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FIELD THEORY OF LARGE AMPLITUDE COLLECTIVE MOTION. A SCHEMATIC MODEL

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FIELD THEORY OF LARGE AMPLITUDE COLLECTIVE MOTION. A SCHEMATIC MODEL

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Полевой подход к описанию коллективных движений с большими амплитудами

При помощи функционального интегрирования выведены уравнения коллективного движения с большими амплитудами для схематической модели. Исходная теория для фермионов переформулируется как теория коллективного бозевского поля. Классические уравнения движения для коллективного поля совпадают с уравнениями Хартри-Фока, зависящими от времени. Их классическое решение мы квантуем при помощи обобщенного ВКБ-метода.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Field Theory of Large Amplitude Collective Motion. A Schematic Model

By using path integral methods we derive the equation for large amplitude collective motion for a schematic two-level model. The original fermion theory is reformulated in terms of a collective (Bose) field. The classical equation of motion for the collective field coincides with the time-dependent Hartree-Fock equation. Its classical solution is quantized by means of the field-theoretical generalization of the WKB method.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

In recent years there is an increasing interest in the study of large amplitude collective motion. So far mainly two approaches have been applied for the description of large amplitude collective motion: the (adiabatic) time-dependent Hartree-Fock [(A) TDHF] method [1] and the generator coordinate method [2]. In refs. [3,4] a rather orthodox formulation of the TDHF method has been proposed in order to go beyond the adiabatic approximation. Its application to the Lipkin model has shown that non-adiabatic effects are important in order to get a positive definite mass parameter [4]. But if one goes beyond the adiabatic approximation, the problem of quantizing the time-dependent classical solution becomes serious, especially, when the TDHF equations can be solved only approximately or numerically, as it is usually the case. Then there is no general receipt how to quantize the TDHF solutions. It is the aim of the present paper to propose a new formulation of the TDHF approximation, which allows to overcome this difficulty. Using path integral techniques [5] we develop a field theoretic approach to large amplitude collective motion. On the classical level our approach is equivalent to the TDHF theory. However, the quantization of the classical solutions can be performed in a unique way, even if these are only approximately known. For simplicity we demonstrate the approach at a schematic model.

Let us mention that path integrals are a very useful tool for the quantization of classical solutions of non-linear field equations ("solitons") [6,12] where they provide us with a natural generalization of semi-classical methods known from ordinary quantum mechanics to field theory.

Recently [7-10] path integrals in collective variables (called hereafter functional approach) have been applied also to the study of nuclear many-body systems. Using path integral techniques we could give a strict foundation of the so-called nuclear field theory [9,10]. (For an introduction into the nuclear field theory see also ref. [14]).

In the functional approach to collective phenomena of interacting Fermi systems one replaces the original fermion theory by an equivalent theory in a collective Bose field. In ref. [9] it was shown that in the static approximation the equation of motion of this collective field reduces to the ordinary (static) Hartree-Fock equation. Further, as we have recently [11] shown for the Lipkin model, within the functional approach the TDHF equation is obtained as the classical equation of motion for the collective field. In the present paper we apply the functional approach to a similar schematic two-level model, the TDHF equation of which is exactly solvable. The derivation of the TDHF equation in the functional approach will give us some new insights in the spirit of the TDHF approximation from the many-body theoretical point of view. The quantization of the TDHF solutions is then performed by means of the field theoretic generalization of the WKB method developed by Dashen, Hasslacher and Neveu [11] (cited hereafter as DHN). This method has proved very successful in the quantization of soliton solutions. In the present paper we shall demonstrate that the generalized WKB method is also very powerful in the quantization of large amplitude collective motion.

The paper is organized as follows: In sec. 2 we present the derivation of the TDHF equation within the functional approach. We start with the generating functional for the fermion Green functions. The two-body interaction is linearized by means of a collective Bose field. The classical equation of motion for this collective field coincides with the TDHF equation. In Sec. 3 the classical

solutions are quantized by means of the generalized WKB method of DHN and are compared with the exact solution. Some concluding remarks are given in Sec. 4.

2. FUNCTIONAL DERIVATION OF THE TDHF EQUATION

In the following we present a field-theoretic derivation of the TDHF equation for a schematic model, whereat we shall investigate the essence of the TDHF approximation from the many-body theoretical point of view.

