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I. Two-Body Forces

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Бабуцидзе Т.Д. и др.

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Кварк-кластерная модель ядра и ядерные силы.
1. Двухчастичные силы

Кварковая модель ядра, где ядро представляется как система бесцветных трехкварковых кластеров-нуклонов, обобщается на случай, когда учитываются, также возбужденные нуклонные и разные Δ-изобарные состояния. На основе заданного локального потенциала кварк-кваркового взаимодействия, учитывающего обмен глюонами на малых расстояниях и конфайнмент на больших расстояниях, в рамках рассматриваемой модели был построен потенциал барион-барионного взаимодействия. Этот потенциал содержит как центральную, так и спин-спиновую и тензорную части, является сугубо нелокальным, что обусловлено обменным характером взаимодействия и имеет радиус действия порядка размеров бариона.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Quark-Cluster Model of Nuclei and Nucleon Forces.
I. Two-Body Forces

A quark model suggested in ref.[1], according to which a nucleus is considered as a system of colourless three-quark clusters-nucleons, is generalized for the case when besides the nucleon states the possibility of producting the excited nucleon and different Δ isobar states is taken into account. Beginning with a given local quark-quark interaction potential, including a gluon exchange at small distances and a confinement at large distances, baryon-baryon interaction potential is constructed in the framework of the model under consideration. This potential contains central, spin-spin and tenzor parts and is basically nonlocal, due to the exchange character of this interaction, and has an interaction radius of the order of the baryon r.m.s. radius.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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Introduction

It has become widely accepted that a nucleon (N)is the three-quark composite object in which any two quarks are in the relative s - state, as to the colour state of this object it belongs to the singlet representation ($\{1^3\}^c$) of $SU^c(3)$ group. If now we accept nucleon model of nuclei we come to a model of the nucleus which is a 3A-quark system of the colour singlet three-quark object (colourless three-quark clusters - nucleons) in a given instant of time. Such a model can be called the quark-cluster model of nuclei by an analogy with the α -cluster model. Indeed: as in the α-cluster all allowed spin and isospin states are occupied,and as a result the corresponding wave function is completely antisymmetrical (the singlet representation $\{1^4\}^{\sigma\tau}$ of $SU^{\sigma\tau}(4)$ group), in nucleon all the allowed colour states are occupied; further the exchange of nucleons with the same spin-isospin quantum numbers is allowed between any two α-clusters analogously the exchange of quarks with the same colour quantum numbers is allowed between two nucleons. In the framework of the model under consideration there is a possibility of the appearance of the three, four and other many-body forces besides the two-body forces arising as a result of the single-quark exchange between two nucleons in a given instant of time. The model described above was suggested in ref.[1].

This model is easily generalized for the case when besides the nucleon states the possibility of the production of the other three-quark objects, e.g., Δ-isobars, excited nucleons N^*, etc., are taken into account. These new types of three-quark objects are the result of the exchange of quarks with the different spin-isospin and radial-orbital quantum numbers, but with the same colour quantum numbers. It is clear that due to the exchange of quarks between cluster states with a "hidden" colour /2/, i.e., states corresponding to the octet representation $\{21\}^c$ of $SU^c(3)$ group can appear. We do not consider here this possibility. Therefor we come to the model according to which a nucleus is the 3A-quark system divided into the colourless three-quark clusters (baryons).

Two-Body Nuclear Forces

The similarity of nuclear forces with forces acting between nouble gas atoms, e.g., between Helium atoms (see, e.g., review article /3/) is well known. In this case He-He forces are the result of the averaging over the electrons from closed shells of an electron-electron interaction from different atoms taking into account an exchange effect. Analogously, baryon-baryon interaction potential can be considered as a result of the averaging of the quark-quark (qq) interaction potential between the quarks from different closed colour shells (colourless three-quark clusters) having the exchange between the clusters with the same colour quarks.

The present paper is devoted to the construction of baryon-baryon (BB) interaction potential on the basis of the model described above. There are different two-baryon state vectors for six-quark system in this model

$$|N_{1/2}N_{1/2}\rangle,\ |\Delta_{3/2}\Delta_{3/2}\rangle,\ |N^*_{1/2}N^*_{1/2}\rangle,\ |\Delta^*_{1/2}\Delta^*_{1/2}\rangle,\ |N^*_{3/2}N^*_{3/2}\rangle,\ |N_{1/2}\Delta_{3/2}+\Delta_{3/2}N_{1/2}\rangle,$$
$$|N_{1/2}N^*_{1/2}+N^*_{1/2}N_{1/2}\rangle,\ |\Delta^*_{1/2}\Delta_{3/2}+\Delta_{3/2}\Delta^*_{1/2}\rangle,\ |N_{1/2}N^*_{3/2}+N^*_{3/2}N_{1/2}\rangle, \tag{1}$$
$$|N_{1/2}\Delta^*_{1/2}+\Delta^*_{1/2}N_{1/2}\rangle,\ |N^*_{1/2}\Delta_{3/2}+\Delta_{3/2}N^*_{1/2}\rangle,\ |\Delta_{3/2}N^*_{3/2}+N^*_{3/2}\Delta_{3/2}\rangle,\ |N^*_{1/2}N^*_{3/2}+N^*_{3/2}N^*_{1/2}\rangle,$$
$$|\Delta^*_{1/2}N^*_{1/2}+N^*_{1/2}\Delta^*_{1/2}\rangle,\ |\Delta^*_{1/2}N^*_{3/2}+N^*_{3/2}\Delta^*_{1/2}\rangle,$$

