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Метод связанных каналов для упругого и неупругого $\pi - {}^4He$ рассеяния

Уравнения метода связанных каналов решаются в данной работе в К-матричном подходе. Это позволяет одновременно рассмотреть упругое и неупругое рассеяние π-мезонов на атомных ядрах. Были вычислены полные упругое и неупругое рассеяния для нескольких каналов реакции ${}^4He(\pi^-,\pi^-)\,{}^4He^*$. Обсуждается также реакция зарядового обмена на ядре ${}^4He$.

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Coupled Channel Method for the $\pi - {}^4He$ Elastic and Inelastic Scattering

Equations of the coupled channel method written in the momentum space are solved in the K-matrix approach. In this way we are able to consider simultaneously the elastic and inelastic pion-nuclear reactions. Calculated total elastic and total inelastic cross sections are given for several outgoing channels of the reaction ${}^4He(\pi^-,\pi^-)\,{}^4He^*$. The single-charge-exchange reaction on ${}^4He$ is discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

Specific forms of the nuclear response in interactions with different intermediate energy (of order $10^2 MeV$) projectiles seem to be extremely rich. With the present advance in running the high-intensity accelerators the pion-nuclear scattering experiments are becoming valuable companion of the classical (e,e'), (p,p'), $({}^4He,{}^4He')$ reactions. The spin-isospin structure of the underlying pion-nucleon amplitude which depends on the pion initial $\vec{k}$ and final $\vec{k}'$ momenta

$$\langle \vec{k}'|f(\omega)|\vec{k}\rangle = A_0 + (\vec{t}\cdot\vec{\tau})A_T + i\frac{\vec{\sigma}\cdot[\vec{k}\times\vec{k}']}{kk'}(A_S + (\vec{t}\cdot\vec{\tau})A_{ST}) \qquad (1)$$

is of intermediate complexity between the "simple" and well-known electromagnetic interaction responsible for the (e,e') scattering and, e.g., N–N elementary amplitude with a more difficult spin-isospin structure. Perspectives of the pion-nuclear inelastic scattering investigations seem to be promising enough to warrant a careful extension of the theories already developed for the pion-nucleus elastic process.

The elastic and inelastic scattering of pions are usually considered separately. The optical model which disregards the virtual nuclear excitations (the so-called coherent scattering approximation, for a review see ref. /1/) is the formalism most frequently used for the description of the π–A elastic scattering; the inelastic process leading to the creation of nuclear excited states was rather successfully analyzed /2/ within the distorted wave impulse approximation (DWIA). In the cases of suspected two-step or multi-step reaction mechanisms (e.g., ${}^{13}_{6}C(\pi^+,\pi^0){}^{13}_{7}N$, ref.[3] or ${}^{16}O(\pi,\pi'){}^{16}O^*(0^+)$, ref.[4] reactions) limited attempts (2-3 channels considered) were made to use the method of coupled channels for the description of inelastic π-A scattering.

In this work we made the first step towards a systematic and simultaneous investigation of both the elastic and inelastic processes including also the charge exchange reactions. A system of coupled integral equations in the momentum space is obtained in Section 2. We solve this system (assuming that the pions propagate on-shell only) in the case of ${}^4He$ for the elastic and several inelastic amplitudes. The underlying nuclear model is described in Section 3. Two points are worth mentioning about the results which are presented and discussed in Section 4. First, we have got a certain insight into the excitation mechanisms of ${}^4He$ by the low energy ($E < 100$ *MeV*) pions. Secondly, we were able to estimate the effects of the virtual nuclear excitations on the elastic cross section in the same energy interval.