The model under consideration consists of $N = 2\Omega$ fermions distributed over two single particle levels, each with degeneracy 2Ω, and interacting via a schematic monopole particle-hole force:

$$H = H_{sp} + H_{tb} , \qquad (1a)$$

where

$$H_{sp} = \epsilon \sum_{\sigma = \pm 1} \sigma \sum_{m=1}^{2\Omega} a^+_{m\sigma} a_{m\sigma} \qquad (1b)$$

and

$$H_{tb} = -V A^+ A \qquad (1c)$$

with

$$A^+ = \sum_{m=1}^{2\Omega} a^+_{m1} a_{m-1} . \qquad (1d)$$

The index σ takes the value $\sigma = 1$ and $\sigma = -1$ for the upper and the lower level, respectively, while the index m labels the degenerate states within each level. The level spacing is 2ϵ, and V denotes the interaction strength.

The generating functional for the fermion Green function is given by the following path integral

$$Z[q, q^+] = \mathfrak{N} \int D a D a^+ e^{i \int dt \{ \mathcal{L}(t) + q^+(t) a(t) + a^+(t) q(t) \}} \qquad (2)$$

where

$$\mathcal{L}(t) = \sum_{m,\sigma} a^+_{m\sigma}(t)(i\partial_t - \sigma\epsilon)a_{m\sigma}(t) + VA^+(t)A(t) \tag{3}$$

is the Lagrangian corresponding to the Hamiltonian (1), q, q^+ are external fermion sources and $\mathcal{N}$ is an irrelevant normalization constant. In order to perform the integration over the fermion variables a^+, a the interaction is linearized by means of a (complex) Bose field ψ

$$e^{iVA^+A} = \mathcal{N}_1 \int D\psi D\psi^+ e^{i\{-\frac{1}{V}\psi^+\psi + \psi^+A + A^+\psi\}}.$$

Introducing the spinor representation

$$a^+_m = (a^+_{m1}, a_{m-1}), \quad q^+_m = (q^+_{m1}, q_{m-1})$$

the generating functional takes the form

$$Z[q,q^+] = \mathcal{N} \int DaDa^+ D\psi D\psi^+ \times$$

$$\times \exp[i\{\int dt\,dt' \sum_m a^+_m(t)G^{-1}[\psi](t,t')a_m(t') +$$

$$+ \int dt[-\frac{1}{V}\psi^+(t)\psi(t) + \sum_m(q^+_m(t)a_m(t) + a^+_m(t)q_m(t))]\}], \tag{5}$$

where G defined by

$$G^{-1}[\psi](t,t') = \begin{pmatrix} i\partial_t - \epsilon & \psi \\ \psi^+ & i\partial_t + \epsilon \end{pmatrix} \delta(t-t') \tag{6}$$

represents the fermion Green function in the "external" field ψ. Integration over the fermion variables yields

$$Z[q,q^+] = \mathcal{N}' \int D\psi D\psi^+ \exp[i\{S[\psi] - \int dt\,dt' q^+(t)G(tt')q(t')\}]. \tag{7}$$

where the new effective action

$$S[\psi] = \int dt\, \mathcal{L}_c[\psi](t) \tag{8a}$$

is given by the collective Lagrangian

$$\mathcal{L}_c[\psi](t) = -\frac{1}{V}\psi^+(t)\psi(t) - i2\Omega\, \mathrm{tr}(\log G^{-1}[\psi])(t,t). \tag{8b}$$

From the least action principle, $\delta S/\delta\psi^+ = 0$, we find the equation of motion of the collective field $\psi(t)$ to be

$$\psi(t) = -2\Omega V\, \mathrm{tr}[\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} G(t,t')\,|_{t=t'-0}]. \tag{9}$$

To solve this equation, we must find the explicit expression for the s.p. Green function $G[\psi](t,t')$ in terms of the collective field ψ by inverting eq. (6). The s.p. Green function can be represented as

$$G[\psi](t,t') = U(t) \begin{pmatrix} -i\theta(t-t') & 0 \\ 0 & i\theta(t'-t) \end{pmatrix} U^{-1}(t'), \tag{10}$$

where $U(t)$ is a unitary matrix which must satisfy

$$[i\partial_t U(t)]U^{-1}(t) = \epsilon\tau_3 - (\psi_1\tau_1 - \psi_2\tau_2),\quad \psi = \psi_1 + i\psi_2, \tag{11}$$

