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Фрагментация глубоколежащих дырочных состояний
в изотопах олова

В недавних экспериментах по (d,t) -реакциям на изотопах олова во всех изучавшихся ядрах были обнаружены резонансно-подобные структуры при энергиях возбуждения 5-7 МэВ, связанные с возбуждением глубоких дырочных состояний $1g_{9/2}$, $2p_{1/2}$, $2p_{3/2}$. В области пика исчерпывается 15-25% полной силы состояния $1g_{9/2}$. В работе теоретически рассчитана фрагментация глубоких дырочных состояний в изотопах олова и проведено сравнение с экспериментом и другими теоретическими расчетами. Расчет выполнен в рамках квазичастично-фононной модели ядра, где фрагментация дырочных состояний является результатом взаимодействия с колебательными и более сложными неколлективными состояниями. Получено, что в области пика сосредоточено 45% силы состояния $1g_{9/2}$, что лучше согласуется с экспериментом, чем у других авторов. Главную роль в фрагментации глубоких дырочных состояний играет взаимодействие с квадрупольными и октупольными колебаниями.

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Fragmentation of the Deep-Lying Hole States
in the Tin Isotopes

The fragmentation of the deep-lying hole neutron states $1g_{9/2}$, $2p_{1/2}$ and $2p_{3/2}$ in the odd tin isotopes is calculated in a quasiparticle-phonon formalism. It is shown that 45% of the total state strength is exhausted around maximum of the strength function of the state $1g_{9/2}$. The main role in the fragmentation of hole states is attributed to the interaction with the quadrupole and octupole vibrations which determines the gross structure of the strength function. The interaction with other phonons spreads the strength of the one-quasiparticle state without changing essentially the behaviour of the strength function.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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The aim of the present paper is to study the fragmentation of the deep-lying neutron hole states in the odd tin isotopes within the semimicroscopic quasiparticle-phonon nuclear model [1,2].

The detection of the resonance-like structures in the pick-up cross sections in different nuclei explains interest in this problem. The first observation of this type have been made while studying the (p,d) -reaction at $E_p = 52.55$ *MeV* on the Mo isotopes [3]. The most extensive and detailed experimental data are on the (d,t) -reaction at several dozen of *MeV* on the Sn isotopes [4,5]. Here at an excitation energy of 5-7 *MeV* a peak with width of about 2 *MeV* has been observed in different isotopes. A DWBA analysis of the angular distributions and the forward-peaked character of the angular distributions allow one to assert that the peak is due to the excitation of the deep-lying neutron hole states $1g_{9/2}$ and 2p. By the estimates of different authors from 15% to 25% of the total strength of the state $1g_{9/2}$ is in the region of peak. The fact that a considerable part of the total strength of the deep-lying hole state is in the energy region with width lesser than the excitation energy is the most unexpected result of these experiments.

From the theoretical point of view it is clear that the fragmentation of single-particle states is due to the interaction of the single-particle motion with other types of nuclear motion and, first of all, with the collective vibrations. This is obvious already from the low-lying excitations. With increasing excitation energy the fragmentation becomes stronger and the state structure more complex. But the quantitative estimates can be obtained only within a certain nuclear model. The fragmentation of the neutron states $1g_{9/2}$ and $2p_{1/2}$, $2p_{3/2}$ in the $^{111-123}Sn$ isotopes has been calculated in paper [6]. However, the calculated spreading of the state $1g_{9/2}$ turned out to be weaker than in the experiment. About 75% of the total state strength is around the distribution maximum as compared to 25% of the experimentally detected.

The model Hamiltonian of the quasiparticle-phonon nuclear model includes besides the average field potential and pairing interactions the factorized multipole and spin-multipole forces which generate in the doubly even nuclei the phonon states with different angular moments and parities. The major part of the model parameters is determined by the experimental data on the low-lying nuclear states. The interaction force of the one-quasiparticle and phonon motion in an odd nucleus is not a new parameter and is determined by the structure of the single-particle spectrum and strength parameters given above (for details, see refs. [1,2]). In recent years the quasiparticle-phonon model is successfully used for the study of nuclear states at intermediate and high excitation energies [8-14]. In particular, the s- and p-wave neutron strength functions in spherical [12] and deformed [13] nuclei have been calculated within this model. The basic initial assumptions of this paper coincide with those of refs. [12,13] The only modification of our paper is inclusion of the isovector part of the long-range effective forces to the Hamiltonian. This causes the change of the matrix element of the quasiparticle-phonon H_{qph} interaction. If before (see formula (3) of ref. [12], or formulae (5) and (6) of ref. [14])

