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Расчеты нейтронных силовых функций четно-четных сферических ядер

В рамках квазичастично-фононной модели ядра рассчитаны нейтронные силовые функции и исследована их спиновая зависимость в нескольких четно-четных сферических ядрах. Результаты расчетов находятся в хорошем согласии с экспериментальными данными.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Calculation of Neutron Strength Functions in Doubly Even Spherical Nuclei

The neutron strength functions are calculated and the spin dependence in several doubly even spherical nuclei is investigated within the quasiparticle-phonon nuclear model. The results of calculation of the s- and p-wave strength functions are in good agreement with the available experimental data. The s-wave strength functions are calculated for two values of spins $J = I_0 \pm 1/2$, and it is shown that the functions S^+ and S^- are, as a rule, close to each other. The results of calculation for the spin dependence of the neutron strength functions agree with the corresponding experimental data.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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INTRODUCTION

The neutron strength functions are important characteristics of nuclei at the excitation energies higher than the neutron binding energy B_n . They can provide the information on the values of a few quasiparticle components of the wave functions of highly excited states. The most complete experimental data on the neutron strength functions are collected in ref. [1] . The experimental data are analyzed in ref. [2] in order to study the spin dependence of the strength functions. Among the recent theoretical papers devoted to the description of the s-neutron strength functions in the spherical nuclei, we should like to mention the calculations within the shell approach to the theory of nuclear reactions [3].

In recent years many investigations have been performed within the quasiparticle-phonon nuclear model [4] to study the fragmentation of a few quasiparticle components of the wave functions at low, intermediate and high excitation energies [5-7] . The neutron strength functions are well described in the odd deformed and spherical nuclei. The fragmentation of the one-phonon states due to the interaction of quasiparticles with phonons has been studied in refs. [8,9]. The partial radiative widths and the giant resonances in doubly even spherical nuclei are well described within the quasiparticle-phonon nuclear model. This paper is aimed at calculating the fragmentation of the one-phonon states in doubly even nuclei within the quasiparticle-phonon model and at obtaining the s- and p-wave neutron strength functions. The calculations are performed for certain spins of the compound-states, thus allowing the study of the spin splitting of the neutron strength functions.

BASIC FORMULAE

The neutron strength functions for a given value of spin J of the compound nucleus are determined by the following relation:

$$S(J) = \sum_i \Gamma_i^{o}(J)/\Delta E, \tag{1}$$

where Γ_i^{o} is the reduced neutron width of the state i ; the summation over i is performed in the energy interval ΔE. In the case when the neutron with orbital momentum ℓ is absorbed by the target - nucleus with spin I_0 , one can find the sum strength function for the states with different spins

$$S_\ell = \sum_{Jj} g(J) S_\ell^{Jj} , \tag{2}$$

where $g(J) = \dfrac{2J+1}{2(2I_0+1)(2\ell+1)}$ is the statistical weight, and S_ℓ^{Jj} is the value of the ℓ-strength function with a given value of spin of the compound-nucleus $\vec{J} = \vec{I}_0 + \vec{\ell} + \vec{1/2} = \vec{I}_0 + \vec{j}$ in the channel j .

Let us find the form of the strength function S_ℓ^{Jj} in the quasiparticle phonon model. The wave functions of the target-nucleus and compound-nucleus have the following form:

$$\Psi(I_0M_0) = \alpha^+_{I_0M_0}|0\rangle \tag{3}$$

$$\Psi_\nu(JM) = \{ \sum_i R_\nu(Ji) Q^+_{JMi} + \sum_{\substack{\lambda_1 i_1 \\ \lambda_2 i_2}} P^{\lambda_1 i_1}_{\lambda_2 i_2} [Q^+_{\lambda_1\mu_1 i_1} Q^+_{\lambda_2\mu_2 i_2}]_{JM} \} |0\rangle , \tag{4}$$

where $\alpha^+ (Q^+)$ is the quasiparticle (phonon) creation operator, $|0\rangle$ is the quasiparticle (phonon) vacuum, and ν is the state number. The secular equation for finding the energies $\eta_{J\nu}$ of the states $\Psi_\nu(JM)$ has the form [8]

$$\mathcal{F}(\eta) \equiv \det | (\omega_{Ji} - \eta_{J\nu})\delta_{ii'} - \sum_{\lambda_1 i_1 \lambda_2 i_2} \frac{U^{\lambda_2 i_2}_{\lambda_1 i_1}(Ji) U^{\lambda_2 i_2}_{\lambda_1 i_1}(Ji')}{\omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2} - \eta_{J\nu}} | = 0. \tag{5}$$

The detailed description of solving this equation and finding the coefficients $R_\nu(Ji)_{Jj}$ is given in refs. [8,10].