(τ_i denotes the Pauli spin matrices). In solving this equation we can proceed as in ref. [7] and parametrize $U(t)$ in terms of Euler angles

$$U(t) = e^{i\frac{\alpha}{2}\tau_3} e^{i\frac{\beta}{2}\tau_2} e^{i\frac{\gamma}{2}\tau_3}. \tag{12}$$

With this ansatz one finds from eq. (11) after straightforward algebraic calculation the following set of differential equations

$$\begin{aligned} -\dot\beta \sin\alpha + \dot\gamma \sin\beta\cos\alpha &= -2\psi_1, \\ \dot\beta\cos\alpha + \dot\gamma\sin\beta\sin\alpha &= -2\psi_2, \\ \dot\alpha + \dot\gamma\cos\beta &= -2\epsilon. \end{aligned} \tag{13}$$

Defining

$$\phi = \sin\beta\, e^{i\alpha} \tag{14}$$

and eliminating the angle γ which is redundant, eqs. (13) can be cast into the form

$$\frac{\dot\phi + 2i\epsilon\phi}{\sqrt{1 - \phi^+\phi}} = i2\psi. \tag{15}$$

Inserting now the explicit expression for G as given by eqs. (10), (12) into the equation of motion (9) and using the definition (14) we get the relation

$$\psi = \Omega V \phi. \tag{16}$$

Combining the last equation with eq. (15) we, finally, obtain the classical equation of motion for the collective field ψ in the form

$$i\dot{\psi} = \psi\left[2\epsilon - \rho\sqrt{1 - \left(\frac{2}{\rho}\right)^2 \psi^+ \psi}\right], \quad \rho = 2\Omega V. \tag{17}$$

It is highly non-linear and coincides with the TDHF equation.

Let us now see how the TDHF approximation is related to the conventional small amplitude perturbation expansion [7,8,9], where the effective action $S[\psi]$ (8) is expanded around a static solution of the equation of motion (9), $\psi_0 = \text{const}^*$. Solving eq. (9) in the static approximation corresponds to the ordinary Hartree-Fock (HF) approximation (see ref. [9]). Defining $\psi = \psi_0 + \psi'$ and introducing the s.p. Green function in the HF approximation $G_0 = G[\psi = \psi_0]$, the expansion of $S[\psi]$ in powers of ψ' yields

$$S[\psi] = \text{const} + S_2[\psi'] + \sum_{n>2} L_n[\psi'], \tag{18}$$

where

$$L_n[\psi'] = -i2\Omega \int dt\, \mathrm{tr}\left\{ \frac{(-)^{n+1}}{n} \left(G \begin{pmatrix} 0 & \psi' \\ \psi'^+ & 0 \end{pmatrix}\right)^n (t,t)\right\} \tag{19}$$

represents a closed fermion loop emitting and absorbing n collective lines, ψ and ψ^+, respectively. The terms linear in ψ' vanish due to eq. (9). The quadratic part of the action,

$$S_2[\psi] = -\frac{1}{2}\int dt\, \psi'^+(t)\psi'(t) + L_2[\psi'],$$

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* The static solution ψ_0 can, of course, also be obtained from eq. (17) with $\dot{\psi} = 0$.

serves for the definition of the free quanta. From the least action principle $\delta S_2[\psi]/\delta\psi' = 0$ one finds the free equation of motion which coincides with the Bethe-Salpeter equation in the RPA (see ref. [9]). By canonical quantization of the collective field ψ' from the free equation of motion one obtains the common RPA phonons as the bare quanta of the theory [9,16]. The higher order fermion loops $L_n[\psi']$, $n > 2$ represent self-interactions of the collective field, i.e., correlations between the RPA phonons which produce anharmonicities in the collective motion. After canonical quantization the small amplitude expansion (18) defines a perturbation theory, which, when performed up to infinite order (what is usually impossible), would yield the exact results of the original fermion theory.

Now it is clear what the TDHF approximation means in the language of the conventional small amplitude perturbation expansion: The TDHF approximation, which is derived from the *full* classical action $S[\psi]$ (see eqs. (8,18)) via the least action principle, corresponds to a summation of the closed fermion loops up to infinite order. However, the summation is performed in the classical approximation, i.e., without having, before the summation is carried out, quantized the harmonic, small amplitude oscillations. In this sense the TDHF method yields a classical description.