2. *FORMALISM OF THE COUPLED CHANNELS*

Starting from the Watson multiple scattering theory, we have obtained the following system of integral equations

$$\langle \vec{Q}'n|F(E)|0\vec{Q}_0\rangle = \langle \vec{Q}'n|v(E)|0\vec{Q}_0\rangle -$$

$$-\frac{1}{(2\pi)^3}\sum_{m=0}^{\infty}\int \frac{\langle \vec{Q}'n|v(E)|m\vec{Q}''\rangle\langle \vec{Q}''m|F(E)|0\vec{Q}_0\rangle}{E-E_m(Q'')+i\epsilon}\frac{d^3Q''}{\mathfrak{M}(Q'')}, \qquad (2)$$

$n = 0,1,2,\ldots$ for the pion-nuclear amplitudes $F(E)$. Here, $\mathfrak{M}(Q'')$ is the pion-nucleus reduced mass. To shorten the notation we use everywhere $E = E(Q_0)$. The potential

$$\langle \vec{Q}'n|v(E)|m\vec{Q}\rangle = A\langle \vec{Q}'n|Wf(\omega)|m\vec{Q}\rangle \qquad (3)$$

may be simply expressed in terms of the π-N amplitude $f(\omega)$, eq. (1), where the functions $A_i \equiv A_i(\vec{k},\vec{k}',\omega)$, $i = 0, S, T, ST$ are constructed from the experimental π-N partial amplitudes (we consider $\ell = 0,1,2$ waves and use the phase-shift parametrization due to Salomon [5]) and π-N form factors, which define the off-energy shell extrapolation of the π-N amplitude. When constructing the potential of eq. (3) we follow the prescription of ref.[6] for the relativistic transformation between the π-N and π-nucleus centre-of-mass systems. In this way we end up (in π-nucleus c.m.s.) with the quantities $Wf(\omega)$, $\vec{Q}$, and $\vec{Q}'$ for the amplitude and the pion initial and final momenta, respectively. No phenomenological corrections for the true pion absorption were considered in eq. (3). The Coulomb interaction effects were included[7] in the $\langle 0|v(E)|0\rangle$ term only.

To solve the system (2) we perform the angular decomposition and then, in the exact Green functions

$$\frac{1}{E(Q_0)-E_m(Q'')+i\epsilon} = \frac{P}{E(Q_0)-E_m(Q'')} - i\pi\delta(E(Q_0)-E_m(Q'')) \qquad (4)$$

retain only the δ-function terms. In this way we are left with an algebraic system of equations. The channel momentum Q_m is defined from the condition $E(Q_0) = E_m(Q_m)$. Similar method was used by Charlton and Eisenberg[8] in the study of elastic pion scattering and we extend it to the many channel case. We do not expect the principal value integrals to be negligible (especially at the low energies). From the previous calculations[1,8] it can be concluded that these terms may modify the elastic cross sections by a factor of about 1.5. The main advantage of the model adopted here is its simplicity and the fact that we obtain estimates of the elastic and inelastic cross sections practically independent of the largely unknown off-energy-shell behaviour of the π-N amplitude. As a matter of fact, the δ-function in RHS of eq. (4) ensures the on-shell propagation of pion only.

If the solution of system (2) is known, the cross section for the pion scattering is given by

$$\frac{d\sigma_{n0}}{d\Omega} = \frac{Q_n}{Q_0} |\langle \vec{Q}_n n | F(E) | 0 \vec{Q}_0 \rangle|^2, \qquad (5)$$

where the nuclear states are labelled by n.

There is an alternative approach, how to go beyond the coherent scattering approximation in the investigation of the elastic scattering. Second order optical potential has been constructed by several authors and applied to the $\pi - {}^4He$ scattering /9/. In this frame all virtual excitations are taken into account in the closure approximation. In other words the nuclear states are supposed to be degenerate with energy of the ground state. At the same time the contributions of all three-, four-,... multi-step processes are neglected. Whereas the natural domain of application for the coupled channel formalism is the low-energy region, the second-order optical model seems to be more appropriate for the higher energies.

3. *NUCLEAR EXCITATION CHANNELS OF* ${}^4He$

The ${}^4He$ nucleus is a very suitable object for testing the validity of the coherent scattering approximation. As it can be seen from *fig. 1* there are only a few nuclear states up to the excitation energy of 30 *MeV* /10/. Among them the two 0^- levels do not count since their coupling to the ground state is zero:

$$\langle \vec{Q}', J^{\pi} = 0^- | v(E) | J^{\pi} = 0^+, \vec{Q} \rangle = 0 \qquad (6)$$

in view of the parity and spin conservation.

