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О скорости μ–e конверсии в реакции $^{16}O(\mu^-,e^-)^{16}O^*$

В работе рассматривается μ–e конверсия на ядре как следствие несохранения мюонного заряда. Цель работы - выяснить целесообразность исследования μ–e конверсии с последующим возбуждением гигантского резонанса в ядре. В качестве лептонной модели используется обобщение стандартной модели Вайнберга-Салама со смешиванием нейтральных лептонов. Вычисление вероятности возбуждения гигантского резонанса проверяется в рамках оболочечной ядерной модели, содержащей частично-дырочные корреляции. Результаты расчетов для O^{16} показывают, что вероятность процесса с возбуждением ядра составляет 13% от вероятности "упругого" процесса, что находится в соответствии с правилом $1/Z$. Отмечается сильная зависимость скорости μ–e конверсии от величины угла Вайнберга θ_w ($\sim \sin^{-4}\theta_w$). При $\sin^2\theta_w \simeq .22$ относительная вероятность "упругого" μ–e -перехода на два порядка выше относительной вероятности $\mu \to e\gamma$ распада.

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On the Rate of Muon-Electron Conversion in the $^{16}O(\mu^-, e^-)^{16}O^*$ Reaction

To elucidate quantitatively the possible importance of the giant resonance excitations in the $\mu^- - e^-$ conversion on light nuclei we have computed the corresponding transition rates within a large-scale nuclear shell model containing the important 2 particle - 2 hole correlations. The result is $R^{GR}_{\mu e} = 13\%$. For the transition to the low-lying collective 0^+0 (6.05 MeV) level we have obtained model-independent value $R_{\mu e}$ (6.05 MeV) = 0.6%.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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Recently the possibilities of the muon number nonconservation have been vividly discussed in the framework of gauge theories. Presently the experimental upper limits for the branching ratios of the exotic decays allowed in such schemes are pushed down substantially; e.g., the branching ratios of an order of 10^{-9} have been quoted as preliminary results /1/ at the Zürich conference, 1977.

In this connection the muon-electron conversion on nuclei provides an exciting possibility of studying the problem since the corresponding branching ratio $B_{\mu e}$ was estimated /2/ to be actually by two orders of magnitude larger than the branching ratio $B_{\mu e\gamma}$ pertaining to the $\mu \to e\gamma$ decay.

The first analysis of the $\mu^- A \to e^- A$ reaction was performed as early as 1958. Feinberg and Weinberg /3/, starting with several plausible assumptions about the nuclear physics involved, found that the branching ratio $B_{\mu e}$ should reach a maximum for the target nuclei in the region of copper. Indeed the transition leaving the target in its ground state is preferred due to the coherence effect. Actually it should be roughly expected that the rate corresponding to the "inelastic" transition $\mu^- A \to e^- A^*$ (accompanied by the nuclear excitation) goes like Z^{-1} (A^{-1}) in comparison with the branching ratio for the "elastic" transition.

The purpose of this note is to extract more specific numerical results for the $\mu^- \, ^{16}O \to e^- \, ^{16}O^{(*)}$ reaction. The price to be paid is the construction of a nuclear model. We think we were able to eliminate for the most part the uncertainties which are usual on this path. The shell model ^{16}O wave functions used here contain the most important two particle-two hole correlations. Excellent numerical results which they provide for the muon capture and the radiative pion capture rates /4/ may serve as an independent check of the accuracy of this model. Let us stress that the model does not suffer from the usual shortcomings of the simple particle-hole shell-model calculations which usually overestimate strongly the weak and electromagnetic characteristics.

The nuclear model used is described in paper /4/. Some relevant details may also be found in ref. /5/ where this model has been used to estimate the (ν, ν') inelastic scattering cross sections capable to provide the information needed for the reconstruction of the tensor and isospin structure of the neutral weak currents.

