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Учет принципа Паули в квазичастично-фононной модели ядра

В квазичастично-фононной модели ядра обычно используются RPA - фононы и квазибозонное приближение и не учитываются корреляции в основном состоянии. Показано, что возможен точный учет принципа Паули. Для случая, когда волновая функция содержит одноквазичастичную и квазичастица-плюс-фонон компоненты, получены точные и приближенные секулярные уравнения. Обсуждено влияние принципа Паули в случае, когда волновая функция содержит и двухфононные компоненты. В обоих случаях происходит сдвиг полюсов в секулярных уравнениях и добавляются члены, связанные с взаимодействием квазичастиц с фононами. Оценено число квазичастиц в основных состояниях и утверждается, что в большинстве деформированных ядер корреляции в основных состояниях невелики. Показано, что в квазичастично-фононной модели ядра можно, когда это необходимо, проводить расчеты с точными перестановочными соотношениями.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Soloviev V.G. **E4 - 12623**

On Inclusion of the Pauli Principle in the Quasiparticle-Phonon Nuclear Model

Usually the RPA-phonons and the quasiboson approximation are used in the quasiparticle-phonon nuclear model, whereas the correlations in the ground state are not taken into account. It is shown that the Pauli principle can exactly be taken into account. The exact and approximate secular equations are obtained for the wave function containing the one-quasiparticle and quasiparticle plus phonon components. The effect of the Pauli principle is discussed, when the wave function contains the one- and two-phonon components. In both the cases the poles are shifted in the secular equations and the quasiparticle-phonon interaction terms are added. The number of quasiparticles in the ground states is estimated. It is stated that in the majority of deformed nuclei the correlations in the ground states are small. It is shown that within the quasiparticle-phonon nuclear model the calculations can be performed with the exact commutation relations, if necessary.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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I. INTRODUCTION

The generalization of the Hartree-Fock Variational Principle [1] suggested by N.N.Bogolubov and then called the Hartree-Fock-Bogolubov Variational Principle [2-4] and the time-dependent selfconsistent field method [5] formulated by him provided to a great extent a foundation of the contemporary theory of atomic nucleus.

N.N.Bogolubov derived the equations of collective vibrations in a general form [5]. In these equations, in order to described the vibrations, the average field and the interaction in the particle-particle and particle-hole channels have been separated in ref. [6]. As a result the system of equations identical with the equations of the finite Fermi-system [7] has been derived. These equations [6] are widely used for describing the collective one-phonon states in atomic nuclei.

A few-quasiparticle components of the nuclear wave functions at low, intermediate and high excitation energies are described within the quasiparticle-phonon nuclear model [8,9]. The quasiparticle-phonon nuclear model takes into account the interactions of quasiparticles with phonons, the one-phonon states rather than single-particle states being used as a basis. One of the most important achievements is the description of the fragmentation (distribution of strength) of single-particle and one-phonon states. The spectroscopic factors of the one-nucleon transfer reactions [10,11] and the neutron and radiative strength functions [12,13] in spherical and deformed nuclei are calculated on the basis of the fragmentation.

In describing the one-phonon states the quasiboson approximation or the RPA are used. There are introduced the phonon operators which, for the deformed nuclei, have the form

$$Q_g^+ = \frac{1}{2} \sum_{qq'} \{ \psi^g_{qq'} A^+(qq') - \phi^g_{qq'} A(qq') \}, \tag{1}$$

where

$$A^{+}(qq') = \frac{1}{\sqrt{2}} \sum_{\sigma} \sigma a^{+}_{q-\sigma} a^{+}_{q'\sigma} \quad \text{or} \quad \frac{1}{\sqrt{2}} \sum_{\sigma} a^{+}_{q\sigma} a^{+}_{q'\sigma} ,$$

$a^{+}_{q\sigma}$ is the quasiparticle creation operator, $(q\sigma)$ - are the quantum numbers of the single-particle state, $\sigma = \pm 1$. The phonon operators satisfy the following commutation relations [14].

$$[Q_g, Q^{+}_{g'}] = \frac{1}{2} \sum_{qq'} (\psi^{g}_{qq'} \psi^{g'}_{qq'} - \phi^{g}_{qq'} \phi^{g'}_{qq'}) -$$

$$- \frac{1}{2} \sum_{q_1 q_2 q_3} (\psi^{g}_{q_1 q_2} \psi^{g'}_{q_1 q_3} - \phi^{g}_{q_1 q_3} \phi^{g'}_{q_1 q_2}) B(q_3 q_2), \qquad (2)$$

$$[Q_g, Q_{g'}] = [Q^{+}_{g'}, Q^{+}_{g}] = 0,$$

where

$$B(q,q') = \sum_{\sigma} a^{+}_{q\sigma} a_{q'\sigma} \qquad \text{or} \qquad \sum_{\sigma} \sigma a^{+}_{q-\sigma} a_{q'\sigma} .$$

The following condition should be fulfilled

$$\langle [Q_g, Q^{+}_{g'}] \rangle = \delta_{gg'} , \qquad (3)$$

in which the averaging is performed over the ground state of a doubly even nucleus.