We have adopted the harmonic oscillator shell model and describe all the odd-parity levels indicated in *fig. 1* as pure 1 particle - 1 hole states. In this model and with the oscillator parameter $a_0 = 1.6$ *fm* a very reasonable energy spectrum was reported earlier /11/. In spite of the fact that the considered excited levels in ${}^4He$ are quasi-bound, their description in terms of the bound state orbitals works usually very well.

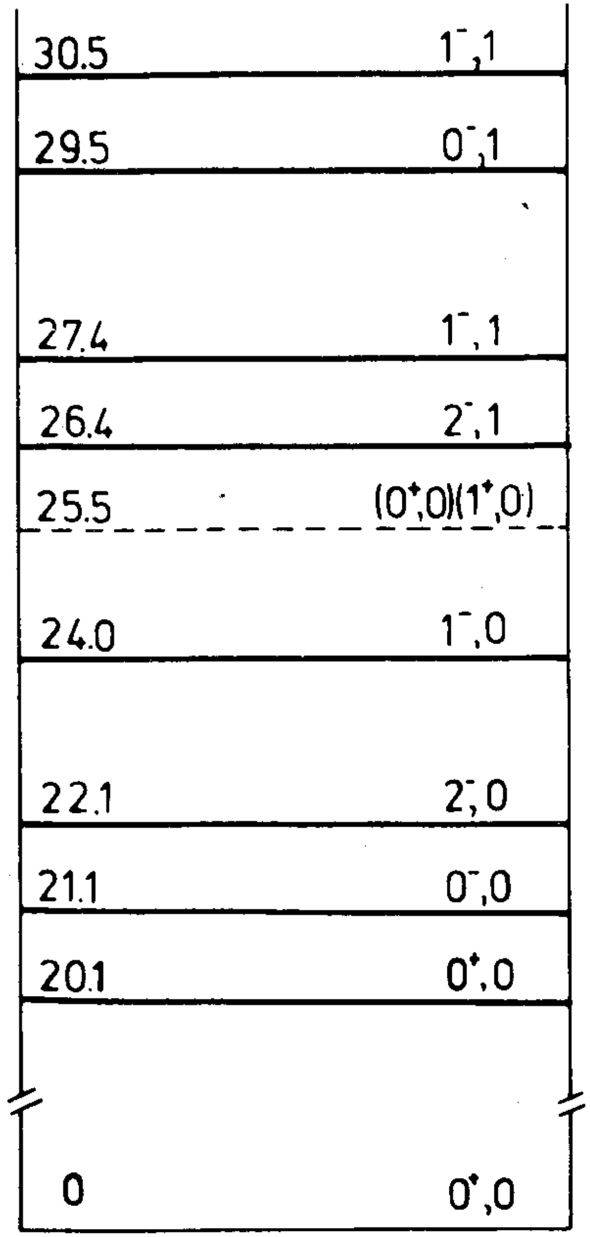

Fig. 1. Level scheme of the ${}^4He$ *nucleus, compiled from refs. 8,9.*

A complete elimination of the centre-of-mass motion spurious effects was performed via the projection technique, which is here (harmonic oscillator orbitals) fully equivalent to the use of internal (e.g., Jacobi) coordinates. Certain amount of configuration mixing due to the nuclear residual interaction may appear for the two $1^-T=1$ states. It is indeed strong in the L-S coupling, it turns, however, into a practically zero-mixing in the jj coupling scheme used here.

We work therefore in the truncated nuclear space n = g.s., $(2^-, 1^-; T=0)$, $(2^-, 1^-_1, 1^-_2; T=1)$; a discussion of the $0^+_2 T=0$ level see below. As for the elastic form factor, we use in the calculation the experimental one, deduced from the elastic $e^- - {}^4He$ scattering /12/, rather than the harmonic oscillator expression.