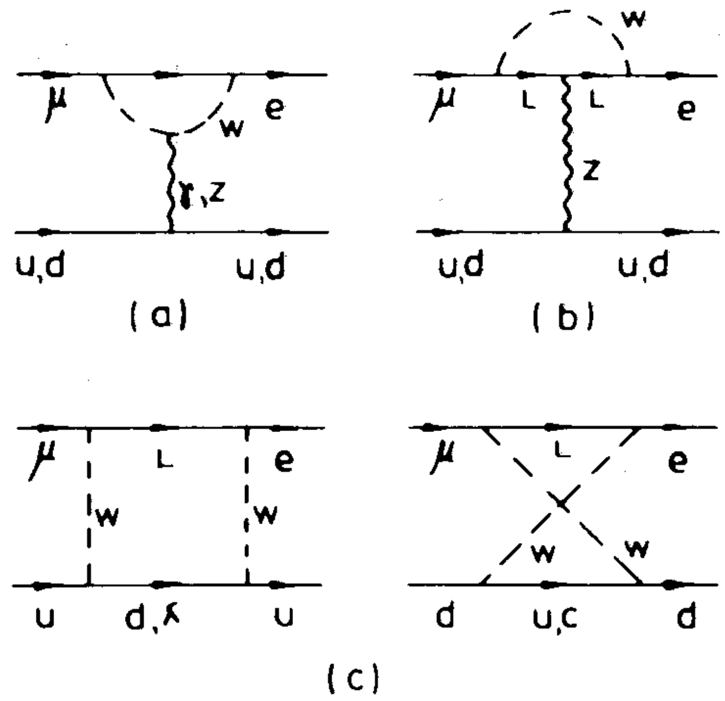
As to leptonic part we consider the SU(2)xU(1) model with the left (L) mixing of neutral leptons /7/

$$\begin{pmatrix} \nu'_1 \\ e \end{pmatrix}_L \begin{pmatrix} \nu'_2 \\ \mu \end{pmatrix}_L \begin{pmatrix} L' \\ \tau \end{pmatrix}_L , \quad \begin{pmatrix} \nu'_1 \\ \nu'_2 \\ L' \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ L \end{pmatrix},$$

where U is the unitary mixing matrix. In this model the Lagrangian of the interaction with intermediate charged boson can be represented in the form

$$\mathcal{L} = \frac{ig}{2\sqrt{2}} \{ \bar{e}\gamma_\alpha (1+\gamma_5) u_{1j} L_j W^- + \bar{\mu}\gamma_\alpha (1+\gamma_5) u_{2j} L'_j W^- + h.c. \}, \quad (1)$$

where as usual $g^2/(8M_W^2) = G/\sqrt{2}$, L_j are the neutral leptons (neutrinos or heavy neutral) and the mixing coefficients u_{ij} satisfy the GIM-mechanism condition $\Sigma_j u_{1j} u_{2j} = 0$. In addition to the realistic model (1) we shall present also some results corresponding to the models with the right /6/ (R) and the both left-right /8/ mixing of the neutral leptons (L-R).



Diagrams of the processes contributing to the muon-electron conversion.

The diagrams contributing to the amplitude of the process under considerations are given in the *figure*. As it has been already shown /6/ the box diagrams (*fig. c*) and Z -exchange diagram (*fig. b*) dominate in the amplitude and one may neglect the contribution of diagrams (*fig. a*). Then the expression for the induced lepton current will reduce to the single term

$$\bar{e}\gamma_\alpha (1+\gamma_5)\mu. \quad (2)$$

The most general form of the single nucleon operator of the nuclear current can be written as

$$\bar{N}'(f_1 \gamma_\alpha + f_2 \gamma_\alpha \gamma_5 + f_3 \sigma_{\alpha\beta} q_\beta) N, \quad (3)$$

where q is the four momentum transfer, and form factors f_i depend on the choice of the lepton model.

For the quarks we supposed the standard SU(2)xU(1) Weinberg-Salam model. We calculated the form factors for u- and d-quarks for the considered lepton models conserving only the leading logarithmic term

$$f_1(u) = \frac{8}{3} K \sin^2\theta_W, \qquad f_2(u) = 0,$$

$$f_1(d) = K(5 - \frac{4}{3}\sin^2\theta_W), \qquad f_2(d) = 5K,$$

$$K = \frac{G}{32\sqrt{2}} \frac{m_\mu^2}{\pi^2 \sin^2\theta_W} u_{1L} u_{2L}^* \frac{m^2}{M_W^2} \ln\frac{m^2}{M_W^2}. \tag{4}$$

For models with right and right-left mixing we have

$$f_1(u) = K(3 + \frac{8}{3}\sin^2\theta_W), \qquad f_2(u) = 3K,$$

$$f_2(d) = K(2 - \frac{4}{3}\sin^2\theta_W), \qquad f_2(d) = 2K, \tag{5}$$

where m is the mass of a heavy neutral lepton. The term with f_3 in these models vanishes. The resulting expressions for the nuclear matrix elements in the non-relativistic limit are given in *Appendix*.