In the RPA two approximations are used: 1) the commutation relation (2) is taken in the form

$$[Q_g, Q^{+}_{g'}] = \delta_{gg'} ; \qquad (4)$$

2) it is assumed that in the ground state of the doubly even nuclei the number of quasiparticles is small, and hence it is assumed that

$$\langle B(qq') \rangle = 0, \qquad (5)$$

or

$$\langle a^{+}_{q\pm} a_{q\pm} \rangle = 0 \qquad (5')$$

When describing the one-phonon states, approximations (4) and (5) are equivalent.

The wave functions in the quasiparticle-phonon nuclear model contain the quasiparticle plus phonon, two phonons and so on components. When describing the wave functions with these components, one should not ignore the fact that the phonons are composed by the superposition of quasiparticle pairs. One also needs to take into account the Pauli principle and to study the cases of application of approximation (4). The exact commutation relations (2) for the two-phonon components have been taken into account in refs. [15,16]. In this paper the Pauli principle is exactly taken into account for the quasiparticle plus phonon components. We consider the effect of the Pauli principle in the deformed nuclei. We also discuss to what extent the use of approximation (5) is valid. To this end we derive the expression for $\langle B(qq') \rangle$ and find its numerical values for different states qq'.

2. THE PAULI PRINCIPLE IN ODD-A DEFORMED NUCLEI

The Hamiltonian of the quasiparticle-phonon nuclear model, taking into account the secular equations for the one-phonon states for the isoscalar multipole forces, has the form

$$H_M = H_v + H_{qv} , \qquad (6)$$

$$H_v = \sum_{q} \epsilon(q) B(qq) - \frac{1}{8} \sum_{\substack{g=\lambda\mu i \\ g'=\mu\lambda i'}} \frac{X^{g} + X^{g'}}{\sqrt{Y_g Y_{g'}}} Q^{+}_{g} Q_{g'} , \qquad (7)$$

$$H_{qv} = \frac{1}{4} \sum_{g} \frac{1}{\sqrt{Y_g}} \sum_{qq'} v_{qq'} f^{g}(qq') \{ (Q^{+}_{g} + Q_g) B(qq') + B(qq')(Q^{+}_{g} + Q_g) \} \qquad (8)$$

We use the following notation: $f^{\lambda\mu}(qq')$ are the matrix elements of the multipole moment λ operator with projection μ, $g=\lambda\mu i$, i is the root number of the secular equation for the one-phonon state, $\epsilon(q) = \sqrt{C^2 + \{E(q)-\lambda_0\}^2}$, $E(q)$ is the single-particle energy, C is the correlation function, λ_0 is the chemical potential, $\epsilon(qq') = \epsilon(q) + \epsilon(q')$, $u_{qq'} = u_q v_{q'} + u_{q'} v_q$, $v_{qq'} = u_q u_{q'} - v_q v_{q'}$, where u_q and $v_{q'}$ are the Bogolubov transformation coefficients,

$$X^{g} = 2 \sum_{qq'} \frac{(f^{g}(qq')u_{qq'})^{2}\epsilon(qq')}{\epsilon^{2}(qq')-\omega_{g}^{2}},$$

$$Y_{g} = \sum_{qq'} \frac{(f^{g}(qq')u_{qq'})^{2}\epsilon(qq')\omega_{g}}{(\epsilon^{2}(qq')-\omega_{g}^{2})^{2}}.$$

The one-phonon state ω_g energy is obtained from the secular equation

$$1-\kappa_{0}^{(\lambda)} X^{g}(\omega_{g}) = 0, \tag{9}$$

where $\kappa_0^{(\lambda)}$ is the isoscalar constant of multipole forces.