where we designated by $N_{1/2}$ and $\Delta_{3/2}$ nucleons and Δ-isobars in the ground states with spin-isospin wave functions corresponding to the irreducible representation (IR) $\{3\}$ of $SU^{\sigma\tau}(4)$ group and with the s - state space wave function. Nucleons and Δ-isobars in the excited states with spin-isospin wave functions, corresponding to the IR $\{21\}$ of $SU^{\sigma\tau}(4)$ group, and with a space wave function, characterized by the non-zero orbital angular momentum are designated by $N^*_{1/2(3/2)}$ and $\Delta^*_{1/2}$. In this basis the BB - interaction potential is a many-row matrix. It is convenient to begin with a shortened basis taking only two state vectors $|N_{1/2}N_{1/2}\rangle$ and $|\Delta_{3/2}\Delta_{3/2}\rangle$. As for the construction of the full matrix it will discussed below.

The state vector of two identical a- and b- baryons is taken in a form

$$|\hat{\Phi}_{ab}\rangle = \frac{1}{\sqrt{2}}(1-P_{ab})|\Phi_{ab}\rangle, \tag{2}$$

where:

$$|\Phi_{ab}\rangle = |\Phi_a(123)\rangle|\Phi_b(456)\rangle|R_{ab}\rangle, \tag{3}$$

$|\Phi_a\rangle$, $|\Phi_b\rangle$ are a- and b-baryon state vectors antisymmetrized over all quark coordinates; $|R_{ab}\rangle$ is the state vector of their relative motion; P_{ab} is the two baryon permutation operator. It is clear that the interaction energy of the BB $\rightarrow$ B'B' transition with fixed values of orbital (L), spin (S) and isospin (I) momenta has the form

$$\hat{W}_{B'B',BB} = -\frac{[1-(-1)^{L'+S'+I'}]}{\sqrt{2}}\langle\Phi^{L'S'I'}_{a'b'}|\sum_{k=1}^{3}\sum_{l=4}^{6}V_{kl}\sum_{i=1}^{3}\sum_{j=4}^{6}P_{ij}|\Phi^{LSI}_{ab}\rangle\frac{[1-(-1)^{L+S+I}]}{\sqrt{2}} \equiv$$
$$\equiv \frac{[1-(-1)^{L'+S'+I'}]}{\sqrt{2}}\,W_{B'B',BB}\,\frac{[1-(-1)^{L+S+I}]}{\sqrt{2}}, \tag{4}$$

where V_{kl} is a qq interaction potential and

$$P_{ij} = P^r_{ij}P^{\sigma\tau}_{ij}P^c_{ij} \tag{5}$$

is (i,j) quarks space, spin-isospin and colour coordinate permutation operator.

Let us take the state vector $|\Phi^{LSI}_{ab}\rangle$ as a product of the colour, spin-isospin and space state vectors:

$$|\Phi^{LSI}_{ab}\rangle = |\{1^3\}\{1^3\}\rangle^c |s_a s_b, t_a t_b; SI\rangle |\Phi_a\rangle |\Phi_b\rangle |R^L_{ab}\rangle. \tag{6}$$

The local qq - potential,in general,has the form

$$V_{kl} = \vec{F}_k \cdot \vec{F}_l \, f_{kl}(\vec{r}_{kl}, \vec{P}_{kl}, \vec{s}_k, \vec{s}_l) = \frac{1}{4} \vec{\lambda}_k \cdot \vec{\lambda}_l \, f_{kl}(\vec{r}_{kl}, \vec{P}_{kl}, \vec{s}_k, \vec{s}_l), \tag{7}$$

here $\vec{\lambda}(\lambda^1, \lambda^2, \ldots, \lambda^8)$ are Gell-Mann matrices and f_{kl} is a some scalar function.

Making use of (4),(6) and (7) we have

$$W_{B'B',BB} = -\sum_{k,i=1}^{3} \sum_{l,j=4}^{6} {}^c\langle\{1^3\}\{1^3\}|\vec{F}_k \cdot \vec{F}_l P^c_{ij} |\{1^3\}\{1^3\}\rangle^c \langle \Phi_{a'} \Phi_{b'} R^{L'}_{a'b'} | f^{S'I';SI}_{kl;ij}(\vec{r}_{kl}, \vec{P}_{kl}) P^r_{ij} |\Phi_a \Phi_b R^L_{ab}\rangle, \tag{8}$$

where

$$f^{S'I';SI}_{kl;ij}(\vec{r}_{kl}, \vec{P}_{kl}) = \langle s_{a'} s_{b'}, t_{a'} t_{b'}: S'I' | f_{kl}(\vec{r}_{kl}, \vec{P}_{kl}, \vec{s}_k, \vec{s}_l) P^{\sigma\tau}_{ij} | s_a s_b, t_a t_b: SI\rangle. \tag{9}$$