$$\langle \alpha_{JM} \| H_{qph} \| [\alpha^+_{jm} Q^+_{\lambda\mu i}]_{JM} \rangle \equiv \Gamma(Jj\lambda i) =$$

$$= \left(\frac{2\lambda+1}{2J+1}\right)^{1/2} \frac{f^{\lambda}_{Jj} v^{(\mp)}_{Jj}}{\sqrt{Y(\lambda i)}}$$

$$Y(\lambda i) = Y_n(\lambda i) + Y_p(\lambda i),$$

then after the inclusion of the isovector long-range forces

$$\Gamma(Jj\lambda i) = \left(\frac{2\lambda+1}{2J+1}\right)^{1/2} \frac{f^{\lambda}_{Jj} v^{(\mp)}_{Jj}}{\sqrt{\mathcal{Y}_n(\lambda i)}},$$

$$\mathcal{Y}_n(\lambda i) = Y_n(\lambda i) + Y_p(\lambda i) \left\{ \frac{\frac{\kappa_0^{(\lambda)} - \kappa_1^{(\lambda)}}{2\lambda+1} X_n^{\lambda i}(\omega)}{1 - \frac{\kappa_0^{(\lambda)} + \kappa_1^{(\lambda)}}{2\lambda+1} X_p^{\lambda i}(\omega)} \right\}^2 \quad (1)$$

where

$$Y_n(\lambda i) = \frac{1}{2}\frac{\partial}{\partial\omega} X_n^{\lambda i}(\omega)\Big|_{\omega=\omega_{\lambda i}} = \frac{1}{2}\frac{\partial}{\partial\omega} \sum^{n}_{j_1 j_2} \frac{(f^{\lambda}_{j_1 j_2} u^{(\pm)}_{j_1 j_2})^2 \epsilon_{j_1 j_2}}{\epsilon^2_{j_1 j_2} - \omega^2}\Big|_{\omega=\omega_{\lambda i}}$$

In the above formulae the following notation is used: f^{λ}_{Jj} is the reduced single-particle matrix element of the multipole or spin-multipole operator

$$u^{(\pm)}_{Jj} = u_J v_j \pm u_j v_J \qquad v^{(\mp)}_{Jj} = u_J u_j \mp v_J v_j$$

the upper sign corresponds to the multipole phonon $Q^+_{\lambda\mu i}$, and the lower one to the spin-multipole phonon; u, v - are the Bogolubov transformation coefficients. $\kappa_0^{(\lambda)}$, $\kappa_1^{(\lambda)}$ are the isoscalar and isovector constants of the long-range forces of multipolarity λ. $\omega_{\lambda i}$, $\epsilon_{j_1 j_2}$ are the energies of the one-phonon and two-quasiparticle states. The indices n(p) show that the given quantity corresponds to neutrons (protons). For instance, the summation runs over the neutron (proton) single-particle spectrum.

As in papers [12,13] the most complicated components of the wave function of an odd nucleus are the "quasiparticle plus phonon" components:

$$\Psi_\nu(JM) = \{ C_{J\nu}\, \alpha^+_{JM} + \sum_{\lambda i j} D^{\lambda i}_j (J\nu) [\alpha^+_{jm} Q^+_{\lambda\mu i}]_{JM} \} \Psi_0 . \quad (2)$$

The equations for the energy $\eta_{J\nu}$ of the state $\Psi_\nu(JM)$ and the expressions for the coefficients $C_{J\nu}$ and $D^{\lambda i}_j$ are well known [1,2,12-13]. The values of η, C and D can easily be found by the numerical methods at modern computers even if the dimensions of the single-particle basis are large. However, it is clear that the wave function (2) cannot pretend to a detailed description of the strength distribution of the one-quasiparticle state over the nuclear spectrum due to its simplicity and only the quantity $\sum_{\nu \in \Delta_E} C^2_{J\nu}$ is correctly described in a certain energy interval. However, the average quantity $\overline{C^2}$ on the interval is rather sensitive to the size of this interval, especially where the state density of the "quasiparticle plus phonon" is not large. The strength function of refs.[21,22] is more stable to the size of the interval and to the model parameters, as though smearing the value of $C_{J\nu}$ for one state over a certain energy region with width Δ. By definition

$$S_J(\eta) = \sum_\nu C^2_{J\nu} \frac{1}{2\pi} \frac{\Delta}{(\eta - \eta_{J\nu})^2 + \Delta^2/4}.$$