Let us consider an odd-A deformed nucleus, the ground and excited states of which are described by the wave function

$$\Psi_{n}(K^{\pi}) = \frac{1}{\sqrt{2}} \sum_{\sigma} \{ C^{n}_{q_0} a_{q_0\sigma} + \sum_{g,q} D^{n}_{gq\sigma} a^{+}_{q\sigma} Q^{+}_{g} \} \Psi_{0}, \tag{10}$$

where

$$Q_{g}\Psi_{0} = 0, \tag{11}$$

and n is the number of the nonrotational state with given K^{π}.

Now we give the following commutation relations:

$$[Q_{g}, a^{+}_{q_0\sigma_0}] = -\frac{1}{\sqrt{2}} \sum_{q} \{ \sigma_0 \psi^{g}_{q_0 q} a_{q-\sigma_0} + \bar{\psi}^{g}_{q_0 q} a_{q\sigma_0} \}, \tag{12}$$

$$[[Q_{g}, a^{+}_{q_0\sigma_0}], Q^{+}_{g_2}] = \sum_{q'} \{ S(gg_2; q_0 q') a^{+}_{q'\sigma_0} + \sigma_0 \bar{S}(gg_2; q_0 q') a^{+}_{q'-\sigma_0} \}, \tag{13}$$

where

$$S(g_1 g_2; q_1 q_2) = -\frac{1}{2} \sum_{q} (\psi^{g_1}_{qq_1} \psi^{g_2}_{qq_2} + \bar{\psi}^{g_1}_{qq_1} \bar{\psi}^{g_2}_{qq_2}), \tag{14}$$

$$S(g_1 g_2; q_1 q_2) = -\frac{1}{2} \sum_{q} (\psi^{g_1}_{qq_1} \bar{\psi}^{g_2}_{qq_2} - \bar{\psi}^{g_1}_{qq_1} \psi^{g_2}_{qq_2}), \tag{14'}$$

with the functions $\psi^{g}_{qq'}$ and $\bar{\psi}^{g}_{qq'}$ differing by the addition of the projections of angular momenta $(K \mp K')$, given in ref. /17/. In what follows we use the notation

$$S_{\sigma\sigma'}(gg'; qq') = \delta_{\sigma\sigma'} S(gg'; qq') + \sigma \delta_{\sigma,-\sigma'} \bar{S}(gg'; qq'), \tag{15}$$

$$S(g,q) = \frac{1}{2} \sum_{q'} \{ (\psi^{g}_{qq'})^{2} + (\bar{\psi}^{g}_{qq'})^{2} \}. \tag{15'}$$

Using the exact commutation relations we obtain the normalization condition of the wave function (10) in the form

$$(C^{n}_{g_0})^{2} + \frac{1}{2} \sum_{gq\sigma} (D^{n}_{gq\sigma})^{2} (1 + S(g,q)) + $$
$$+ \frac{1}{2} \sum_{\substack{gq\sigma \\ g'q'\sigma'}}{}' D^{n}_{gq\sigma} D^{n}_{g'q'\sigma'} S_{\sigma\sigma'}(gg'; qq') = 1. \tag{16}$$

For $S = 0$ condition (16) becomes the ordinary expression used in ref. /17/ for the description of the nonrotational levels in deformed nuclei.

Now we calculate the average value of H_M over state (10) and obtain

$$(\Psi^{*}_{n} H_{M} \Psi_{n}) = \epsilon(q_0)(C^{n}_{q_0})^{2} + \frac{1}{2} \sum_{gq\sigma} (\epsilon(q) + \omega_{g})(D^{n}_{gq\sigma})^{2} -$$
$$- \frac{1}{2} C^{n}_{q_0} \sum_{g_2 q_2 \sigma_2} D^{n}_{g_2 q_2 \sigma_2} [\frac{v_{q_0 q_2}}{\sqrt{Y_{g_2}}} f^{g_2}(q_0 q_2) + \sum_{g} \frac{1}{\sqrt{Y_g}} \xi_{\sigma_2}(gg_2; q_0 q_2)] +$$
$$+ \frac{1}{2} \sum_{\substack{g_2 q_2 \sigma_2 \\ g'_2 q'_2 \sigma'_2}} D^{n}_{g_2 q_2 \sigma_2} D^{n}_{g'_2 q'_2 \sigma'_2} [(\epsilon(q_2) + \omega_{g_2}) S_{\sigma_2\sigma'_2}(g_2 g'_2; q_2 q'_2) - \tag{17}$$
$$- \frac{1}{8} \sum_{g=\lambda'_2\mu'_2 i} \frac{X^{g} + X^{g'_2}}{\sqrt{Y_g Y_{g'_2}}} S_{\sigma_2\sigma'_2}(g_2 g'_2; q_2 q'_2)].$$