4. RESULTS AND DISCUSSION

In *Table 1* the total elastic and total inelastic cross sections are displayed for the ${}^4He(\pi^-,\pi^-){}^4He^*$ reaction. The results obtained by solving eq. (2) are compared with the PWIA cross sections. Under PWIA we mean the procedure, where the expressions $\langle \vec{Q}_n\, n\,|v(E)|\, 0\, \vec{Q}_0\rangle$ are

Table 1

Total elastic and inelastic $\sigma(J^\pi T)$ cross sections for the ${}^4He(\pi^-,\pi^-){}^4He^{(*)}$ reaction in mbarn.

E(MeV)	40		51		60		75	
	eq.2	PWIA	eq.2	PWIA	eq.2	PWIA	eq.2	PWIA
σ_{el}	22.36	47.59	32.49	58.75	40.30	69.37	64.10	109.70
$\sigma(2^-,0)$	0.33	0.34	0.95	1.00	1.67	1.83	3.69	4.58
$\sigma(1^-,0)$	0.26	0.27	0.80	0.89	1.43	1.67	3.21	4.30
$\sigma(2^-,1)$	0.03	0.03	0.13	0.13	0.26	0.28	0.72	0.84
$\sigma(1^-,1)$	2.15	2.30	3.73	4.19	4.83	5.65	6.78	8.78

Table 2

Inclusive cross sections (in mbarn) of the $\pi^- + {}^4He$ reaction

E(MeV)	40	51	60	75
		Coupled-channel calculation		
σ_{el}	22.36	32.49	40.30	64.10
σ_{exch}	2.18	3.86	5.09	7.50
$\sigma_{in} = \sum_{n\neq 0} \sigma_{n,0}$	4.95	9.47	13.28	21.90
$\sigma_r = \sigma_{tot} - \sigma_{el}$	35.56	48.51	60.54	94.87
$\sigma_{abs} = \sigma_r - \sigma_{in}$	30.61	39.04	47.26	72.97
σ_{tot}	57.93	80.95	100.83	158.97
		Optical model calculation		
σ^0_{el}	22.42	32.54	40.44	64.68
σ^0_r	30.78	39.59	48.30	76.38
σ^0_{tot}	53.20	72.13	88.73	141.06
		Experimental data /13/		
σ^{exp}_{el}		29.50	36.69	46.78
σ^{exp}_r		33.59	52.30	89.60
σ^{exp}_{tot}		63.09	88.99	136.38

used in eq. (5) instead of the matrix elements of F(E). Cross sections calculated for the two $(1^-,1)$ levels are summed in *Table 1*.

Whereas excitations of the specific channels are given in *Table 1*, some inclusive characteristics of the $\pi^- + {}^4He$ reaction are displayed in *Table 2*. For the sake of comparison we present here the total elastic cross section, too. The total cross section of the (π^-, π^0) reaction is denoted as σ_{exch}. Under σ_{in} we understand the sum of all the total cross sections for inelastic ${}^4He(\pi^-,\pi^-){}^4He^*$

reaction going to the 1p-1h nuclear excited states. Total cross section for the reaction $\pi^- + {}^4He \to$ "anything", total reaction cross section and total inelastic cross section for all channels, which were not taken explicitly into account, are denoted as σ_{tot}, σ_r and σ_{abs}, respectively. The quantities $\sigma^{\circ}_{e\ell}$, σ°_{r} and σ°_{tot} were obtained from eq. (2) neglecting the coupling to the excited states. Finally, $\sigma^{exp}_{e\ell}$, σ^{exp}_{r} and σ^{exp}_{tot} were deduced from the energy dependent phase-shift analysis /13/ of experimental data. A typical statistical error of these quantities is 10%, σ^{exp}_{tot} and consequently σ^{exp}_{r} may, however, be biased by a considerable systematical error (see Binon et al. /13/). In the following, we comment in some detail on the results presented in *Tables 1* and *2*.