Introducing the proper Jakob coordinates in the six-quark c.m. frame one has

$$\begin{aligned} \mathcal{J}^{i,j}_{k,l} &\equiv \langle \Phi_{a'} \Phi_{b'} R^{L'}_{a'b'} | f^{S'I';SI}_{kl;ij} P^r_{ij} | \Phi_a \Phi_b R^L_{ab}\rangle = \\ &= \int d\vec{r}\, R^{L'*}_{a'b'}(\vec{r}) \Big\{ \int d\vec{R}_{kl} d\vec{r}_{kl} d\vec{x} d\vec{y}\, \Phi^*_{a'}(\vec{r}_1,\vec{r}_2,\vec{r}_3;\vec{R}_{a'}) \Phi^*_{b'}(\vec{r}_4,\vec{r}_5,\vec{r}_6;\vec{R}_{b'}) \cdot \\ &\cdot f^{S'I';SI}_{kl}(\vec{r}_{kl}, \vec{P}_{kl}) P^r_{ij} \left[\Phi_a(\vec{r}_1,\vec{r}_2,\vec{r}_3;\vec{R}_a) \Phi_b(\vec{r}_4,\vec{r}_5,\vec{r}_6;\vec{R}_b)\right] \Big\} R^L_{ab}(P^r_{ij} \vec{r}), \end{aligned} \tag{10}$$

where

$$\vec{r}_{kl} = \vec{r}_k - \vec{r}_l, \quad \vec{R}_{kl} = \frac{1}{2}(\vec{r}_k + \vec{r}_l), \quad \vec{x} = \vec{r}_i - \vec{r}_{i'}, \quad \vec{y} = \vec{r}_j - \vec{r}_{j'},$$
$$\vec{r} = \vec{R}_a - \vec{R}_b; \quad (i,i' \neq k;\ j,j' \neq l).$$

Let us use an identity[4] *

$$R^L_{ab}(P^L_{ij}\vec{r}) = \left(\frac{\beta^2}{\pi}\right)^3 \int \exp\left[-\beta^2 (P^r_{ij}\vec{r} - i\vec{S})^2\right] \exp\left[\beta^2 (\vec{r}' - i\vec{S})^2\right] R^L_{ab}(\vec{r}') d\vec{S} d\vec{r}', \tag{11}$$

which is held for an arbitraty value of parameter β with $Re\,\beta^2 > 0$. This parameter has to be taken for convenience. As a result we have

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*The authors would like to thank V.S.Skhirtladze who has pointed out on the possibility of using this relation.

$$\mathcal{J}^{ij}_{kl} = \int d\vec{r} d\vec{r}' R^{L'*}_{a'b'}(\vec{r}') \langle \vec{r}' | V^{kl;ij}_{B'B',BB} | \vec{r}\rangle R^L_{ab}(\vec{r}), \tag{12}$$

where

$$\begin{aligned} \langle \vec{r}' | V^{kl;ij}_{B'B',BB} | \vec{r}\rangle &= \left(\frac{\beta^2}{\pi}\right)^3 \int d\vec{R}_{kl} d\vec{r}_{kl} d\vec{x} d\vec{y} \left[\Phi_{a'}(\vec{r}', \vec{R}_{kl}, \vec{r}_{kl}, \vec{x}) \Phi_{b'}(\vec{r}', \vec{R}_{kl}, \vec{r}_{kl}, \vec{y})\right]^* \cdot \\ &\cdot f^{S'I';SI}_{kl}(\vec{r}_{kl} \vec{P}_{kl}) \int d\vec{S} P^r_{ij} \left\{\Phi_a(\vec{r}', \vec{R}_{kl}, \vec{r}_{kl}, \vec{x}) \Phi_b(\vec{r}', \vec{R}_{kl}, \vec{r}_{kl}, \vec{y}) \exp\left[-\beta^2(\vec{r}' - i\vec{S})^2\right] \cdot \right. \\ &\cdot \exp\left[\beta^2(\vec{r} - i\vec{S})^2\right] \end{aligned} \tag{13}$$

are the elements of the BB $\rightarrow$ B'B' transition potential. It is easy to see that for a given (kl), e.g., for (kl)=(14) we have three types of the different matrix elements of the potential:

$$V^{14;14}; \quad V^{14;15} = V^{14;16} = V^{14;24} = V^{14;34};$$
$$V^{14;25} = V^{14;26} = V^{14;35} = V^{14;36}; \tag{13'}$$

We should like to emphasize that the effective BB potential we have obtained is basically nonlocal. This is connected with the exchange character of the BB interaction and that we have not used the adiabatic approximation (see below).