Using the variational principle, we derive the following system of equations:

$$(\epsilon(q_2) + \omega_{g_2} - \eta_n) D^{n}_{g_2 q_2 \sigma_2} - \frac{1}{2} C^{n}_{q_0} [\frac{v_{q_0 q_1}}{\sqrt{Y_{g_2}}} f^{g_2}(q_0 q_2) + \sum_{g} \frac{\xi_{\sigma_2}(gg_2; q_0 q_2)}{\sqrt{Y_g}}] + \tag{18}$$

$$+\frac{1}{2}\sum_{g_2'q_2'\sigma_2'} D^n_{g_2'q_2'\sigma_2'}\{(\epsilon(q_2)+\omega_{g_2}+\epsilon(q_2')+\omega_{g_2'}-2\eta_n)S_{\sigma_2\sigma_2'}(g_2g_2';q_2q_2')$$

$$-\frac{1}{8}\sum_g\Big[\frac{X^g+X^{g_2'}}{\sqrt{Y_gY_{g_2'}}}S_{\sigma_2\sigma_2'}(g_2g;q_2q_2')+\frac{X^g+X^{g_2}}{\sqrt{Y_gY_{g_2}}}S_{\sigma_2\sigma_2'}(g_2'g;q_2q_2')\Big]\}=0, \tag{19}$$

$$(\epsilon(g_0)-\eta_n)C^n_{q_0}-\frac{1}{2}\sum_{g_2q_2\sigma_2}D^n_{g_2q_2\sigma_2}\{\frac{v_{q_0q_2}}{\sqrt{Y_{g_2}}}f^{g_2}(q_0q_2)+\sum_g\frac{\xi_{\sigma_2}(gg_2;q_0q_2)}{\sqrt{Y_g}}\}=0,$$

where

$$\xi_{\sigma_2}(gg_2;q_0q_2)=$$

$$=\sum_q\{v_{qq_0}[S_{gg_2}(qq_0;q_2q)+\sigma_2\bar{S}_{gg_2}(qq_0;q_2q)]+$$
$$+v_{qq_2}[S_{gg_2}(qq_2;qq_0)+\sigma_2\bar{S}_{gg_2}(qq_2;qq_0)]\}, \tag{20}$$

$$S_{gg_2}(q_1q_2;q_3q_4)=-\frac{1}{2}f^g(q_1q_2)\sum_{q_1}(\psi^g_{q'q_3}\psi^{g_2}_{q'q_4}+\bar{\psi}^g_{q'q_3}\bar{\psi}^{g_2}_{q'q_4})+$$
$$+\frac{1}{2}\bar{f}^g(q_1q_2)\sum_{q'}(\psi^g_{q'q_2}\bar{\psi}^{g_2}_{q'q_4}-\bar{\psi}^g_{q'q_3}\psi^{g_2}_{q'q_4}), \tag{21}$$

$$\bar{S}_{gg_2}(q_1q_2,q_3q_4)=-\frac{1}{2}f^g(q_1q_2)\sum_{q'}(\psi^g_{q'q_3}\bar{\psi}^{g_2}_{q'q_4}-\bar{\psi}^g_{q'q_3}\psi^{g_2}_{q'q_4})+$$
$$+\frac{1}{2}\bar{f}^g(q_1q_2)\sum_{q'}(\psi^g_{q'q_3}\psi^{g_2}_{q'q_4}+\psi^g_{q'q_3}\psi^{g_2}_{q'q_4}). \tag{21'}$$

Thus, the exact consideration of the Pauli principle in the quasiparticle plus phonon components of the wave function (10) leads to a complex system of equations (18) and (19). These equations become the equations of the

quasiboson approximation if S and ξ are assumed to be equal to zero. To solve the aforesaid system, one has to find the function $D^n_{gq\sigma}$ from eq. (18); this means that one should perform diagonalization in the space $g_2q_2\sigma_2$, i.e., to diagonalize the matrix of the rank of about 10^3. Then the function $D^n_{gq\sigma}$ is substituted into eq. (19), the solution of which gives the level energies η_n of an odd-A deformed nucleus. The solution of the problem is very complicated in comparison with the quasi-boson approximation and one should find the approximate solutions.

We should like to mention a property of eqs. (18) and (19). To solve them, one should perform the summation over all roots of the secular equations for the one-phonon states. That is, beyond the scope of the quasi-boson approximation one should not restrict himself to the characteristics of those phonons which enter into the wave function (10), but calculate all the roots of the secular equation for each necessary phonon with given $\lambda\mu$. Note, that in the quasiparticle-phonon nuclear model this fact is of no difficulty, since the full phonon space is calculated.