It appears that the dominant inelastic transitions are those leading to the excitation of the dipole and spin-dipole "giant" isovector resonances (GR) in ^{16}O. We have computed the ratio $R_{\mu e}^{GR} = W^{GR}/W^{el}$ of the rates corresponding to the inealastic (GR) and elastic $\mu - e$ conversion processes. Using the Weinberg and Feinberg[3] expression

$$W^{el} = 16 m_\mu Z_{eff}^4 \alpha^5 |f_I|^2 Z |F_{NN}|^2 \tag{6}$$

for the $\mu - e$ elastic conversion rate we obtain the ratio $R_{\mu e}^{GR}$ in the form

$$R_{\mu e}^{GR} = (|f_1| Z |F_{NN}|)^{-2} \int |l_\mu^* J_\mu| d\Omega_{\vec{q}_e}, \tag{7}$$

where l_μ and J_μ are the leptonic and hadronic currents, respectively, and the integration should be performed over the orientations of the final electron impulse. The explicit formulae used for calculation of the ratio $R_{\mu e}^{GR}$ are given in *Appendix*.

The numerical results for the ratios $R_{\mu e}^{GR}(J^\pi T = 1^- 1)$ and $R_{\mu e}^{GR}(J^\pi T = 2^- 1)$ are displayed in *Table 1*. It should be noted that the results depend only very weakly on the choice of the leptonic model and on the value of $\sin^2\theta_W$. Actually both W^{GR} and W^{el} exhibit almost the same (strong, c.f. *Table 2*) dependence on these factors, which is eliminated only on their ratio.

Table 1

Ratio $R_{\mu e}^{GR} = W(\mu\, ^{16}O \to e\, ^{16}O^*) / W(\mu\, ^{16}O \to e\, ^{16}O)$ of the inelastic to elastic scattering rates. Results are given for different nuclear excited states (including the "giant" spin-dipole resonances) separately. For comparison we present in parenthesis also the calculated rates obtained within the simple "one-particle one-hole" configuration mixing model $(J_f^\pi = 1^-)$ of the final nuclear states (third row)

$J_f^\pi T_f$		$R_{\mu e}^{GR} = W(\mu\, ^{16}O \to e\, ^{16}O^*)/W(\mu\, ^{16}O \to e\, ^{16}O)$					$\sum_{i=1}^{5} R_{\mu e}^{GR}(i)$
$1^- 1$	E (MeV)	12.5	17.4	20.8	23.0	26.5	
	R^{GR}	0.004	0.01	0.02	0.03	0.02	0.08
	R^{GR}(ph)	(0.004)	(0.007)	(0.007)	(0.05)	(0.05)	(0.12)
$2^- 1$	E (MeV)	13.1	18.1	18.7	19.7	23.3	
	R^{GR}	0.012	0.001	0.004	0.03	0.008	0.05
						Total$^{(GR)}$	0.13

Table 2

Ratio $B_{\mu e}/B_{\mu e\gamma}$ of relative rates of the $\mu\,^{16}O \to e\,^{16}O$ and $\mu \to e\gamma$ processes is given for the different leptonic model and two values of the Weinberg angle.

Mixing	$\sin^2\theta_W = 0.22$	$\sin^2\theta_W = 3/8$
left	26.0	10.0
right-left	2.2	0.65
right	53.6	16.6

To show the importance of the correct nuclear wave functions we list also the numbers for $R^{GR}_{\mu e}(\Gamma\ 1)$ calculated in a simple 1 particle-1 hole shell model. A strong overestimation ($\approx$50%) of the inelastic nuclear excitations is obtained in this case. From our experience[4] we may, however, conclude that the result of *Table 1*, $R^{GR}_{\mu e}(\text{total}) = 13\%$ is final and can be considered as a realistic estimate of the effect. Though small as expected, it shows that the GR excitations in the conversion experiments on the light nuclei may be of certain importance if precise calculations are to be done.