The strength function S_ℓ^{Jj} is expressed through the coefficient $R_\nu(Ji)$ defining the contribution of the one-phonon component to the wave function of the state ν and is

$$S_\ell^{Jj} = \frac{\Gamma^o_{s.p.}}{\Delta E} \sum_{\nu \Delta E} \gamma_\nu^2 (Jj) \tag{6}$$

$$\gamma_\nu^2 (Jj) = \left| \sum_i R_\nu(Ji)\Phi_i \right|^2 \quad \Phi_i = \sum_n u_{n\ell j} \psi^{Ji}_{n\ell j, n_o \ell_o I_o}. \tag{7}$$

Here $u_{n\ell j}$ are the Bogolubov transformation coefficients, $\psi^{Ji}_{n\ell j, n_o\ell_o I_o}$ are the phonon amplitudes which can be find by solving equations in the RPA [10]; the single-particle states are denoted by $n\ell j$, where n is the principal quantum number of the single-particle state. It is seen from eqs. (6) and (7) that the value of S_ℓ^{Jj} depends, first, on the contribution of the one-phonon components $R_\nu(Ji)$ to the wave function (4), second, on the value of the amplitude $\psi^{Ji}_{n\ell j, n_o\ell_o I_o}$ entering into the phonon Q^+_{JMi}, and third, on the coefficient $u_{n\ell j}$. The value of $u_{n\ell j}$ is close to unity for the highly-lying particle states and small for the hole states. To evaluate the reduced single-particle widths $\Gamma^o_{s.p.}$ in the case of the Saxon-Woods potential, we use the semiempirical formula from ref. [11]

$$\Gamma^o_{s.p.} = 2kR \frac{\hbar^2}{MR^2\sqrt{E}} (1 + 6{,}7 d^2), \tag{8}$$

where k is the neutron wave number, R is the nuclear radius, and d is the diffuseness parameter of the Saxon-Woods potential. For the nuclei under consideration $\Gamma^o_{s.p.} \approx \frac{50\,\text{keV}}{A^{1/3}}$.

Let us calculate the strength function with the Lorentz weight function by the following formula

$$\gamma^2_{Jj}(\eta) = \sum_\nu \gamma_\nu^2(Jj)\rho(\eta-\eta_\nu) = \frac{1}{2\pi} \sum_\nu \gamma_\nu^2(Jj) \frac{\Delta}{(\eta-\eta_{J\nu})^2 + \Delta^2/4}, \tag{9}$$

where Δ is the interval near the energy $\eta_{J\nu}$ over which the averaging is performed. We use the theorem of residues and express $\gamma^2_{Jj}(\eta)$ through the contour integral around the poles, which are the solutions of eq.(5), and after some transformations, we (see ref.[4]) have

$$\gamma^2_{Jj}(\eta) = \frac{1}{\pi}\,\mathrm{Im}\,\frac{\Phi\,(\eta + i\Delta/2)}{\mathcal{F}\,(\eta + i\Delta/2)}, \qquad (10)$$

where $\Phi^2 = \sum_{ii'} M_{ii'} \Phi_i \Phi_{i'}$, and $M_{ii'}$ are the minors of the determinant in eq.(5). When calculating the strength functions S_ℓ^{Jj} , one should change

$$\sum_{\nu\,\Delta E} |\gamma^2_\nu(Jj)| \to \int_{\Delta E} d\eta\, \gamma_{Jj}(\eta).$$

The model Hamiltonian includes the average field as the Saxon-Woods potential, the pairing interaction, the isoscalar and isovector multipole and spin-multipole forces. The values of the pairing constants have been fixed by the experimental data for the pairing energies. The constants of the quadrupole and octupole forces have been chosen so as to reproduce the experimental data for the 2^+_1 and 3^-_1 levels in the calculations with the wave function (4). We use the parameters given in paper [6] ; therefore there are no free parameters when calculating the neutron strength functions.