Let us derive the approximate equations. To this end, in eq. (18) we shall preserve only the diagonal terms in the space $g_2q_2\sigma_2$ since the nondiagonal corrections are small, alternating and strongly fluctuate. As a result we have

$$D_{g_2q_2\sigma_2}\frac{C^n_{q_0}}{2}\frac{\frac{v_{q_0q_2}}{\sqrt{Y_{g_2}}}f^{g_2}(q_0q_2)+\sum_g\frac{\xi_{\sigma_2}(gg_2;q_0q_2)}{\sqrt{Y_g}}}{(\epsilon(q_2)+\omega_{g_2}-\eta_n)(1+S(g_2q_2))-\frac{1}{8}\sum_i\frac{X^{\lambda_2\mu_2i}+X^{g_2}}{\sqrt{Y_{\lambda_2\mu_2i}Y_{g_2}}}S(\lambda_2\mu_2i,g_2,q_2q_2)}. \tag{22}$$

It is possible to use the equation $1/2(X^{\lambda_2\mu_2i}+X^{g_2})\ 1/\kappa$. Substitute this value $D^n_{g_2q_2\sigma}$ into eq. (19) and get the following secular equation for finding the energies:

$$\epsilon(q_0)-\eta_n-$$
$$-\frac{1}{4}\sum_{g_2q_2}\frac{\frac{(v_{q_0q_2}f^{g_2}(q_0q_2))^2}{Y_{g_2}}(1+S(g_2,q_2))^2}{(\epsilon(q_2)+\omega_{g_2}-\eta_n)(1+S(g_2q_2))-\frac{1}{4}\sum\frac{1}{\kappa^{(\lambda_2)}}\frac{S(\lambda_2\mu_2i,g_2;q_2q_2)}{\sqrt{Y_{\lambda_2\mu_2i}Y_{g_2}}}}\ 0. \tag{23}$$

In the numerator we have left the diagonal numbers only.

Thus, due to the Pauli principle the quasiparticle q_2 plus phonon poles are shifted and additional terms appear in the quasiparticle-phonon interation. When calculating the shift of the quasiparticle q_2 and phonon $g_2 = \lambda_2 \mu_2 i_2$ pole, one should calculate all the roots i of secular equation (9) for the phonons $\lambda_2 \mu_2$. The shift is the larger, the stronger the collectiveness of the phonon. The values of $S(g,q)$ determined by formula (15') are the larger in absolute value, the stronger the violation of the Pauli principle in the corresponding quasiparticle plus phonon component of the wave function (10). If

$$S(g_0, q_0') = -1$$

then, as is seen from eq. (16), the component $g_0 q_0'$, forbidden by the Pauli principle, is absent in the normalization condition of the wave function.

It is necessary to know in what cases the Pauli principle should strictly be taken into account in the calculations with the wave function (10), and when the quasiboson approximation can be used. For this purpose we should find the roots of secular equation (23), and calculate the strength functions of $E\lambda$- and $M\lambda$-transitions, the cross sections of the reactions with electrons and hadrons, and so on.

3. PAULI PRINCIPLE IN EVEN-EVEN DEFORMED NUCLEI

In ref.[15] the Pauli principle has been taken into account in the two-phonon components of the wave functions of even-even deformed nuclei. The model Hamiltonian is taken in the form of (6), (7) and (8), and the double commutator is used

$$[[Q_{g_1}, Q_{g_2}^+], Q_{g_3}^+] = \sum_g \{ K(gg_1 g_2 g_3) Q_g^+ + \tilde{K}(gg_1 g_2 g_3) Q_g \}, \qquad (24)$$

where

$$K(gg_1 g_2 g_3) = -\frac{1}{2} \sum_{\substack{q_1 q_2 \\ q_3 q_4}} (\psi_{q_1 q_2}^{g_1} \psi_{q_1 q_3}^{g_2} - \phi_{q_1 q_3}^{g_1} \phi_{q_1 q_2}^{g_2})(\psi_{q_4 q_2}^{g_3} \psi_{q_4 q_3}^{g} + \phi_{q_4 q_3}^{g_3} \phi_{q_4 q_2}^{g}). \qquad (25)$$

Here we do not distinguish between $\psi_{qq'}^{g}$ and $\bar{\psi}_{qq'}^{g}$.