Only for the sake of simplicity the weighting function is taken in the Lorentzian form. Besides a large stability to the model parameters and to the size of the averaging interval, the strength function has one more advantage, i.e., it can be calculated directly without solving the secular equation and finding the structure of each state $\Psi_\nu(JM)$[21]. This is valid for more complex than (2) wave functions of odd nuclei and also for the wave functions of doubly even nuclei[2], what makes the method of strength functions a powerful tool in calculatiing, for instance, the widths of the giant resonances and radiative strength functions[9,11]. However, in theory there appears a new parameter Δ, the choice of which is caused, as a rule, by technical reasons. In spherical nuclei for Δ we have used the values from the interval 0.4-2.0 *MeV*. A possible physical interpretation of Δ will be given in what follows. It should be mentioned that in these calculations the use of the method of strength functions is not necessary. It mainly provides a clear representation of the results.

Now let us discuss the parameters of the model Hamiltonian. The single-particle energies and wave functions are calculated with the Saxon-Woods potential, the parameters of which being chosen on the basis of the results of papers[15] and given in the *Table*. In calculations we have used the code REDMEL which realizes the numerical method of solving the Schrödinger equation suggested in paper[16]. The code REDMEL allows one to calculate correctly the position and wave functions of the quasibound states with a relatively small width. The *Table* gives also the pairing interaction constants G_N and G_Z which have been determined by the experimental values of pairing energies[17]. The constants of the dipole forces are chosen according to the position of the giant dipole resonance in the even Sn isotopes[18] and from the condition that $\omega_{11}=0$, what allows one to exclude with good accuracy the influence of the spurious state which is caused by the breaking of the translational invariance[19]. The constants of the quadrupole and octupole interactions are chosen so as to fit the theoretical values of the 2^+_1 and 3^-_1 level energies to the experimental ones. Theoretical energies have been calculated taking into account the admixtures of the two-phonon components to their structure[20]. The RPA values of the 2^+_1 and 3^-_1 level energies in the even tin isotopes at these values of the constants are by 200-300 *keV* larger than the experimental ones. The values of $\kappa_0^{(\lambda)}$ with $\lambda > 3$ are taken less by a factor of 1.5 than it follows from the estimates of monograph[22]. At these values of the constants the low-lying collective states with $\lambda > 3$ do not appear. The isovector constants $\kappa_1^{(\lambda)}$ have been calculated by using the relation

$$\kappa_1^{(\lambda)} = -\kappa_0^{(\lambda)} \times 0.2(2\lambda + 3). \qquad (3)$$

The relation (3) is a renormalized version of the estimates given by O.Bohr and B.Mottelson[22]. The renormalization has been performed so that the ratio of the constants of the quadrupole forces $\kappa_1^{(2)}/\kappa_0^{(2)}$ could provide a correct position of the isovector quadrupole resonance. The value of this ratio from ref.[22] is too large. For the spin-multipole constants we have used the formula[10,23]

$$\kappa_1^{(\lambda)} = -4\pi \frac{28}{\langle r^{\lambda-1}\rangle^2} \frac{MeV}{fm^{2\lambda-2}}; \qquad \kappa_0^{(\lambda)} = 0.$$

It should be noted that the above values of the constants have already been used for the calculation of the radiative strength functions and the E1 and M1 -resonances in spherical doubly even nuclei, a satisfactory agreement with experiment being obtained /9/. The strength functions have been calculated at $\Delta = 0.5$ *MeV*.

The fragmentation of the hole states $1g_{9/2}$, $2p_{1/2}$ and $2p_{3/2}$ has been calculated for the $^{111-121}Sn$ isotopes within the excitation energies from 0 to 10 *MeV*. For the $^{111-115}Sn$ isotopes we have used the single particle scheme of levels corresponding to the mass number A =115, whereas for the $^{117-121}Sn$ isotopes the calculations have been performed with scheme A=121 (see the *Table*).