The basis function of the IR $\{1^3\}$ of $SU^c(3)$ group has been used for calculation of the matrix element ${}^c\langle\{1^3\}\{1^3\}|\vec{F}_k \cdot \vec{F}_l P^c_{ij}|\{1^3\}\{1^3\}\rangle^c$. Keeping in mind that according to our assumptions colour octet cluster states have to be neglected we omit the states appearing besides the singlet states after the action of the operator P^c_{ij} on the state vector $|\{1^3\}\{1^3\}\rangle^c$. As a result we have:

$${}^c\langle\{1^3\}\{1^3\}|\vec{F}_k \vec{F}_l P^c_{ij}|\{1^3\}\{1^3\}\rangle^c = \begin{cases} 1/9, & (i,j)=(k,l) \\ -1/18, & i=k,\ j\neq l \\ & i\neq k,\ j=l \\ 1/36, & (i,j)\neq(k,l) \end{cases} \tag{14}$$

The direct calculations show us, that

$$P^{\sigma\tau}_{ij} \begin{bmatrix} |\frac{1}{2}\frac{1}{2}, \frac{1}{2}\frac{1}{2}; SI\rangle \\ |\frac{3}{2}\frac{3}{2}, \frac{3}{2}\frac{3}{2}; SI\rangle \\ \cdots\cdots \\ \cdots\cdots \end{bmatrix} = \begin{bmatrix} a_{11}, & a_{12}, & \ldots \\ a_{21}, & a_{22}, & \ldots \\ \cdots & & \\ \cdots & & \end{bmatrix}_{SI} \begin{bmatrix} |\frac{1}{2}\frac{1}{2}, \frac{1}{2}\frac{1}{2}; SI\rangle \\ |\frac{3}{2}\frac{3}{2}, \frac{3}{2}\frac{3}{2}; SI\rangle \\ \cdots\cdots \\ \cdots\cdots \end{bmatrix}. \tag{15}$$

Table

SI	i	a_{i1}	a_{i2}	a_{i3}	a_{i4}	a_{i5}	a_{i6}	a_{i7}	a_{i8}	a_{i9}	a_{i10}	a_{i11}	a_{i12}	a_{i13}	a_{i14}	a_{i15}	
00	1	$\frac{7}{9}$	$\frac{4}{9}$	$-\frac{2}{9}$	$\frac{\sqrt{2}}{9}$	$\frac{\sqrt{2}}{9}$		$-\frac{2\sqrt{2}}{9}$									
	2	$\frac{4}{9}$	$\frac{1}{9}$	$\frac{4}{9}$	$-\frac{2\sqrt{2}}{9}$	$-\frac{2\sqrt{2}}{9}$		$\frac{4\sqrt{2}}{9}$									
10	1	$-\frac{1}{27}$	$\frac{4\sqrt{5}}{27}$	$-\frac{10}{27}$	$\frac{5\sqrt{2}}{27}$	$\frac{\sqrt{10}}{27}$		$\frac{8\sqrt{2}}{27}$	$\frac{4\sqrt{2}}{27}$	$-\frac{8\sqrt{2}}{27}$				$\frac{10\sqrt{2}}{27}$			
	2	$\frac{4\sqrt{5}}{27}$	$\frac{1}{27}$	$\frac{4\sqrt{5}}{27}$	$-\frac{2\sqrt{10}}{27}$	$-\frac{2\sqrt{2}}{27}$		$\frac{4\sqrt{10}}{27}$	$\frac{2\sqrt{10}}{27}$	$-\frac{4\sqrt{10}}{27}$				$-\frac{4\sqrt{10}}{27}$			
01	1	$-\frac{1}{27}$	$\frac{4\sqrt{5}}{27}$	$-\frac{10}{27}$	$\frac{\sqrt{10}}{27}$	$\frac{5\sqrt{2}}{27}$		$\frac{8\sqrt{2}}{27}$			$-\frac{8\sqrt{2}}{27}$		$\frac{4\sqrt{2}}{27}$		$\frac{10\sqrt{2}}{27}$		
	2	$\frac{4\sqrt{5}}{27}$	$\frac{1}{27}$	$\frac{4\sqrt{5}}{27}$	$-\frac{2\sqrt{2}}{27}$	$-\frac{2\sqrt{10}}{27}$		$\frac{4\sqrt{10}}{27}$			$-\frac{4\sqrt{10}}{27}$		$\frac{2\sqrt{10}}{27}$		$-\frac{4\sqrt{10}}{27}$		
11	1	$\frac{31}{81}$	$\frac{20}{81}$	$\frac{22}{81}$	$-\frac{5\sqrt{10}}{81}$	$\frac{5\sqrt{10}}{81}$	$\frac{40}{81}$	$-\frac{14\sqrt{2}}{81}$	$\frac{4\sqrt{10}}{81}$	$-\frac{4\sqrt{2}}{81}$	$-\frac{4\sqrt{2}}{81}$	$-\frac{32}{81}$	$\frac{4\sqrt{10}}{81}$	$\frac{14\sqrt{2}}{81}$	$\frac{14\sqrt{2}}{81}$	$\frac{4\sqrt{2}}{81}$	
	2	$\frac{20}{81}$	$\frac{1}{81}$	$\frac{20}{81}$	$-\frac{2\sqrt{10}}{81}$	$-\frac{2\sqrt{10}}{81}$	$\frac{20}{81}$	$\frac{20\sqrt{2}}{81}$	$-\frac{2\sqrt{10}}{81}$	$-\frac{20\sqrt{2}}{81}$	$-\frac{20\sqrt{2}}{81}$	$\frac{20}{81}$	$\frac{2\sqrt{10}}{81}$	$-\frac{20\sqrt{2}}{81}$	$-\frac{20\sqrt{2}}{81}$	$\frac{20\sqrt{2}}{81}$	

The coefficients a_{mn} for fixed values of (SI) in NN channel defined in the basis (1) are shown in the table. If the shortened basis is used this coefficients must be normalized again. Taking into account the formula (15), BB interaction potential can be written as a matrix

$$V_{BB} = \begin{bmatrix} a_{11} V_{NN,NN} \ , a_{21} V_{NN,\tilde{\Delta}\tilde{\Delta}}, \cdots \\ a_{12} V_{\Delta\Delta,\tilde{N}\tilde{N}} \ , a_{22} V_{\Delta\Delta,\Delta\Delta}, \cdots \\ \cdots\cdots\cdots\cdots\cdots \end{bmatrix} . \qquad (16)$$

where $\tilde{N}$ ($\tilde{\Delta}$) denotes a cluster with nucleon (Δ -isobar) space function, but with spin 3/2 (1/2) and isospin 3/2 (1/2).