4.1. *Elastic Scattering*

From the comparison of $\sigma^{\circ}_{e\ell}$ and $\sigma_{e\ell}$ (see *Table 2*) we may conclude that the elastic cross section is affected very weakly by the virtual nuclear excitations in the energy interval considered. The calculated total elastic cross sections are in reasonable agreement with the experimental data.

The elastic differential cross sections are slightly lowered for the forward and backward angles and the minimum is somewhat filled in when the channel coupling is considered. The changes amount to about 2-5 per cent. It is interesting to note that the same trends are observed in the elastic differential cross sections when the channel coupling is "turned on" as if a usual phenomenological term of the true pion absorption were included in the optical model /14/. Further, PWIA is clearly inadequate for the elastic scattering. Moreover, assuming $F(E) \approx v(E)$ the amplitude $\langle \vec{Q}'0|F(E)|0\vec{Q}\rangle$ badly violates the two-body π-A unitarity condition, whereas there are no troubles with the unitarity if the system (2) is solved (no matter whether the channel coupling is retained or "turned off").

4.2. 1p-1h *Excitations*

From *Table 1* it can be concluded that the cross sections of the inelastic pion scattering increases with energy more rapidly than the elastic one, reaching $\approx$20-25 per cent of the elastic cross section for the pion energies of 60-70 *MeV*. PWIA results become progressively worse with the increasing pion energy. Most strongly are excited the (1^-, 1) levels of ^{4}He, their quantum numbers correspond to the famous giant dipole resonance. The relative weakness of the transition g.s. $\to$ (2^-,1) is specific for the pion inelastic scattering on ^{4}He: since the 2^-, 1 level is a pure $S=1$ state, the above transition is controlled by the term A_{ST} in the amplitude (1). Neglecting all the non-resonant π-N p-waves, it holds $|A_S|^2 \approx |A_T|^2 \approx 4|A_{ST}|^2$.

In the first series of our numerical work we have also studied the possible effects of the positive parity 0^+_2 $T=0$ "breathing mode" level at 20.2 *MeV*. The corresponding wave function was calculated via solution of the hyperspherical harmonics equation by Žofka and Sotona /15/. They have shown that this wave function provides an electric form factor in good agreement with the experimental data. From our results we have seen that inclusion of the $0^+_2 0$ level into the coupled channel π-^{4}He calculation has a minor effect only. The integrated cross section is always ($T_\pi \le 75$ *MeV*) similar to that of the strongly suppressed 2^-1 (26.4 *MeV*) level. We have then omitted the $0^+_2 0$ level from our further computer runs.

4.3. *Charge-Exchange Reaction*

From our calculations we can make some predictions about the pion charge-exchange reaction on ^{4}He . Since it holds $\sigma[{}^4He(\pi^-,\pi^-){}^4He(T=1)] \approx \sigma[{}^4He(\pi^-,\pi^\circ){}^4H(T=1)]$, the inclusive charge-exchange cross section, σ_{exch}, can be easily evaluated as a sum of all isovector transition cross sections. The cross section σ_{exch}, reaching several millibarns for the pion energies considered should be fully accessible for the present day experimental technique. In *fig. 2*, the angular distribution of π° mesons is displayed for the inclusive (π^{-}, π°) reaction. It is interesting to note that almost all π° mesons are going out into the backward hemisphere.

To understand qualitatively the shape of the $^{4}He(\pi^{-},\pi^{\circ})$ angular distribution it is enough to recall that here we cannot expect a coherent-type distribution typical for the charge-exchange process connected with the creation of an isobar-analogue state as, e.g., in the $^{3}He(\pi^{-},\pi^{\circ})$ ^{3}H reaction. In our case the nuclear transition form factors contain the spherical Bessel functions $j_{\ell}(qr)$ with $\ell \neq 0$ only. As a result we have indeed the angular distribution of *Figure 2* suppressed at the small angles.