Before the discussion of our results let us consider the role of the effective force isovector components for the correct description of the state structure of odd nuclei since in doubly even nuclei it is very essential at high excitation energies /24/. *Figure 1* shows the results of calculation of the strength function $S_{9/2}(\eta)$ (upper diagram) and of the values of C_ν^2 (lower diagram) for the states $1g_{9/2}$ in ^{119}Sn. The solid line is the results obtained with $\kappa_1^{(2)} = \kappa_1^{(3)} = 0$ and the dashed line is the results with $\kappa_1^{(2)} = -1.4\kappa_0^{(2)}$ and $\kappa_1^{(3)} = -1.8\kappa_0^{(3)}$ (other forces have not been taken into account in this calculation). The distribution of values of C_ν^2 over different states Ψ_ν

Table

The parameters of the Saxon-Woods potential and the pairing interactions constants

A	N,Z	r_0 fm	V_0 MeV	æ fm^2	α fm^{-1}	$G_{N,Z}$ MeV
115	N=67	1.28	44.28	0.413	1.613	0.134
	Z=49	1.24	54.5	0.347	1.587	0.182
121	N=71	1.28	43.2	0.413	1.613	0.122
	Z=51	1.24	59.9	0.346	1.587	0.136

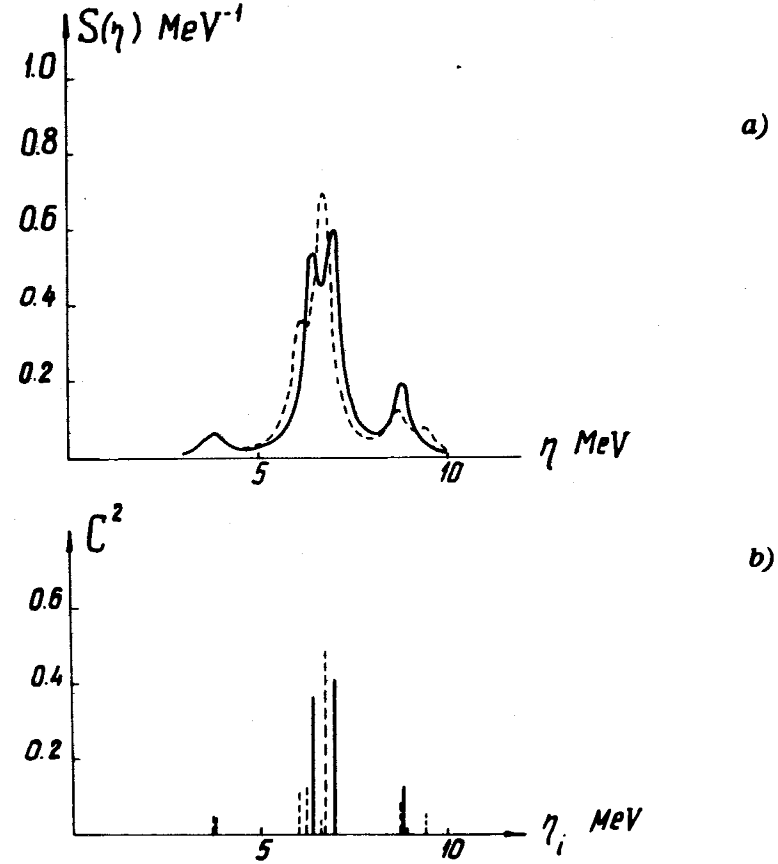

Fig. 1. Calculation of the fragmentation of the $1g_{9/2}$ *state in* ^{119}Sn. *a) the strength function* $S_{9/2}(\eta)$, *b) contribution of one-quasiparticle components* $C^2_{9/2\nu}$ *to individual states* $\Psi_\nu(9/2^+)$. *The solid line is the calculation with* $\kappa_1^{(2)} = \kappa_1^{(3)} = 0$. *The dashed line is the calculation with* $\kappa_1^{(2)} = -1.4\kappa_0^{(2)}$ *and* $\kappa_1^{(3)} = -1.8\kappa_0^{(3)}$.

changes noticeably. One may say that the state $1g_{9/2}$ is more spread when the isovector forces are taken into account in the Hamiltonian. The difference between the strength functions calculated in both the cases is weaker and the quantity $\int_{\Delta E} S(\eta)d\eta$ changes slightly at $\Delta E \sim$

~1 *MeV*. This is in agreement with the results of the analysis performed for the s-wave neutron strength functions[25].