To elaborate further on the problem of the parameter dependence we have computed also the relative rates for the $\mu\,^{16}O \to e\,^{16}O$ and $\mu \to e\gamma$ processes. Let again $B_{\mu e\gamma}$, be the branching rate of the $\mu \to e\gamma$ decay and $B_{\mu e} = W^{el}/W^{capt}$, where W^{el} is given by eq. (6) and W^{capt} is the usual μ^- capture rate. The ratio $B_{\mu e}/B_{\mu e\gamma}$ exhibits a very strong ($\sim\sin^{-4}\theta_W$) dependence on the numerical value of the Weinberg angle. Some examples are given in *Table 2* where both the current value ($\sin^2\theta = 0.22$) and earlier estimate[3] ($\sin^2\theta_W = 3/8$) have been used. In this calculation $\ln(M_W^2/m^2) \approx 3.5$ for the L-model and $\ln(M_W^2/m^2) = 5$ for the R-L and R models. The difference

of the ratios for the R-L and R models is connected with the strong dependence of the $\mu \to e\gamma$ rate on the choice of the leptonic model.

In *Table 3* we display the rate of the $\mu - e$ conversion which can be compared with the experimental data. The latest upper limit on $B_{\mu e}$ was obtained at SIN[10]. Their result for sulphur is $B^{exp}_{\mu e} < 1.5 \cdot 10^{-10}$.

Table 3

Branching ratio $B_{\mu e} \cdot 10^{10}$ of the μ–e conversion on ^{16}O.

$(u_{1L}u^*_{2L})^2$	Model	m = 1 GeV	m = 1.5 GeV
0.25	L	1.1	5.5
	R, R-L	2.5	11.4
0.10	L	0.4	2.2
	R, R-L	1.0	4.5

The charged boson mass is $M_W = 60$ *GeV*.

Our computations presented in *Table 3* were performed for ^{16}O we have, however, $B_{\mu e}(^{32}S) = 3/2\,B_{\mu e}(^{16}O)$.

In conclusion we would like to argue that the traditional inclination[2,9] in favour of the μ–e conversion search on heavier elements (like copper) is not compulsory: The ratio $B_{\mu e}/B_{\mu e\gamma}$ is actually only twice larger for Cu when compared with the case of ^{16}O. Therefore it cannot be excluded that experimentally favourable light target nuclei could also be suggested. As for our example of ^{16}O we do not discuss here its feasibility as an actual target. Our aim was to consider an example where uncertainties connected with the nuclear structure information can be safely minimized.

Simultaneously we want to call attention to the possible study of an interesting partial transition. In the $\mu - e$ conversion the $J^{\pi}\, T = 0^{+}0$ (6.05 *MeV*) collective level may be excited in ^{16}O. In a calculation performed without reference to the particular nuclear model /5/ , using the experimental value of the E0 transition rate, we have obtained $R_{\mu e}\,(0^{+}(g.s.) \to 0^{+}(6.05)) = 0.6\%$. Since the monopole radiation $0^{+} \to 0^{+}_{g.s.}$ in the lowest order is forbidden, this level decays predominantly by the emission of the electron pairs. This specific feature may facilitate the experimental investigation.

We would like to thank Dr. S.M.Bilenky for stimulation, and also him and Dr. R.A.Eramzhyan for interesting discussions.

APPENDIX

1. The matrix elements of the hadronic current can be written in the form

$$M_4 \equiv \langle J_f M_f | J_4 | J_i M_i \rangle =$$

$$= 4\pi \sum_{LM} Y^{*}_{LM}(\Omega_{\vec{q}_e})\,(2J_f + 1)^{-1/2} \begin{bmatrix} J_i & L & J_f \\ M_i & M & M_f \end{bmatrix} T(L)\,,$$

$$M_{\mu} \equiv \langle J_f M_f | J_{\mu} | J_i M_i \rangle = \qquad \text{(A.1)}$$