2. NEUTRON STRENGTH FUNCTIONS

It is known that in the dependence of the neutron strength functions on mass number A, there are maxima and minima. So, for instance, the s-wave strength function S_0 has maximum in the region $A \sim 55$ and decreases more than by an order of 2 when passing to the region $A \sim 100$. Figure 1 shows the behaviour of the one-quasi-particle state energy, $3s_{1/2}$ as a function of A , for the Saxon-Woods potential reckoned from the neutron binding energy B_n . The strength function S_0 has the maximal value when the state $3s_{1/2}$ is near B_n . The deeper the one-quasiparticle state $3s_{1/2}$, the lesser the value of S_0 .

The interaction between the quasiparticles producing the collective one-phonon states, is responsible for the fragmentation of the two-quasiparticle components over some one-phonon states. The value of $\frac{1}{2}|\psi^{Ji}_{n\ell j,\, n_0\ell_0 I_0}|^2$ approximately determines the contribution of the two-quasiparticle state $\{n\ell j, n_0\ell_0 I_0\}$ to the one-phonon state i . Figure 2 shows how the strength of the two-quasiparticle state $\{2d_{5/2}, 3s_{1/2}\}$

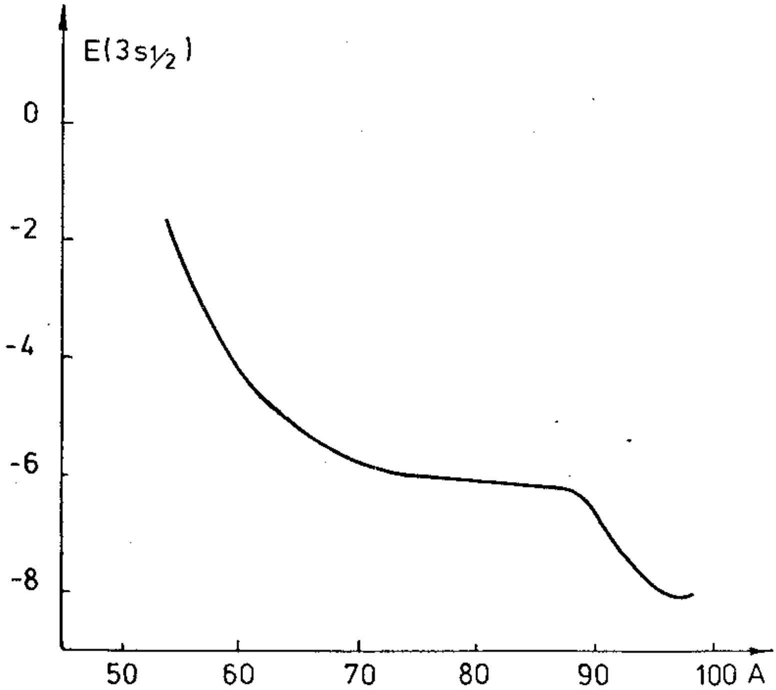

Fig.1. Dependence of the $3s_{1/2}$ one-quasiparticle state energy on A .

is distributed among the quadrupole phonons in ^{96}Mo . The main strength of this state is concentrated at 3.4 MeV; there are notable components at lower energies. The states with an energy higher than 5 MeV exhaust 1.5% of strength of this state. The contribution of the component $\{2d_{5/2}, 3s_{1/2}\}$ to the one-phonon state near $B_n = 9.15$ MeV is close to zero. Therefore, the calculation of the strength functions in the one-phonon approximation gives very small values.