As one can see from (9) and (13) we need the exact expression for $f_{\kappa e}(\vec{r}_{\kappa e}, \vec{P}_{\kappa e}, \vec{s}_\kappa, \vec{s}_e)$ function for the construction of BB interaction. Let us use the expression for qq interaction suggested in ref./5/ and generally accepted substitutions

$$\frac{1}{r_{\kappa e}} \to f_{\kappa e}(r_{\kappa e}), \quad \frac{1}{r^3_{\kappa e}} \to \frac{1}{r_{\kappa e}} \frac{\partial}{\partial r_{\kappa e}} \left(\frac{1}{r_{\kappa e}}\right) \to -\frac{1}{r_{\kappa e}} \frac{\partial}{\partial r_{\kappa e}} f_{\kappa e}(r_{\kappa e}),$$
$$\delta(\vec{r}_{\kappa e}) = -\frac{1}{4\pi} \Delta_{r_{\kappa e}} \left(\frac{1}{r_{\kappa e}}\right) \to -\frac{1}{4\pi} \Delta_{r_{\kappa e}} f_{\kappa e}(r_{\kappa e}), \qquad (17)$$

with the demand of the translational invariance the function $f_{\kappa e}$ is reduced to

$$f_{\kappa e}(\vec{r}_{\kappa e}, \vec{P}_{\kappa e}, \vec{s}_\kappa, \vec{s}_e) = f_{\kappa e}(r_{\kappa e}) - \frac{1}{2m^2} f_{\kappa e}(r_{\kappa e}) \left[-\vec{P}^2_{\kappa e} + \hat{r}_{\kappa e}(\hat{r}_{\kappa e} \cdot \vec{P}_{\kappa e}) \vec{P}_{\kappa e}\right] +$$
$$+ \frac{1}{4m^2}\left[1 + \frac{8}{3} \vec{s}_\kappa \cdot \vec{s}_e\right] \Delta_{r_{\kappa e}} f_{\kappa e}(r_{\kappa e}) + \frac{1}{2m^2}\left[3\vec{l}_{\kappa e} \cdot \vec{S}_{\kappa e} + \{6(\vec{r}_{\kappa e} \cdot \vec{s}_\kappa)(\vec{r}_{\kappa e} \cdot \vec{s}_e - 2\vec{s}_\kappa \cdot \vec{s}_e)\}\right] \cdot \qquad (18)$$
$$\cdot \frac{1}{r_{\kappa e}} \frac{\partial}{\partial r_{\kappa e}} f_{\kappa e}(r_{\kappa e}), \qquad \hat{r} = \frac{\vec{r}}{r},$$

where $f_{\kappa e}(r_{\kappa e})$ is taken in a form/6/

$$f_{\kappa e}(r_{\kappa e}) = \frac{g^2}{r_{\kappa e}} \left[1 - \left(\frac{r_{\kappa e}}{r_0}\right)^{n+1}\right], \quad n \geq 1. \qquad (19)$$

A Coulomb-like interaction term in (19) at small separations is associated with massless vector mesons(gluons) exchange and the second term exhibits a "confinement" interaction at large separation.

An inner state space wave function of a baryon is taken in the form

$$\Phi_{a(b)} = \left[\frac{\alpha^2_{a(b)}}{\pi}\right]^{3/2} \left(\frac{4}{3}\right)^{3/4} \exp\left\{-\alpha^2_{a(b)} \sum_{i\in a(b)} (\vec{r}_i - \vec{R}_{a(b)})^2\right\}, \tag{20}$$

where parameter $\alpha^2_{a(b)}$ is determined from r.m.s. radius of a baryon $\sqrt{\langle(\vec{r}_i-\vec{R}_{a(b)})^2\rangle} = \frac{1}{\sqrt{2}\,\alpha_{a(b)}} = r_{a(b)}$.