The wave function is taken in the form

$$\Psi_n(K^\pi) = \{ \sum_i R_i^n Q_g^+ + \frac{1}{\sqrt{2}} \sum_{g_1 g_2} P_{g_1 g_2}^n Q_{g_1}^+ Q_{g_2}^+ \} \Psi_0 . \qquad (26)$$

After the same calculations as in the previous section, we get the secular equation

$$\det \| (\omega_i - \eta_n) \delta_{ii'} - W_{ii'} \| = 0, \qquad (27)$$

$$W_{ii'} = \sum_{g_1 g_2} \frac{(U_{g_1 g_2}(\lambda\mu i) + V_{g_1 g_2}(\lambda\mu i))(U_{g_1 g_2}(\lambda\mu i') + V_{g_1 g_2}(\lambda\mu i'))}{(\omega_{g_1} + \omega_{g_2} - \eta_n)(1 + \frac{K(g_1 g_2 g_2 g_1)}{1 + \delta_{g_1 g_2}}) - \frac{1}{4(1+\delta_{g_1 g_2})} \sum_{i_0} \{ \frac{K(g_1 g_2 g_2 \lambda_1 \mu_1 i_0)}{\kappa^{(\lambda_1)} \sqrt{Y_{\lambda_1 \mu_1 i_0} Y_{g_1}}} + \frac{K(g_1 g_2 g_1 \lambda_2 \mu_2 i_0)}{\kappa^{(\lambda_2)} \sqrt{Y_{\lambda_2 \mu_2 i_0} Y_{g_2}}} \}} . \qquad (28)$$

with the functions U and V given in ref.[16] In ref.[16] a more general case has been considered, when the isoscalar and isovector multipole forces are taken into account.

Due to the Pauli principle the two-phonon poles are shifted and additional term appears in the quasiparticle-phonon interaction. It is shown in ref.[16] that the shift towards the increase in the two-phonon pole energies in eq. (28) is considerable for the first collective phonons ω_{g_1} and ω_{g_2} and very large for $g_1 = g_2$. The largest shift is given by the first equal phonons $g_1 = g_2$ for $i_0 = 1$; a further summation over $i_0 = 1$ results in the decrease of this shift. In some cases, when one phonon is the most collective low-lying phonon and the other is the collective phonon forming the giant resonance, the shifts may appear to be not small and be of (0.1-0.4) MeV. In the majority of cases the shifts of poles are small.

It is necessary to study the effect of the Pauli principle on the energies of nonrotational states, on the strength functions of $E\lambda$-transitions and on the cross section of excitation of giant resonances. First, of all it is necessary to study the effect of the Pauli principle on the position and properties of the low-lying states with large two-phonon components. The verification of the Axeal-Brink hypothesis about the giant resonances on the exited states is of much interest.

4. ON THE NUMBER OF QUASIPARTICLES IN THE GROUND STATE

Let us derive the expression $\langle B(qq')\rangle$, estimate the number of quasiparticles in the ground state, and study the fulfillment of the following condition:

$$\langle B(qq')\rangle \simeq 0,$$

which is used in all the calculations. This should be done in order to investigate when the RPA-phonons can be used in the quasiparticle-phonon nuclear model or a modification is needed.

Using formula (11) the wave function Ψ_0 of the ground state of a doubly even nucleus has been derived in ref. [14] This wave function determined as vacuum for the phonons of multipolarity $\lambda\mu$, has the form

$$\Psi_0^{(\lambda\mu)} = \frac{1}{\sqrt{N'}} \exp\{-\frac{1}{4}\sum_i \sum_{qq'} \sum_{q_2q_2'} (\psi^{-1})^{\lambda\mu i}_{qq'} \phi^{\lambda\mu i}_{q_2q_2'} A^+(qq')A^+(q_2q_2')\}\Psi_{00}, \quad (29)$$

where N' is the normalization factor, and

$$\alpha_{q\sigma}\Psi_{00} = 0. \quad (30)$$

Since the condition

$$Q_g\Psi_0 = 0$$

should be fulfilled for the phonons of any multipolarity $\lambda\mu$, then

$$\Psi_0 = \Psi_0^{(\lambda_1\mu_1)}\Psi_0^{(\lambda_2\mu_2)} \dots \Psi_0^{(\lambda_n\mu_n)}$$

and it can be written as

$$\Psi_0 = \frac{1}{\sqrt{N}} \exp\{-\frac{1}{4}\sum_g \sum_{qq', q_2q_2'} (\psi^{-1})^g_{qq'} \phi^g_{q_2q_2'} A^+(qq')A^+(q_2q_2')\}\Psi_{00}. \quad (31)$$