The quasiparticle-phonon nuclear model takes into account the interactions of quasiparticles with phonons, which result in the fragmentation of strength of the one-phonon states over many nuclear levels. As a result the strength of the two-quasiparticle state $\{2d_{5/2}, 3s_{1/2}\}$ is distributed in a wider energy interval including B_n, as compared to the one-phonon approximation. Figure 3 shows the functions $\gamma^2_{J\,1/2}(\eta)$ for $J=2$ and $J=3$ calculated by formula (10). The interaction of quasiparticles with phonons results in that a part of strength of the two-quasiparticles state is transferred upward so that at the excitation energy of 9 MeV the functions $\gamma^2_{J\,1/2}(\eta)$ have rather large values. Just the values of these components $\{2d_{5/2}, 3s_{1/2}\}$ at the energy equal to B_n define the values of S_0^{Jj}.

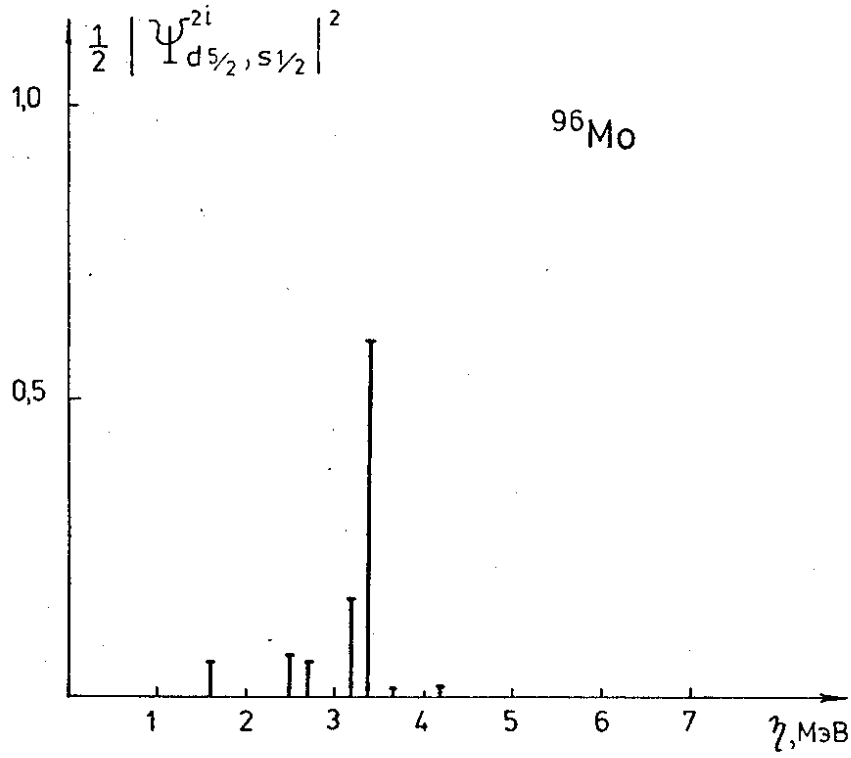

Fig.2. Distribution of the two-quasiparticle component $\{2d_{5/2}, 3s_{1/2}\}$ over the roots of quadrupole phonons in ^{96}Mo.