The calculations show that static part of $NN\to NN$, $\Delta\Delta\to\Delta\Delta$ and $NN\leftrightarrows\Delta\Delta$ transitions potential in $\vec{r}$-representation has the form

$$\langle\vec{r}\,'|V_{B'B',BB}|\vec{r}\rangle = \frac{g^2}{2}\left[\frac{16\alpha'^3\alpha^3}{3\pi(\alpha'^2+\alpha^2)^3}\right]^{3/2} (\alpha'\alpha)^{3/2}\sqrt{\alpha'^2+\alpha^2}\,\times$$

$$\times\exp\left[\frac{9}{8}(\alpha'^2+\alpha^2)\vec{r}\vec{r}\,'\right]\left\{\exp\left[-\left(\frac{3}{16}\alpha'^2+\frac{27}{16}\alpha^2\right)\vec{r}\,'^2-\left(\frac{27}{16}\alpha'^2+\frac{3}{16}\alpha^2\right)\vec{r}^{\,2}\right]\times\right.$$

$$\left.\times\left[F^c(\vec{r}\,',\vec{r})+\frac{\alpha'^2+\alpha^2}{4m^2}\left(g+\frac{8}{3}\vec{S}_a\vec{S}_b\right)F^{ss}(\vec{r}\,',\vec{r})+\frac{\alpha'^2+\alpha^2}{2m^2}F^t(\vec{r},\vec{r}\,')\right]\right\}+\{\vec{r}\rightleftarrows\vec{r}\,'\};$$

$$F^c(\vec{r}\,',\vec{r}) = \frac{2}{3\sqrt{\alpha'^2+\alpha^2}}\left[-\frac{1}{|\vec{r}-\vec{r}\,'|}+\begin{pmatrix}\dfrac{9|\vec{r}-\vec{r}\,'|}{4r_0^2}\\ \dfrac{27|\vec{r}-\vec{r}\,'|^2}{8r_0^3}\end{pmatrix}\right]-\frac{8}{\sqrt{5\pi}}\left[\Phi\left(\frac{1}{2},\frac{3}{2};-\frac{90(\alpha'^2+\alpha^2)}{80}|3\vec{r}-\vec{r}\,'|^2\right)+\right.$$

$$+\begin{pmatrix}\dfrac{5}{4r_0^2(\alpha'^2+\alpha^2)}\Phi\left(-\frac{1}{2},\frac{3}{2};-\frac{9(\alpha'^2+\alpha^2)}{80}|3\vec{r}-\vec{r}\,'|^2\right)\\ \dfrac{45\sqrt{5\pi}}{32r_0^3(\alpha'^2+\alpha^2)^{3/2}}\Phi\left(-1,\frac{3}{2};-\frac{9(\alpha'^2+\alpha^2)}{80}|3\vec{r}-\vec{r}\,'|^2\right)\end{pmatrix}-\frac{2}{\sqrt{\pi}}\left[\Phi\left(\frac{1}{2},\frac{3}{2};-\frac{9(\alpha'^2+\alpha^2)}{16}|\vec{r}+\vec{r}\,'|^2\right)+\right.$$

$$+\begin{pmatrix}\dfrac{1}{r_0^2(\alpha'^2+\alpha^2)}\Phi\left(-\frac{1}{2},\frac{3}{2};-\frac{9(\alpha'^2+\alpha^2)}{16}|\vec{r}+\vec{r}\,'|^2\right)\\ \dfrac{1}{4r_0^3(\alpha'^2+\alpha^2)^{3/2}}\Phi\left(-1,\frac{3}{2};-\frac{9(\alpha'^2+\alpha^2)}{16}|\vec{r}+\vec{r}\,'|^2\right)\end{pmatrix},$$

$$F^{ss}(\vec{r}\,',\vec{r}) = \frac{4}{27(\alpha'^2+\alpha^2)^{3/2}}\left[\frac{8\pi}{9}\delta(\vec{r}-\vec{r}\,')+\begin{pmatrix}\dfrac{1}{r_0^2|\vec{r}-\vec{r}\,'|}\\ \dfrac{9}{2r_0^3}\end{pmatrix}\right]+\frac{64}{45\sqrt{5\pi}}\left[\exp\left\{-\frac{9(\alpha'^2+\alpha^2)}{80}|3\vec{r}-\vec{r}\,'|^2\right\}+\right.$$

$$+\begin{pmatrix}\dfrac{5\Phi\left(\frac{1}{2},\frac{3}{2};-9(\alpha'^2+\alpha^2)/80\cdot|3\vec{r}-\vec{r}\,'|^2\right)}{4r_0^2(\alpha'^2+\alpha^2)}\\ \dfrac{45\sqrt{5\pi}}{16r_0^3(\alpha'^2+\alpha^2)^{3/2}}\end{pmatrix}+\frac{4}{9\sqrt{\pi}}\left[\exp\left\{-\frac{9(\alpha'^2+\alpha^2)}{16}|\vec{r}+\vec{r}\,'|^2\right\}+\begin{pmatrix}\dfrac{\Phi\left(\frac{1}{2},\frac{3}{2};-\frac{9(\alpha'^2+\alpha^2)}{16}|\vec{r}+\vec{r}\,'|^2\right)}{r_0^2(\alpha'^2+\alpha^2)}\\ \dfrac{3\sqrt{\pi}}{2r_0^3(\alpha'^2+\alpha^2)^{3/2}}\end{pmatrix}\right],$$

$$F^t(\vec{r}\,',\vec{r}) = \frac{2}{27(\alpha'^2+\alpha^2)^{3/2}}T_{12}(\widehat{\vec{r}-\vec{r}\,'})\left[\frac{4}{9|\vec{r}-\vec{r}\,'|^3}+\begin{pmatrix}\dfrac{1}{r_0^2|\vec{r}-\vec{r}\,'|}\\ \dfrac{1}{r_0^3|\vec{r}-\vec{r}\,'|^2}\end{pmatrix}\right]+\frac{8(\alpha'^2+\alpha^2)}{225\sqrt{5\pi}}(3\vec{r}-\vec{r}\,')^2T_{12}(\widehat{3\vec{r}-\vec{r}\,'})\times$$