Although highly non-linear, the classical equation of motion (18) can be solved analytically. In polar coordinates

$$\psi = R e^{i\theta}.$$

Its solution reads

$$\psi = \frac{\rho}{2}\chi e^{-i\omega(\chi)t + i\theta_0}. \tag{20}$$

It describes oscillations of the collective field ψ with a frequency

$$\omega(\chi) = 2\epsilon - \rho\sqrt{1-\chi^2}, \tag{21}$$

which depends on the amplitude χ. (θ_0 is the initial phase of $\tilde{\psi}$). The constancy of the amplitude χ of the classical solution (20) is a consequence of the fact that our model interaction (1c) does not change the number of particle-hole excitations. To make this clear, we note that from the least action principle applied to eq. (5) the fermion structure of the composite field ψ follows to be $\psi = VA$. Therefore the amplitude of the classical solution $\tilde{\psi}$ (20) can be related to the expectation value of the operator A^+A, which measures the number of particle-hole excitations, via

$$\chi^2 = \frac{1}{\Omega^2} |\langle A^+ A \rangle|^* . \tag{22}$$

In the classical solution (20) of the equation of motion (17) the amplitude χ of the collective field ψ is still an undetermined parameter. The quantization of the classical solution, which is performed in the next section, will fix this amplitude.

3. *QUANTIZATION OF THE CLASSICAL SOLUTION*

In the following the classical solution (20) is quantized using the field-theoretic generalization of the WKB method developed by Dashen, Hasslacher and Neveu [12]. For a review of this method see ref. [13]. In the DHN method, the energy eigenvalues are obtained from the poles of the propagator

$$G(E) = \mathrm{tr}\frac{1}{H - E} = i\,\mathrm{tr}\int_0^\infty dT\, e^{i(E-H)T} =$$

$$= i\int_0^T dT\, e^{iET} \int d\psi_0 K(\psi_0, \psi_0),^{**} \tag{23}$$

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*Strictly speaking, the amplitude χ is related to the expectation value of the operator A in a "coherent" state such that $\psi = V\langle A\rangle$.

** Here $\int d\psi_0$ means an ordinary integral over the complex variable $\psi_0 = \psi(t_0)$ (t_0 - fixed).

where the Feynman propagation kernel

$$K(\psi'', \psi') = \langle \psi'' | e^{-iHT} | \psi' \rangle$$

is conveniently represented as a path integral

$$K(\psi'', \psi') = \int D\psi D\psi^* e^{iS(\psi'', \psi', T)} \tag{24}$$

with

$$S(\psi'', \psi', T) = \int_0^T dT\, \mathcal{L}_c[\psi](t) \tag{25}$$

being the action along the trajectory $\psi(t)$ connecting the initial and final field configuration, $\psi(t=0) = \psi'$ and $\psi(t=T) = \psi''$, respectively. In the semi-classical quantization given by the WKB method one considers small fluctuations along the classical path (in our case in the θ degree of freedom), i.e., the functional integral (24) is calculated in the stationary phase approximation which is known to be

$$K(\psi'', \psi') = e^{iS_{cl}(\psi'', \psi', T)} \left| \frac{\partial^2 S_{cl}(\psi'', \psi', T)}{\partial\theta''\partial\theta'} \right|^{1/2} , \tag{26}$$

where S_{cl} is the action along the classical path $\psi(t) = \tilde{\psi}(t)$ (20) between the field configurations ψ' and ψ''. The second factor contains the quantum fluctuations. The trace in eq. (23) implies a summation over all classical orbits with the same T differing in the fundamental periods (per cycle)

$$\tau = \frac{T}{m} = \frac{2\pi}{\omega(\chi)} , \quad m = 1, 2, 3, \dots , \tag{27}$$

where m is the number of traverse of the same orbit. The classical action, $S_{cl}(T)$, which is evaluated in the appendix, is given by

$$S_{cl}(T) = \mathcal{L}_c[\tilde{\psi}]T \tag{28a}$$

with

$$\mathcal{L}_c[\tilde{\psi}] = \tilde{\mathcal{L}}_c(\chi) = \frac{\rho^2}{4V}[1 - \sqrt{1-\chi^2}\,]^2 \tag{28b}$$