Now we calculate

$$\langle B(qq')\rangle = (\Psi_0^* B(qq')\Psi_0) =$$

$$= \sum_g \sum_{q_2q_2'q''} (\psi^{-1})^g_{q_2q_2'} \phi^g_{q''q} (\Psi_0^* A^+(q_2q_2')A^+(qq')\Psi_0) =$$

$$= \sum_{gg_1g_2} \sum_{q_2q_2'q''} (\psi^{-1})^g_{q_2q_2'} \phi^g_{q''q} \phi^{g_2}_{qq''} \psi^{g_2'}_{q_2q_2'} (\Psi_0^* Q_{g_2} Q_{g_2'} \Psi_0) =$$

$$= \sum_{gg'} \sum_{q_2q_2'q} (\psi^{-1})^g_{q_2q_2'} \psi^{g'}_{q_2q_2'} \phi^g_{q''q} \phi^{g'}_{qq''} = \sum_g \sum_{q''} \phi^g_{qq''}\phi^g_{q''q'}. \quad (32)$$

We can write as

$$\langle B(qq')\rangle = \sum_{\lambda\mu} \langle B(qq')\rangle_{\lambda\mu}, \quad (33)$$

$$\langle B(qq')\rangle_{\lambda\mu} = \sum_{iq''} \phi^{\lambda\mu i}_{qq''} \phi^{\lambda\mu i}_{q''q'}, \quad (34)$$

$$\langle \alpha^+_{q\pm} \alpha_{q'\pm} \rangle = \frac{1}{2}\langle B(qq')\rangle. \quad (35)$$

These formulae are derived with the commutation relations (2). It follows from the condition of orthonormalization of the one-phonon states that

$$\frac{1}{2}\sum_{qq'} (\psi^g_{qq'}\psi^{g'}_{qq'} - \phi^g_{qq'}\phi^{g'}_{qq'}) - \frac{1}{2}\sum_{q_1q_2q_3} (\psi^g_{q_1q_2}\psi^{g'}_{q_1q_2} - \phi^g_{q_1q_3}\phi^{g'}_{q_1q_2})\langle B(q_3q_2)\rangle = \delta_{gg'} \quad (36)$$

To calculate the functions $\langle B(qq')\rangle_{\lambda\mu}$, one should take into account all the roots of secular equation (9). This is of no difficulty within the quasiparticle-phonon nuclear model. The functions $\langle B(qq')\rangle_{\lambda\mu}$ for $q \neq q'$ should be small. This is due to the fact that if $K \neq K'$, then conserving the quantum number K

$$\langle B(qq')\rangle_{\lambda\mu} = 0 \quad \text{if} \quad q \neq q' \quad K \neq K'.$$

For $q \neq q'$ and $K = K'$ sum (34) is alternating and the $\langle B(qq')\rangle_{\lambda\mu}$-value is small. This is confirmed by the calculations, according to which $\langle B(qq')\rangle < 10^{-4}$ at $q \neq q$, $K = K'$. Taking into account that $\langle B(qq')\rangle_{\lambda\mu}$ is small, condition (36) can be rewritten as follows:

$$\frac{1}{2}\sum_{qq'} (\psi^g_{qq'}\psi^{g'}_{qq'} - \phi^g_{qq'}\phi^{g'}_{qq'})(1 - \frac{1}{2}\langle B(qq)\rangle - \frac{1}{2}\langle B(q'q')\rangle) = \delta_{gg'}. \quad (37)$$

The preliminary calculations performed show that for the phonons $\lambda=2$, $\mu=2$.

$$\langle \alpha^{+}_{q\pm} \alpha_{q\pm} \rangle_{22} < 0.1 . \quad (38)$$

So, in ^{166}Er only for two values of q the $\langle \alpha^{+}_{q\pm} \alpha_{q\pm} \rangle_{22}$ - values lie in the interval 0.05-0.10. The rest values are less than 0.05. The quantities $\langle \alpha^{+}_{q\pm} \alpha_{q\pm} \rangle$ take the largest values for the single-particle levels q, lying near the Fermi energy. For the octupole phonons

$$\langle \alpha^{+}_{q\pm} \alpha_{q\pm} \rangle_{3K} < 0.05 . \quad (39)$$

and the sum

$$\langle \alpha^{+}_{q\pm} \alpha_{q\pm} \rangle_{22} + \sum_{K} \langle \alpha^{+}_{q\pm} \alpha_{q\pm} \rangle_{3K} < 0.1 . \quad (40)$$

It may be expected that in the majority of deformed nuclei the number of quasiparticles in the ground states is small and the RPA-phonons can be used.