$$\times\left[\Phi\left(\frac{5}{2},\frac{7}{2};-\frac{9(\alpha'^2+\alpha^2)}{80}|3\vec{r}-\vec{r}\,'|^2\right)+\begin{pmatrix}\dfrac{5}{4(\alpha'^2+\alpha^2)r_0^2}\Phi\left(\frac{3}{2},\frac{7}{2};-\frac{9(\alpha'^2+\alpha^2)}{80}|3\vec{r}-\vec{r}\,'|^2\right)\\ \dfrac{15\sqrt{5\pi}}{16(\alpha'^2+\alpha^2)^{3/2}r_0^3}\Phi\left(1,\frac{7}{2};-\frac{9(\alpha'^2+\alpha^2)}{80}|3\vec{r}-\vec{r}\,'|^2\right)\end{pmatrix}+\right.$$

$$+\frac{(\alpha'^2+\alpha^2)}{30\sqrt{\pi}}(\vec{r}+\vec{r}\,')^2T_{12}(\widehat{\vec{r}+\vec{r}\,'})\left[\Phi\left(\frac{5}{2},\frac{7}{2};-\frac{9(\alpha'^2+\alpha^2)}{16}|\vec{r}+\vec{r}\,'|^2\right)+\right. \tag{21}$$

$$\left.+\begin{pmatrix}\dfrac{\Phi\left(\frac{3}{2},\frac{7}{2};-\frac{9(\alpha'^2+\alpha^2)}{16}|\vec{r}+\vec{r}\,'|^2\right)}{r_0^2(\alpha'^2+\alpha^2)}\\ \dfrac{3\sqrt{\pi}\,\Phi\left(1,\frac{7}{2};-\frac{9(\alpha'^2+\alpha^2)}{16}|\vec{r}+\vec{r}\,'|^2\right)}{2r_0^3(\alpha'^2+\alpha^2)^{3/2}}\end{pmatrix}\right],$$

$$T_{12}(\hat{r}) \equiv 6(\hat{r}\cdot\vec{S}_a)(\hat{r}\cdot\vec{S}_b)-2\vec{S}_a\cdot\vec{S}_b\,; \qquad (\hbar=c=1),$$

where the upper row in the brackets corresponds to the case n=1 in the formula (19) and the lower one - to the case n=2; $\Phi(\alpha,\beta,z)$ is a degenerate hypergeometrical function.

The expression (21) allows us to estimate the value of the BB interaction radius. According to the leading exponent for NN interaction radius r_{NN} is obtained $r_{NN}\simeq\frac{1}{\sqrt{2}\,\alpha}=r_N$, i.e., the nucleon radius. Other factors in (21) will increase this value. To exhibit the exact behaviour of the effective potential at large distances and at small separations where the existence of a repulsive core is expected one needs to factorized this potential. At present this problem is being studied.

The matrix elements which are not shown in (16) correspond to the excitation of the baryon radial and orbital degrees of freedom. For their construction suitable wave functions of baryon excited states have to be taken instead of the ground state wave function (20).

It's easy to generalize the potential (21) in the case when the baryon ground state wave function is taken as a superposition of the functions of the type (20). Such wave functions provide the better description of the baryon form-factors at a wide range of transfer momentum. It is clear that such generalization will increase NN interaction radius r_{NN} and will give the better description of the potential behaviour at large distances because of an extention of the two baryon distribution functions ovelap.

As it was mentioned above we have not constrained the baryon center mass motion at constructing the potential (21), in particular we have not used an adiabatic approximation - have not fixed a relative distance between the oluster during the interaction. The adiabatic approximation in the case of BB interaction is not founded, because a mass of a quark being exchanged between baryons must be of the order 1/3 of baryon mass at non-relativistic treatment of the problem. As a result the quark exchange leads to the substential alternation of the two baryon relative radius-vector $\vec{r}$. Nevertheless we consider such approximation which leads to a local potential allowing us to make some qualitative conclusions on a potential behaviour. This approximation had been used in ref./7/ for the construction of short range NN interaction on the basis of the nucleon quark model.

For a construction of a BB interaction potential in the adiabatic approximation some approximation has to be done in the exact expression (10). Namely, $P^r_{ij}\ \vec{r}$ must be replaced by $\vec{r}$ in a wave function $R^L_{ab}(P^r_{ij}\ \vec{r})$ and $\vec{R}_a$ and $\vec{R}_b$ must be replaced by $1/2\ \vec{Z}$ and $-1/2\ \vec{Z}$, correspondingly ($\vec{Z}$ is a fixed relative radius-vector of baryons) in the wave functions Φ_a and Φ_b. As a result for elements of the local potential we have

$$V^{k\ell,ij}_{B'B',BB}(\vec{r}) = \Big\{ \int d\vec{R}_{k\ell}\, d\vec{r}_{k\ell}\, d\vec{x}\, d\vec{y}\, \Phi^*_{a'}(\vec{r}_1,\vec{r}_2,\vec{r}_3;\tfrac{1}{2}\vec{Z})\Phi^*_{b'}(\vec{r}_4,\vec{r}_5,\vec{r}_6;-\tfrac{1}{2}\vec{Z}) \times f^{s'I',sI}_{k\ell;ij}(\vec{r}_{k\ell},\vec{P}_{k\ell})\, P^r_{ij}\left[\Phi_a(\vec{r}_1,\vec{r}_2,\vec{r}_3;\tfrac{1}{2}\vec{Z})\Phi_b(\vec{r}_4,\vec{r}_5,\vec{r}_6;-\tfrac{1}{2}\vec{Z})\right]_{\vec{Z}=\vec{r}} . \tag{22}$$