Now let us discuss the results on the fragmentation of hole states. The function $S_{9/2}(\eta)$ in the nuclei $^{111,115,119,121}Sn$ is shown in *Fig. 2*. The left column (*Fig. 2a*) of the diagram represents the results of calculation taking into account the quadrupole and octupole phonons only and the right column (*Fig. 2b*) represents the results obtained with the whole phonon space. Let us analyse *Figure 2b*. The maximum of the function $S_{9/2}(\eta)$ is shifted down with respect to the position of the one-quasiparticle level $1g_{9/2}$ by several

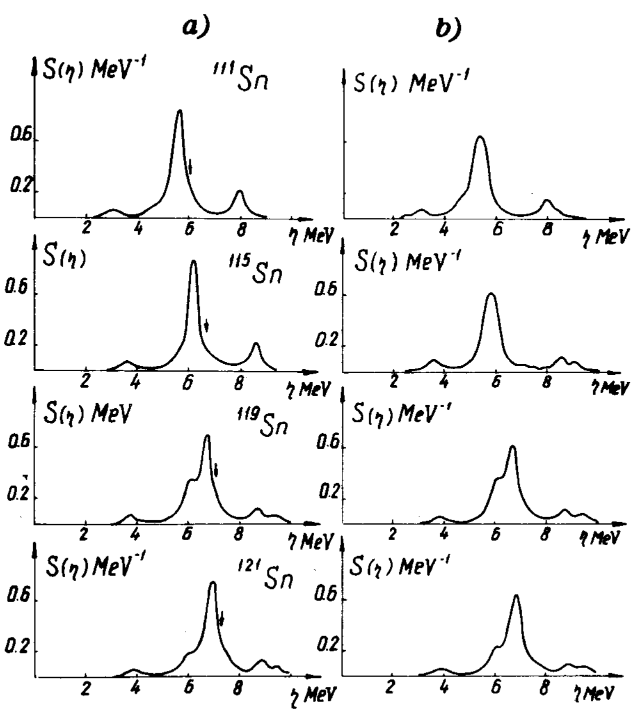

Fig. 2. The strength function $S_{9/2}(\eta)$ *in the* Sn *isotopes. a) calculation with the quadrupole and octupole phonons only; b) the calculation with large phonon space; The arrow in Fig. a) shows the position of the one-quasiparticle* $1g_{9/2}$ *state.*

hundred *keV* but nevertheless it is higher than it was established experimentally. Of course, this is due to the single particle scheme of levels. In the region of 0-10 *MeV* 90% of the total state $1g_{9/2}$ strength is exhausted, from 43% to 49% being exhausted in different isotopes in the interval $\Delta E = 1$ *MeV* whose center coincides with the maximum of $S_{9/2}(\eta)$. The latter value decreases with increasing A. In ^{115}Sn 45% of the state $1g_{9/2}$ strength is exhausted around the peak, whereas in the experiment about 25% of the strength is detected in the peak. There are 15 states with $J^{\pi} = 9/2^{+}$ in the same interval, 9 of them having $C_{\nu}^{2} > 1\%$ (what can be compared with 8 states $9/2^{+}$ identified experimentally in the region of peak[5]). It should be noted that the full width at the half maximum (FWHM) of the main peak in all the nuclei exceeds 500 *keV* (i.e., the value of Δ) and ranges from 600 to 750 *keV*. This is due to the influence of the phonons of higher momenta ($\lambda > 3$), mainly of multipole type (i.e., with $I^{\pi} = 4^{+}, 5^{-}, 6^{+}$). Indeed, the curves $S_{9/2}(\eta)$ in *Fig. 2a* have the same structure as in *Fig. 2b*, but the FWHM of their peaks is of 500 *keV* (i.e., is equal to the parameter Δ). In *Fig. 2a* more than 50% of the state strength is concentrated in the interval $\Delta E = 1$ *MeV* around maximum of the function $S_{9/2}(\eta)$. From the two cases considered above, one can see that the exclusion of phonons with $\lambda > 3$ results in a sharp decrease in the number of states around the peak and in the increase of concentration of the $1g_{9/2}$ state strength in individual states. This is exhibited most clearly in ^{119}Sn, where in the interval 6.2-7.2 *MeV* only 3 states instead of 11 remain after the exclusion of phonons of higher momenta. These three states, however, concentrate 70% of the whole state $1g_{9/2}$ strength whereas 11 states only 52%; the maximal value of C_{ν}^{2} for individual states increases from 48% to 59%. Thus, one may assert that the interaction with collective quadrupole and octupole phonons is the most important in the strength distribution of hole states determining the gross structure of the strength function $S_{J}(\eta)$. The role of the rest phonons is to additionally smear the picture formed by the interaction with the quadrupole and octupole vibrations.