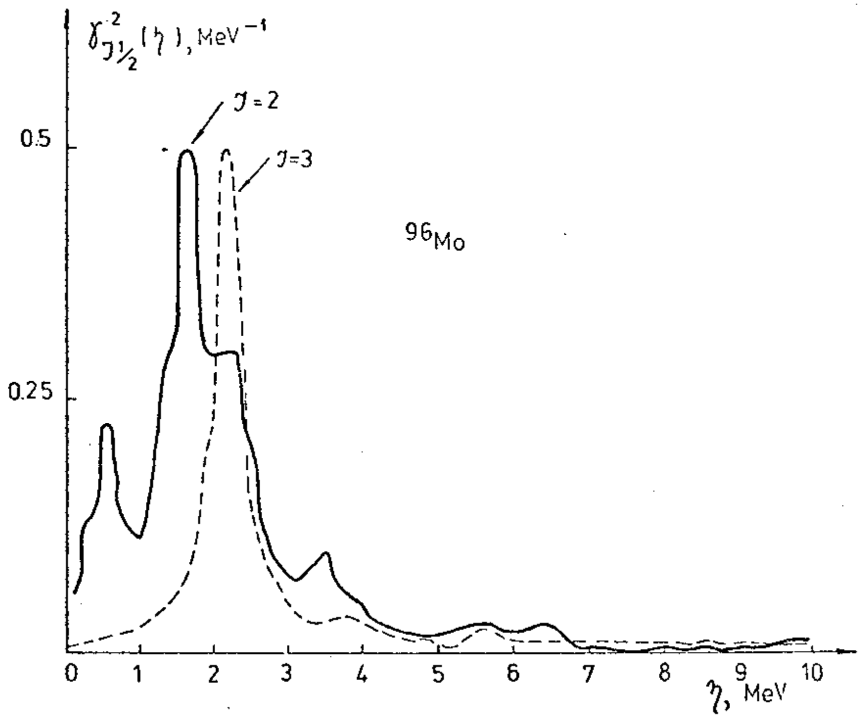

Fig.3. Strength function $\gamma^2_{J\,1/2}(\eta)$ for ^{96}Mo. The solid line is $\gamma^2_{2\,1/2}(\eta)$. The dashed line is $\gamma^2_{3\,1/2}(\eta)$.

We have calculated the s- and p-wave strength functions by formulae (2),(6) and (7) for those spherical nuclei for which the single-particle states $3s_{1/2}$, $2p_{1/2}$, $2p_{3/2}$ and for Nd $3p_{1/2}$ and $3p_{3/2}$ are the bound single-particle states of the Saxon-Woods potential.

Table 1 gives the experimental data /1,12/ and the results of our calculations of the s-wave strength function. It is seen from Table 1 that the calculations describe rather well the experimental data and represent correctly the behaviour of S_0 as a function of A. The results of our calculations

Table 1

S-wave strength functions of even-even spherical nuclei

Target	Spin and parity of target	B_n, MeV	Ref.	$10^4 \times S_0$	
				Experim.	Calc. with ΔE=0,6 MeV
^{53}Cr	3/2$^-$	9.72	1	5,03±2,06	4,5
^{61}Ni	3/2$^-$	10,6	1	3,0±0,8	2,5
^{73}Ge	9/2$^+$	10,2	1	1,5±0,4	1,6
^{87}Sr	9/2$^+$	11,1	1	0,26±0,06	0,88
^{91}Zr	5/2$^+$	8,63	1	0,9±0,3	0,6
^{95}Mo	5/2$^+$	9,15	12	0,48±0,1	0,5
^{97}Mo	5/2$^+$	8,64	12	0,37±0,15	0,8

Table 2

P-wave strength functions of even-even spherical nuclei

Target	Spin and parity of target	B_n, MeV	Ref.	$10^4 \times S_1$	
				Experim.	Calc. with ΔE=0,6 MeV
^{53}Cr	3/2$^-$	9,72	1	0,081±0,051	0,08
^{61}Ni	3/2$^-$	10,6	1	–	0,10
^{144}Nd	7/2$^-$	7,82	12	1,2±0,5	1,6

for nuclei with A ~ 90 are very close to those of ref. [3]. Table 2 shows the results of calculation for the p-strength functions S_1 which are also in good agreement with experiment. In ^{54}Cr the large value of S_0 and the small value of S_1 are described simultaneously. The results of our calculation depend weakly on the choice of the averaging interval ΔE, and in all nuclei under consideration S_ℓ change not more than by 20% with increasing ΔE up to 1 MeV. The results of calculation are more sensitive to the change of quadrupole and octupole constants. The change of the constants within 5% (which influences essentially the position of the 2^+_1 and 3^-_1 levels) changes S_ℓ not more than by 30%. An exception is the nucleus ^{74}Ge in which the anharmonic effects are very strong. The reliability of the obtained results can be verified by the calculation of the radiative strength functions. The values of the M1-radiative strength functions at E_γ = 7.7 MeV and 7.6 MeV equal to $\langle k(M1)\rangle$ = $=8\cdot 10^{-9}$ MeV^{-3} for ^{92}Zr and $\langle k(M1)\rangle = 10\cdot 10^{-9}$ MeV^{-3} for ^{96}Mo are given in ref. [13]. Our calculations with the wave function (4) by the method described in ref. [7] with the constants we have used when calculating S_ℓ, give the values $\langle k(M1)\rangle = 6.6\cdot 10^{-9}$ MeV^{-3} for ^{92}Zr and $\langle k(M1)\rangle = 7.0\cdot 10^{-9}$ MeV^{-3} for ^{96}Mo. This is in agreement with the experimental data having uncertainty factor of about 2 [13]. It should be noted that though the strength functions are investigated experimentally for a long time, the results obtained by various groups have a rather large dispersion, especially at minimum.