By using the expression (20) for the wave function we have obtained

$$V_{NN}(\vec{r}) = g^2\left(\tfrac{2}{3}\right)^{13/2}\frac{\alpha}{3\sqrt{\pi}}\, e^{-\frac{4\alpha^2r^2}{3}}\sum_{i=1}^{3} a_i \Big\{ -9\Phi(\tfrac{1}{2},\tfrac{3}{2};-b_i^2\alpha^2r^2) + \begin{pmatrix} \frac{9\cdot 2}{3(\alpha r_0)^2}\Phi(-\tfrac{1}{2},\tfrac{3}{2};-b_i^2\alpha^2r^2) \\ \frac{9\sqrt{\pi}}{16(\alpha r_0)^3}\Phi(-1,\tfrac{3}{2};-b_i^2\alpha^2r^2) \end{pmatrix} + \frac{3}{4}(9+\tfrac{8}{3}\vec{S}_a\cdot\vec{S}_b)\frac{\alpha^2}{m^2}\left[e^{-b_i^2\alpha^2r^2} + \begin{pmatrix} \frac{2\Phi(\frac{1}{2},\frac{3}{2};-b_i^2\alpha^2r^2)}{3(\alpha r_0)^2} \\ \frac{2\sqrt{\pi}}{\sqrt{6}(\alpha r_0)^3} \end{pmatrix}\right] + \tfrac{1}{5}T_{12}(\hat{r})\frac{\alpha^2}{m^2}\alpha^2r^2B_i^2\left[\Phi(\tfrac{5}{2},\tfrac{7}{2};-b_i^2\alpha^2r^2) + \begin{pmatrix} \frac{2}{3(\alpha r_0)^2}\Phi(\tfrac{3}{2},\tfrac{7}{2};-b_i^2\alpha^2r^2) \\ \frac{2\sqrt{\pi}}{\sqrt{6}(\alpha r_0)^3}\Phi(1,\tfrac{7}{2};-b_i^2\alpha^2r^2) \end{pmatrix}\right] , \tag{23}$$

$$a_1 = a_3 = 1; \quad a_2 = -2; \quad b_1^2 = \tfrac{1}{6}; \quad b_2^2 = \tfrac{25}{24}, \quad b_3 = \tfrac{8}{3} .$$

for NN interaction. Here upper and lower rows in the brackets have the same meaning as in the formula (21).

We should like to point out that the qq potential which corresponds to a square confinement does not contribute into the spin-spin part of the NN interaction. According to ref./7/ in which only this term is taken into account, the main contribution in the NN potential at small distances comes from the spin-spin part. The difference between these two results is connected solely with the main assumption we have used, that the interaction between the clusters proceeds via the exchange of quarks with the same colour (see formulae (13') and (14)).

Keeping in maind the importance of the derivation of the NN interaction repulsive core it is interesting to look at the short range behaviour of the potential (23). It is easy to see that up to the first order of r^2 we have

$$V_{NN}(r) \underset{r\to 0}{\sim} \Big\{ \left[\frac{\alpha^2r^2}{4} + \begin{pmatrix} \frac{2\alpha^2r^2}{3(\alpha r_0)^2} \\ \frac{5\sqrt{2\pi}\,\alpha^2r^2}{4\sqrt{3}(\alpha r_0)^3} \end{pmatrix}\right] + \frac{3\alpha^2}{4m^2}(9+\tfrac{8}{3}\vec{S}_a\cdot\vec{S}_b)\left[-\frac{3\alpha^2r^2}{4} + \begin{pmatrix} \frac{\alpha^2r^2}{6(\alpha r_0)^2} \\ 0 \end{pmatrix}\right] + \frac{3\alpha^2r^2\alpha^2}{20m^2}T_{12}(\hat{r})\left[1 + \begin{pmatrix} \frac{2}{3(\alpha r_0)^2} \\ \frac{\sqrt{2\pi}}{\sqrt{3}(\alpha r_0)^3} \end{pmatrix}\right]\Big\} . \tag{24}$$

According to the formula (16) the expressions (23) and (24) must be multiplied by a coefficient a_N and taking its values from table I we come to the conclusion that for the odd states (I=0, S=1 or I=1, S=0) the central and tenzor part give the attraction at short distances; as to the spin-spin part it is allways repulsive at n=2 in (19), and it is repulsive at n=1 only for the value of the linear confinement radius $r_0 < 2/3\ r_N$. For even states (I=0, S=0 or I=1, S=1) the situation is opposite.

Note that if we estimate the NN interaction radius by the leading exponent in the expression (23) we get that r_{NN} is of order $\sqrt{3/2}\ r_N$.

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