In the quasiparticle-phonon nuclear model the ground state is vacuum for the phonons with all $\lambda\mu$ therefore the applicability of the RPA phonons can finally be concluded after calculating the sum

$$\sum_{\lambda\mu} \langle \alpha^{+}_{q\pm} \alpha_{q\pm} \rangle_{\lambda\mu} .$$

The number of quasiparticles is estimated in the ground state of spherical nuclei. The multipole phonons with $I^{\pi} = 1^{-}, 2^{+}$ and 3^{-} and the spin-multipole phonons with $I^{\pi} = 1^{+}$ and 2^{-} are calculated. The largest number of quasiparticles in the ground state is generated by the 2^{+} phonons for the subshells lying near the Fermi energy. So, for the tin isotopes the largest value for the 2^{+} phonons is 0.07. The total value of the number of quasiparticles in the ground state generated by five phonons mentioned above for the subshells lying near the Fermi energy, is 0.02-0.10. In the tellurium isotopes this value is 0.12. Therefore, within the quasiparticle-phonon nuclear model the Sn and Te isotopes can be calculated without taking into account the number of quasiparticles in the ground states. For the spherical nuclei lying near the transition nuclei, it is necessary to calculate the number of quasiparticles in the ground states; it is obviously very large.

5. CONCLUSION

The effect of the Pauli principle, the improvement of the RPA and correlations in the ground states have been considered in many papers, for instance, in refs. /18-24/. Our aim is to study the effect of the Pauli principle and the correlations in the ground states in the calculations within the quasiparticle-phonon nuclear model. It is necessary to investigate in what cases one can work in the RPA and when the Pauli principle or the correlations in the ground states should be taken into account.

The calculation of all the roots of the secular equation for the phonons of multipolarity $\lambda\mu$ when only one root is used is the main difficulty. The space of the one-phonon states is the basis in the quasiparticle-phonon nuclear model. This means that the phonons with multipolarity λ up to $\lambda = 7$ or 8 are calculated (in spherical nuclei the spin-multipole phonons are also calculated), and all the roots of the secular equations are calculated for each multipolarity. The number of roots is determined by the number of single-particle levels of the average field in the neutron and proton systems. Therefore, this difficulty is not important; it results in the increasing computational time, which is very large sometimes.

In the calculations with the wave functions containing the quasiparticle-plus-phonon or two-phonons components, the effect of the Pauli principle gives more complex secular equations having the pole shifts in comparison with the equations with the RPA-phonons. The effect of the Pauli principle may turn to be essential in the calculation of the structure of low-lying states, the wave functions of which have large quasiparticle plus phonon or two phonons components, and of the giant resonances on the exited states. The fragmentation of the one-quasiparticle and one-phonon states and the corresponding strength functions with the RPA-phonons can be calculated without taking into account the Pauli principle.

Within the quasiparticle-phonon nuclear model one can easily calculate the number of quasiparticles or correlations in the ground states. These calculations are performed for the spherical and deformed nuclei. In most of the deformed nuclei the number of quasiparticles in the ground states in small and we may use the RPA-phonons.

For the solution of the nuclear many-body problem,one should first of all take into account those terms of the interaction and those components of the wave function which determine the calculated properties. The approximate method of calculation should give the most accurate value of this nuclear property. At each stage of solving the problem it is important to avoid the calculation with higher accuracy than the accuracy of final approximate results. This is due to the fact that almost any overestimated accuracy of calculation needs much computational time. Nothing can be obtained gratis when solving such a complicated problem.

A great success in the solution of the many-body nuclear problem is attributed to the approximate rather than exact consideration of the conservation laws. For the solution of the problem one should take into account the conservation laws in the same approximation. It is the fundamental work on quasiaverages /25/ by N.N.Bogolubov that allowed a mathematically correct statement of the problem taking approximately into account the conservation laws. Just the approximate fulfillment of the laws of conservation of the angular momentum, number of particles, the conditions of translational and Galilean invariance, the Pauli principle and so on gave the approximate solutions of such complicated problems, which could hardly be solved otherwise.

The quasiparticle-phonon nuclear model allows the calculations taking exactly into account the Pauli principle. The latter should be taken into account only when it is necessary. In most the cases the calculations can be performed with the RPA-phonons. This conclusion relates also to the approximate consideration of the translational and Galilean invariance, and the different conditions of self-consistency.

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