4.4. Other Excitations

Having obtained the solution of Eq. (2), we calculated σ_{tot} from forward elastic amplitude using the optical theorem. Further, we evaluated the cross sections $\sigma_r = \sigma_{tot} - \sigma_{e\ell}$ and $\sigma_{abs} = \sigma_{tot} - \sigma_{e\ell} - \sigma_{in}$. The last quantity cha-

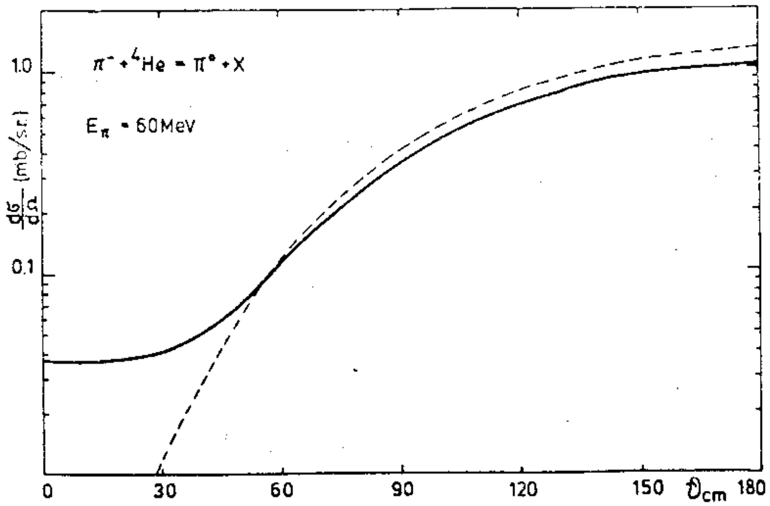

Fig. 2. Differential cross section of π° mesons in the g.s. $\rightarrow (J^{-},1)$ transitions, - - - - = PWIA, ——— = coupled channel calculation.

racterises the escape of pion from the elastic channel to all channels, which have not been taken into account explicitly in solving Eq. (2). In a consistent theory, σ_{abs} would be build up from two contributions: excitations of more complex nuclear states in $^{4}He(\pi^{-},\pi^{-})$ $^{4}He^{*}$ reaction and the true pion absorption, whereas σ_r is the total cross section for all inelastic processes. It can be seen from *Table 2,* that σ_{abs} is comparable with $\sigma_{e\ell}$ exceeding the cross section for 1p-1h excitations considerably. The calculated σ_{tot} does not agree very well with σ_{tot}^{exp}, nevertheless the ratio $\sigma_{e\ell}/\sigma_r$ is much closer to the experimental value than $\sigma_{e\ell}^{\circ}/\sigma_r^{\circ}$. We can conclude that the coupled channel method yields a more realistic balance between elastic and inelastic processes than the simple optical model.

It is tempting to consider σ_{abs} as an approximation of the true pion absorption cross section, since the more complex nuclear states are excited probably very weakly in the $^{4}He(\pi^{-},\pi^{-})$ $^{4}He^{*}$ reaction at our energies. Indeed, we do not include explicitly the true pion absorption into the potentials (3), nevertheless a portion of pion absorption is contained in the experimental π-N amplitudes, which we have used in the calculation. Certainly a word of caution is in order especially in connection with our neglect of the principle value part of the Green function. It is well known that σ_{tot} is much more sensitive to the details of the adopted model than are $\sigma_{e\ell}$, or other cross sections for excitation of specific channels. Therefore, more definite conclusions concerning the physical meaning of σ_{abs} may be drawn only after the complete calculations will be performed.

4.5. Concluding Remarks

As a further development, we are going to consider also the two-particle two-hole nuclear excitations in our coupled channel calculations. We do not think that these configurations might affect strongly the elastic or single-charge-exchange cross sections for the scattering of low-energy pions. However, extending in this way the space of nuclear configurations, we expect to obtain reliable results also at pion energies close to the Δ_{33} resonance. Perhaps even more interesting will be the possibility to calculate the double-charge-exchange cross sections, since the $T=2$ excitations appear in the 2p-2h nuclear subspace. Such more complete calculations, including also the principal value integrals in the Green functions (4), are in progress. We postpone the detailed comparison of the existing $\pi-{}^4He$ experimental data [13,16] with the coupled channel calculations until completion of this work.

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