$$= 4\pi \sum_{\ell m\, LM} Y^{*}_{\ell m}(\Omega_{\vec{q}_e})(2J_f + 1)^{-1/2} \begin{bmatrix} 1 & \ell & L \\ \mu & m & M \end{bmatrix} \begin{bmatrix} J_i & L & J_f \\ M_i & M & M_f \end{bmatrix} S(\ell, L)\,,$$

where J_i, M_i and J_f, M_f stand for the total spin and its projection of the nuclear initial and final state, respectively. The Condon-Shortley phase convention was used for the spherical harmonics $Y_{\ell m}$ and the symbols $\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$ stand for the Clebsch-Gordan coefficients. The quantities T and S are defined as follows:

$$T(L) = i^{L} f_1\, C(0LL) + \sum_r i^{r} \frac{|\vec{q}|}{2M} f_2 \begin{bmatrix} 1 & L & r \\ 0 & 0 & 0 \end{bmatrix} C(1rL) + i^{L+1} \frac{f_2}{M} D(0LL)\,,$$

$$S(\ell, L) = i^{\ell+1} f_2\, C(1\ell L) -$$

$$- \sum_r i^{r+1} \frac{|\vec{q}|}{\sqrt{2}\,M} f_1 \sqrt{3(2r+1)} \begin{bmatrix} 1 & r & \ell \\ 0 & 0 & 0 \end{bmatrix} W(11Lr;\, 1\ell)\, C(1\ell L) +$$

$$+ i^{L+1} \frac{|\vec{q}|}{2M} f_1 \begin{bmatrix} 1 & \ell & L \\ 0 & 0 & 0 \end{bmatrix} C(0LL) - i^{\ell} \frac{f_1}{M} D(1\ell L)\,. \qquad \text{(A.2)}$$

Here $\vec{q}$ is the 3-momentum transfer, M is the nucleon mass and the symbol $W(\ldots;\ldots)$ is the Racah coefficient. We have introduced here the following reduced matrix elements:

$$C(0LL) = \langle J_f \| \sum_{k=1}^{A} \frac{1}{2}(1 + \tau_3(k))\, j_L(|\vec{q}|\, r_k)\, Y_L(\Omega_{\vec{r}_k}) \| J_i \rangle\,,$$

$$C(1\ell L) = \langle J_f \| \sum_{k=1}^{A} \frac{1}{2}(1 + \tau_3(k))\, j_{\ell}(|\vec{q}|\, r_k) \sum \begin{bmatrix} 1 & \ell & L \\ \mu & m & M \end{bmatrix} \sigma_{\mu} Y_{\ell m}(\Omega_{\vec{r}_k}) \| J_i \rangle\,,$$

$$D(0LL) = \langle J_f \| \sum_{k=1}^{A} \frac{1}{2}(1 + \tau_3(k))\, j_L(|\vec{q}|\, r_k)\, Y_L(\Omega_{\vec{r}_k})(\vec{\sigma} \cdot \vec{\nabla}) \| J_i \rangle\,,$$

$$D(1\ell L) = \langle J_f \| \sum_{k=1}^{A} \frac{1}{2}(1 + \tau_3(k))\, j_{\ell}(|\vec{q}|\, r_k) \sum \begin{bmatrix} 1 & \ell & L \\ \mu & m & M \end{bmatrix} Y_{\ell m}(\Omega_{\vec{r}_k}) \nabla_{\mu} \| J_i \rangle\,. \qquad \text{(A.3)}$$

2. The final expression for the calculated ratio of inelastic to elastic transition rate $R^{GR}_{\mu e}$ eq. (7) is

$$R^{GR}_{\mu e} = \frac{2.25}{(|f_1|\, Z\, |F_{NN}|)^2} \frac{1}{2J_i + 1} \sum_{M_i, M_f} (M_4 M_4^{+} + M_{\mu} M_{\mu}^{+})\left(1 - \frac{E^*}{m_{\mu}}\right).$$

Using the quantities S and T of eq. (A.2) it assumes the form

$$R^{GR}_{\mu e} = 2.25 \times 4\pi (|f_1| Z | F_{NN}|)^{-2} (1 - \frac{E^*}{m_\mu})[\sum_L |T(L)|^2 + \sum_{\ell L} |S(\ell,L)|^2]$$

where E^* is the nuclear excitation energy and the numerical coefficient 2.25 is due to the quark counting.

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