3. SPIN DEPENDENCE OF s-WAVE STRENGTH FUNCTIONS

The spin dependence of the neutron strength functions has been much discussed in the literature. The most complete experimental data are in ref. [2]. In the case when s-neutron is absorbed by nucleus with an odd number of nucleons with spin I_0, the compound-states with $J = I_0 \pm 1/2$ are excited. For each value of spin J the strength function can have the values of S^+ and S^- ($S^{\pm} = S_0^{I_0 \pm 1/2, 1/2}$) unequal between themselves. The statistical analysis of the experimental data performed in ref. [2] shows that for the majority of nuclei $S^+ = S^-$ and the deviations from this fact are purely accidental. However, in some nuclei S^+ differs noticeably from S^-. As a rule, such a difference takes place for the nuclei in which the averaging is performed over a small number of resonances. Because of poor statistics the accuracy of the determination of $S^{\pm}$ is small. Rather keen experiments on the transmission of polarized neutrons through a polarized nuclear target allowing the direct determination of the difference $S^{+}-S^{-}$, have been performed in refs.[14,15] for some nuclei of the rare-earth region. Within the experimental error it is obtained that $S^{+}=S^{-}$.

It is easy to calculate the values of S^{+} and S^{-} within our approach. As it is seen from fig.3 $\gamma^2_{J\,1/2}(\eta)$ have different form depending on the excitation energy for different values of J. Therefore, whether the values of S^{+} and S^{-} will coincide or not depends on the ratio of $\gamma^2_{J\,1/2}(\eta)$ for two values of J in the B_n region. The quantities $\gamma^2_{I_0\pm 1/2,\,1/2}$ are deterimed by different matrix elements, and therefore, they may differe from each other. Table 3 shows the results of calculations for S^{+} and S^{-} and for the quantity $a = 2\,\dfrac{S^{+}-S^{-}}{S^{+}+S^{-}}$ which characterizes the divergence of S^{+} from S^{-}. As is seen from table 3 there is no strong difference between S^{+} and S^{-} though there is a certain spin dependence for Sr and Mo. For ^{96}Mo the calculation gives the value a =0.27, whereas the analysis of the experimental data in this nucleus[2] gives the value a =1.2. It should be noted that the analysis has been performed for 6 resonances with $J = I_0 - 1/2$ only. Therefore, the determination of a is not accurate. The strongest spin dependence is obtained for ^{88}Sr. Unfortunately, there are no experimental data on $S^{\pm}$ in this nucleus.

Based on the available experimental data and the results of calculations performed, we may conclude that for the majority of nuclei $S^{+} = S^{-}$. But for some nuclei, due to their individual properties, this regularity can be distorted. A further experimental study of the spin dependence of the neutron strength functions is necessary with better statistics and in a wide range of mass numbers. It is advisable to search for deviation of S^{+} from S^{-} in nuclei with the number of nucleons N close to the values N=50,82, as in these nuclei there are observed many deviations from the statistical regularities. It is also necessary to calculate the values of $S^{\pm}$ for a large number of nuclei.

Table 3

Spin dependence of s-wave strength functions

Compound-nucleus	$S^+ \times 10^4$	$S^- \times 10^4$	a
^{54}Cr	4,64	4,37	0,06
^{62}Ni	2,46	2,4	0,025
^{88}Sr	0,7	1,1	-0,44
^{92}Zr	0,65	0,58	0,11
^{96}Mo	0,55	0,42	0,27
^{98}Mo	0,86	0,75	0,24

CONCLUSION

The calculations of the neutron strength functions performed in this paper and in refs. /5,6/ differ essentially from the standard calculations within the optical nuclear model. Within the quasiparticle-phonon nuclear model the fragmentation of the one- and two-quasiparticle states is calculated over many nuclear levels. The values of the corresponding components of the one- and two-quasiparticle states at the excitation energy near B_n determine the values of the neutron and partial radiative strength functions.

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