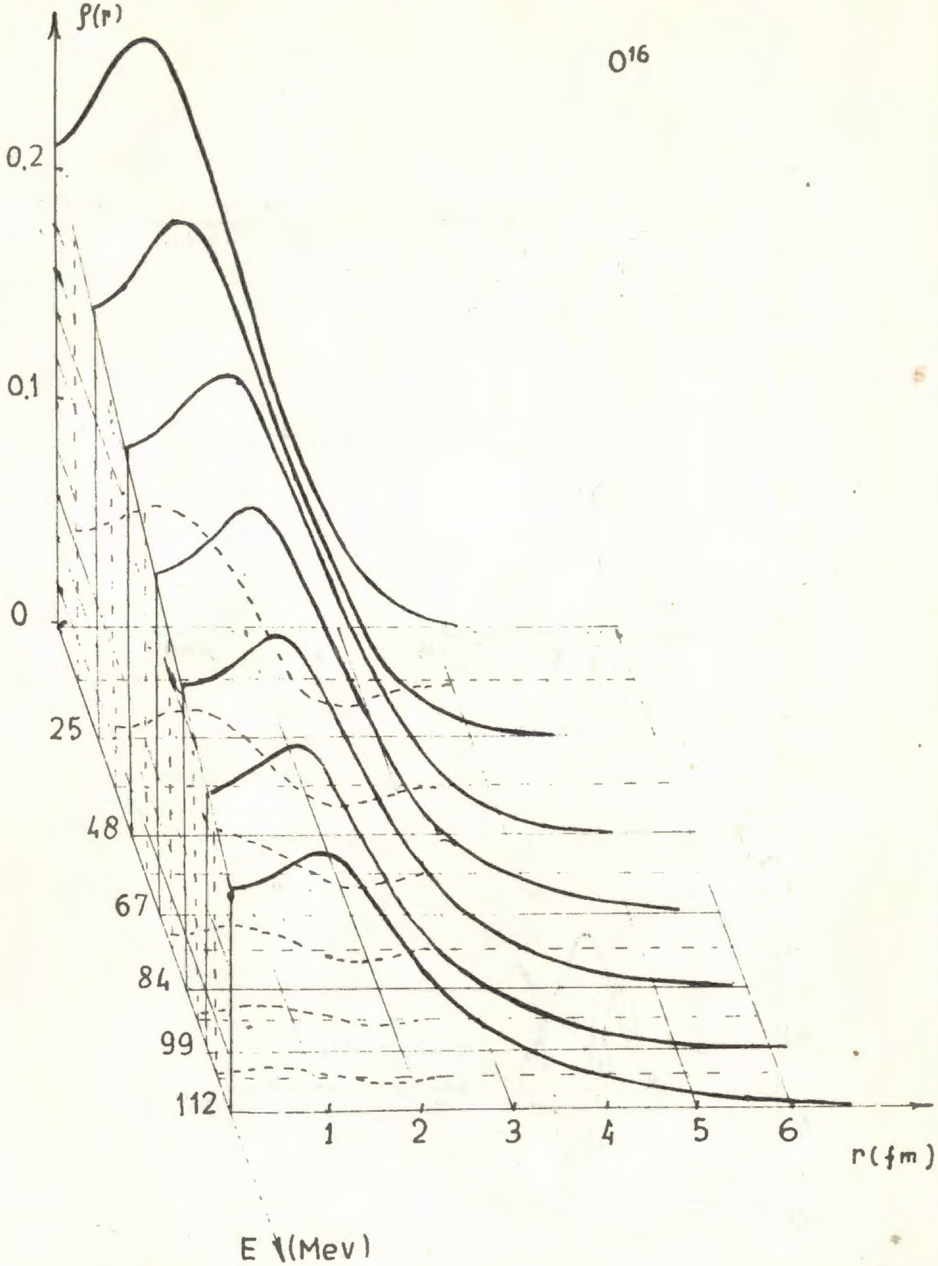

Fig.3. Density distribution for various excitation energies for ^{16}O.

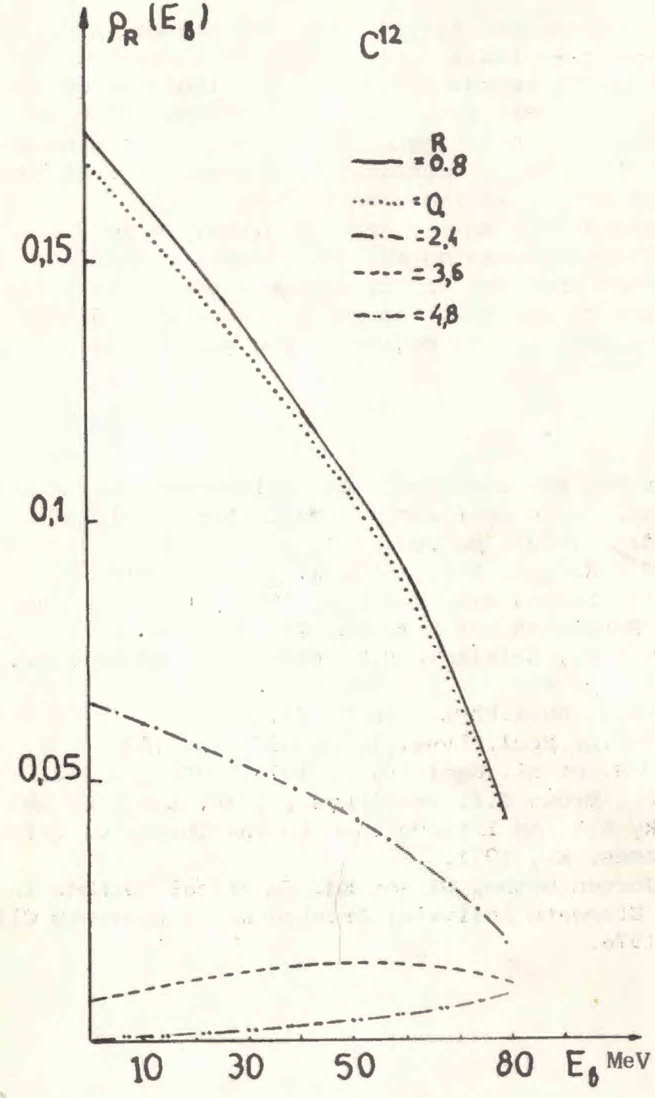

Fig.4. $n_r(E)$ change in density with increasing exitation energy at the centre of the nucleus $(r=0)$, at the maximum of density $(r=0.8$ Fm $)$, at $r=2,4$ and for $r = 3.6$ Fm for ^{12}C.

=2.25 fm decreases by a factor of 2. In the same energy region, nuclear diffuseness increases from 4.5 fm to 6.5 fm.

As an illustration, fig. 4 shows the change in density with increasing exitation energy in the ^{12}C nucleus (at its centre and at the boundary). It is seen that the density descreases by 4 times with increasing energy by 80 MeV at the centre and the point equal to the mean-square nuclear radius and the density increases by 6 times at a distance equal to the double mean-square radius.

The transitional density decreases very rapidly as a function of the increasing exitation energy. This accounts for the effect that 90% of the monopole sum is exhausted by the region of the first monopole resonance and only several per cent, in the region of the second one.

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