Clearly we have $S_{cl}(T = m\tau) = mS_{cl}(\tau)$, and for the propagator $G(E)$ (23) we get

$$G(E) = i \sum_m \int d\chi \int_0^\infty m\, d\tau\, A(\chi)\delta(\tau - \frac{2\pi}{\omega(\chi)}) \times$$

$$\times \exp[i(E\tau + S_{cl}(\tau))m]\Delta, \qquad (29)$$

where

$$\Delta = \int_0^{2\pi} d\theta \left|\left[\frac{\partial^2 S_{cl}(\theta'',\theta',T)}{\partial\theta''\partial\theta'}\right]_{\theta'=\theta''=\theta}\right|^{1/2}, \qquad (30)$$

$$A(\chi) = \left| \frac{d(\frac{2\pi}{\omega(\chi)})}{d\chi} \right|,$$

and the δ-function enforces the constraint (27). We can now immediately integrate over τ. The remaining integration over χ is performed in the stationary phase approximation. The phase

$$W(\chi) = (E + \vec{\mathcal{L}}_c(\chi))\tau(\chi) \qquad (31)$$

becomes stationary for

$$W' = \frac{dW(\chi)}{d\chi} = 0. \qquad (32)$$

The last relation defines χ as a function of the energy E. The result of the χ integration is then

$$G(E) = i \sum \left(\frac{2\pi i}{W'' m}\right)^{1/2} m\, e^{imW} \cdot A \cdot \Delta, \qquad (33)$$

where we have to put for χ the value obtained from the stationary phase condition (32), $\chi = \chi(E)$.

What remains to be done is to calculate the quantity Δ defined by eq. (30). For this we need the change of the classical action when varying the initial and final phase θ' and θ'', respectively, for a fixed T (for

a closed periodic orbit we have $\theta' = \theta''$). Of course, variation of the initial and final phase for fixed T will also change the amplitude $\chi \to \bar{\chi}$. The classical motion on an orbit with this new amplitude $\bar{\chi}$ has a fundamental period (per cycle) $\bar{\tau}$ given by eq. (27)

$$\bar{\tau} = \frac{2\pi}{\omega(\bar{\chi})},$$

which is connected with the fundamental period τ of the closed orbit ($\theta' = \theta''$) via

$$2\pi T = 2\pi m\tau = (2\pi m + \theta'' - \theta')\bar{\tau}. \qquad (34)$$

With the last relation we find from eq. (28) for the action of the orbit with slightly different initial and final phase ($\theta' \neq \theta''$) and amplitude χ

$$S_{cl}(\theta'',\theta',T) = \mathcal{L}_c(\bar{\chi}(\bar{\tau}))T =$$

$$= \frac{2\pi m\tau}{4V}\left[\rho - 2\epsilon + \frac{\theta'' - \theta' + 2\pi m}{\tau}\right]^2. \qquad (35)$$

Hence the factor Λ (30) is given by

$$\Lambda = 2\pi \left| \frac{-\omega(\chi)}{2Vm} \right|^{1/2}. \qquad (36)$$

Inserting this value for Λ in eq. (33) we are left with a geometric series. Summing up this series, what yields

$$G(E) = i2\pi A\left[\frac{2\pi i|\omega(\chi)|}{2|V|W''}\right]^{1/2} \frac{e^{iW}}{1 - e^{iW}}, \qquad (37)$$

we find that the propagator $G(E)$ has poles at energies $E = E_n$ which satisfy

$$W(E_n) = 2\pi n, \qquad n - \text{integer} \qquad (38)$$

With $W(E) = W(\chi(E))$ defined by eqs. (31), (32) the quantization condition (38) takes the form

$$\chi_n^2 = (\chi(E_n))^2 = 1 - (1 - \frac{n}{\Omega})^2. \qquad (39)$$

Finally, inserting this value of χ into the stationary phase condition (32) the quantized energies measured from the energy of the state with $n=0$ follow to be

$$E_n = n\{2\epsilon - \rho(1 - \frac{n}{2\Omega})\}. \tag{40}$$

The quantum number n, obviously, corresponds to the number of fermions excited to the upper level.