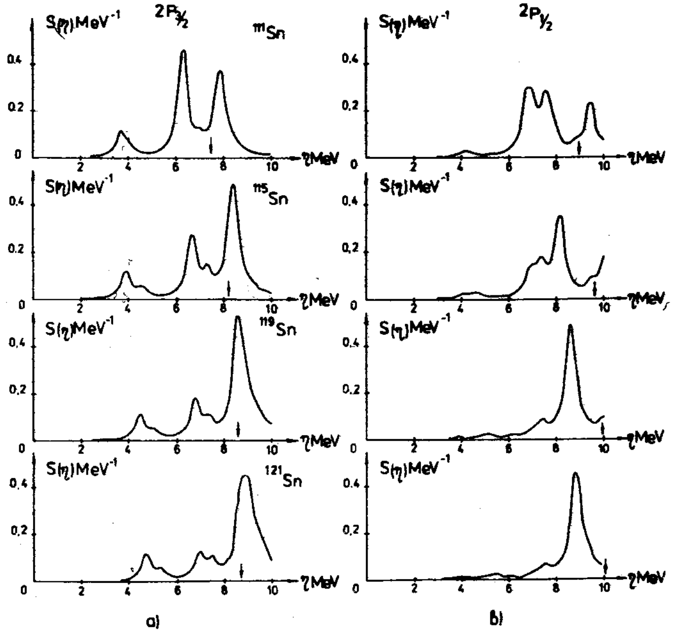

Fig. 3. The strength functions of the $2p_{1/2}$ *and* $2p_{3/2}$ *states in the* Sn *isotopes. Calculation with large phonon space. a)* $S_{3/2}(\eta)$ *; b)* $S_{1/2}(\eta)$. *The arrows in the figure show the position of the one-quasiparticle levels a)* $2p_{3/2}$ *; b)* $2p_{1/2}$.

Figure 3 shows the strength functions $S_{1/2}(\eta)$ and $S_{3/2}(\eta)$ describing the fragmentation of 2p hole states. One can see that the 2p states are spread stronger than the $1g_{9/2}$ state. In the excitation energy region studied a much lesser part of the strength of these states is exhausted than for $1g_{9/2}$, what is, first of all, due to their larger excitation energy. In the interval of 0-10 *MeV* about 80% of the $2p_{3/2}$ state strength is exhausted (this value changes slightly when passing from one nucleus to another). At the same time for the $2p_{1/2}$ state the value of $\int_0^{10} S_{1/2}(\eta)\, d\eta$ decreases rapidly with increasing A (from 70% in ^{111}Sn to 50% in ^{121}Sn). It is shown in ref. /5/ that about 11% of the $2p_{1/2}$ and $2p_{3/2}$ state strength is exhausted in the interval from 4.8 to

6.1 *MeV* in ^{115}Sn. Our calculation gives the value of about 13% what is in agreement with experiment.

The above-mentioned results show that the fragmentation of the $1g_{9/2}$ hole state in our calculation is much stronger than in paper /6/, though it is not yet sufficient for a satisfactory description of experiment. Since the model used in paper /6/ is very close to the quasiparticle-phonon model by its main assumptions and the parameters of both the models can be directly compared, it is useful to compare more thoroughly these models and to study the reason of deviation of the results.

The model of paper /6/ takes exactly into account the interaction of quasiparticles with the quadrupole and octupole vibrations only. However, the strength of this interaction is a free parameter independent of either the characteristics of the single-particle spectrum or the characteristics of the phonons. Such quantities as the strength of interaction of quasiparticles with phonons, the phonon energy, the one-quasiparticle state energies (including the $1g_{9/2}$ state) were independent fitting parameters. The latter were chosen so as to describe satisfactorily the low-lying states in the odd tin isotopes and the position of maximum of the $1g_{9/2}$ state strength. The comparison of the results thus obtained with our values shows that the density of the one-quasiparticle neutron states in ref. /6/ is noticeably smaller than that obtained with the Saxon-Woods potential with the parameters from the *Table*; the used quadrupole and octupole phonon energies in ref. /6/ are always lower than the experimental ones by 50-600 *keV* (in our RPA calculations, these values are higher than the experimental ones by 200-300 *keV*). The last fact has two consequences. First, having taken into account the anharmonic corrections, our model gives correct values of the 2^+_1 and 3^-_1 level energies in the even tin isotopes, whereas the model of ref. /6/ gives much lower values of these energies. Second, since the decrease of the phonon energies results in their large collectiveness, the interaction of quasuparticles with phonons in ref. /6/ should be stronger than in ours. But the things are different. Assuming that our superfluid factors $v^{(-)}_{j_1 j_2}$ and multipole single-particle matrix elements and those of paper /6/ are the same, the difference in the matrix elements of the quasiparticle-phonon interaction means the difference in the values of $B_\lambda (m\omega_0/h)^{\lambda/2}$ and $\mathcal{Y}_n^{-1/2}(\lambda 1)$ (see formula (1) of this paper and (2.2) of ref. /6/). In Sn isotopes they vary in the limits:

$$B_2 (\frac{m\omega_0}{h}) = 0.03 \text{ keV} \cdot \text{fm}^{-2}, \quad \mathcal{Y}_n^{-1/2}(21) = 0.05 \div 0.07 \text{ keV} \cdot \text{fm}^{-2}$$

$$B_3 (\frac{m\omega_0}{h})^{3/2} = 0.0055 \text{ keV} \cdot \text{fm}^{-3}, \quad \mathcal{Y}_n^{-1/2}(31) = 0.008 \div 0.007 \text{ keV} \cdot \text{fm}^{-3}.$$

Thus, our interaction is stronger by a factor of 1.5-2. Besides, the quantity $B_\lambda (m\omega_0/h)^{\lambda/2}$ decreases slowly with increasing A as $A^{-\lambda/6}$, while $\mathcal{Y}_n^{-1/2}(\lambda 1)$ changes irregularly from an isotope to isotope as it depends on the specific features of the single-particle scheme, the position of the neutron chemical potential and the phonon energy $\omega_{\lambda 1}$. In the light Sn isotopes out interaction is stronger and maximum is achieved in ^{113}Sn.

The interaction with noncollective three- and five-particle states has been taken into account in paper /6/ by introducing an artificial width of the states by the weight Lorentz function *. This allows one to describe the state strength distribution by the smooth function which is analogous to our strength function. The width of the Lorentz curve $\Gamma(E)$ is calculated by formula

$$\Gamma(E) = 2\pi\rho(E) < v^2 > , \qquad (4)$$

where $\rho(E)$ is the density of non-collective states at energy E, which has been calculated in combinatorial way; $<v^2>$ is the mean square of the matrix element of the one-quasiparticle-non-collective states interaction. Thus, the value of $\Gamma(E)$ in contrast with Δ is not a parameter and changes with changing excitation energy. Hence, the parameter Δ can be interpreted as a characteristic taking effectively into account the interaction with the "quasiparticle plus two phonons" states. Let us compare $\Gamma(E)$ with the parameter Δ. At the energy of an order of the one-quasiparticle state $1g_{9/2}$ energy, the value of $\Gamma(E)$, calculated by (4) with the data from ref. /6/ is of 130-180 *keV*, but in ref. /6/ this value was added by an arbitrary value of 150 *keV*. Therefore, in the calculations the value of $\Gamma(E) \simeq 300$ *keV* was used. It is much lesser than our value of $\Delta = 500$ *keV*. We think the value of $\Gamma(E)$ to be too low for the following reasons. First, the underestimated density of one-quasiparticle states in an unfilled shell should result in a too low value of $\rho(E)$. Second, in our calculations the interaction with phonons $\lambda > 3$ results in increasing Δ by 150-250 *keV*, what in the order of magnitude coincides with $\Gamma(E)$ (4), but we did not include the five-quasiparticle states which has been taken into account in $\Gamma(E)$. From the aforesaid one may conclude that in ref. /6/ the strength of quasiparticle-phonon interaction is underestimated, what causes a weak spreading of the hole states.

The results of our paper show that even a simplest version of the quasiparticle-phonon model describes qualitatively the fragmentation of the deep hole states. An additional spreading may be expected after introducing the "quasiparticle plus two phonons" components into the wave function. These components should be taken into account to describe correctly the radiative strength functions in odd nuclei. But the problem becomes more complicated. There are as yet only first calculations along this line /26/.

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* The same method was used in paper /7/, only the weight function was chosen in a Gaussian form.

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