A characteristic feature of the semiclassical solution is that the amplitude χ of the collective field ψ (see eq. (20)) is state-dependent. (In contrary to this, in the RPA we have a fixed amplitude $\chi \ll 1$). For the state with $n=0$, which is the ground state if $\rho < 2\epsilon$ (all particles are in the lower level), we have $\chi = 0$. With increasing n the amplitude χ steadialy increase up to it reades its maximal value $\chi = 1$ for $n = \Omega$ (the half of the particles are in the upper level). If n further increases, the amplitude χ decrease and for $n = 2\Omega$ becomes again zero (all particles are in the upper level). This behaviour of χ is easily understood if one considers that the amplitude is related to the expectation value of the operator A^+A defined in eq. (1d), via eq. (22).

Let us now compare the result of the WKB quantization (40) with the exact excitation energies as follow from group theory *:

$$E_n^{exact} = n\{2\epsilon - \rho[1 - \frac{n}{2\Omega}(1 - \frac{1}{n})]\}. \tag{41}$$

The error obtained after semi-classical quantization of the TDHF solutions for the excitation energies is of the order $1/2\Omega$ if the effective coupling strength ρ is considered to be of the order one.

4. SUMMARY AND CONCLUSION

In this paper we have proposed a field-theoretic approach to large amplitude collective motion, which is based on the functional approach and semiclassical quantization. It has been shown within a schematic model that the classical equation of motion of the functional approach is equivalent to the TDHF equation. The derivation of the TDHF equation within the functional approach exhibits the spirit of the TDHF approximation. It makes clear that the TDHF approximation implies summing up all closed loop diagrams in the classical approximation. Quantization of the classical solutions by means of the field-theoretic generalization of the WKB method has given a fairly good agreement with the exact energy eigenvalues for an arbitrary large interaction strength. This includes also such situations where in the static Hartree-Fock picture the system undergoes a phase transition to a "deformed" ground state/8/.

The fairly good agreement obtained here makes the functional approach together with the WKB quantization a very promising method for the study of large amplitude collective motion, for instance in very soft nuclei. It suggests further investigations of more realistic models within the here proposed approach.

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* Our model Hamiltonian (1) can be expressed by the generators of the SU(2) group.

APPENDIX

Calculation of the Classical Action

The effective action $S[\psi]$ is defined by eq. (8). We have to calculate the Lagrange function (8b), $\mathcal{L}_c[\psi]$, for the classical solution $\psi = \tilde{\psi}$ given by eq. (20). The first term of $\mathcal{L}_c[\psi]$ is trivial. The second term can be calculated by expanding the logarithm as in eq. (18). This yields

$$-i\,tr(\log G^{-1}[\tilde{\psi}])(t,t') = -i\,tr(\log G_0^{-1})(t,t') + g(t,t'),$$

where *

$$g(t,t') = i\sum_k \frac{1}{k}(\lambda F)^k(t,t') \tag{A.1}$$

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*Multiplication is understood here in the functional sense what implies integration over intermediate times.

with

$$\lambda = (\rho\chi)^2,$$

$$F(t,t') = \int dt_1\, G_+(t t_1) e^{-i\omega t_1} G_-(t_1, t') e^{i\omega t'} ,$$

and

$$G_\pm(t,t') = \mp i\theta(\pm[t-t'])e^{\pm i\epsilon(t-t')}$$

are the unperturbed s.p. Green functions. The function $F(t,t')$ depends only on $t-t'$, thus it is convenient to Fourier transform eq. (A.1)

$$g(t t') = \int \frac{d\Omega}{2\pi} e^{-i\Omega(t-t')} e(\Omega)$$

yielding

$$g(\Omega) = i \sum_{k=1}^{\infty} \frac{1}{k}(\lambda F(\Omega))^k .$$

Differentiation with respect to λ transforms the sum into a geometric series, and we get

$$g(\Omega) = i \int_0^\lambda d\lambda' \frac{F(\Omega)}{1 - \lambda' F(\Omega)} .$$

Reversing the Fourier transformation and performing then the λ-integration yields

$$g(t,t')|_{t=t'} = \sqrt{(\epsilon - \frac{\omega}{2})^2 + \rho^2\chi^2} .$$

Omitting the (infinite) constant, $-itr(\log G_0^{-1})(t,t)$, we find for the classical action

$$S_{cl}(T) = \mathcal{L}_c[\vec{\psi}]T = \frac{\rho^2}{4V}(1-\sqrt{1-\chi^2})^2 T. \qquad (A.2)$$

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