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Средняя поляризация отдачи в ядерном захвате мюона

Обсуждается теория средней поляризации ядра отдачи в захвате мюона ядрами с произвольным спином. Анализ проводится с целью определения вклада слабых взаимодействий в величину поляризации, с учетом роли сверхтонкой структуры мезоатома. Средняя поляризация отдачи вычисляется из канонического представления переходной матрицы плотности, которая построена при помощи матрицы плотности для начального состояния системы и амплитуды ядерного перехода данной в мультипольном разложении. Рассматривается случай прямого перехода в основное состояние дочернего ядра, а также каскадного перехода, где при захвате мюона образуется возбужденное состояние ядра, которое посредством электромагнитного распада переходит в основное состояние. На примере ядра ^{9}Be проанализированы одновременные прямой и каскадный переходы. Оказывается, что для захвата мюона в ядре ^{9}Be участие каскадного перехода приводит к потушению величины средней поляризации ядра отдачи. Эффект этот влияет на определение формфакторов слабого взаимодействия.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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On the Average Recoil Polarization in the Nuclear Muon Capture

Theory of average recoil polarization in the nuclear muon capture is discussed for nuclei with arbitrary spin. Formulae are derived in cases of direct and cascade nuclear transitions or both. Illustrative examples are shown to indicate the weak interaction interests in this observable.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

The experimental investigation of *nuclear* muon capture has two principal aims [1] - first, to study the muon-nucleon weak coupling constants in a nuclear environment and hence to find out if these are different from the ones characteristic of muon interacting with a free nucleon; second, given the weak interaction Hamiltonian, to explore nuclear physics aims, for example, by learning about transition matrix elements or understanding collective ("giant") isovector excitation modes. With the advent of meson factories and sharper experimental tools, experiments of increasingly difficult nature are designed, besides the transition strength measurements. The angular correlation measurements via the polarization recoil asymmetry of the daughter nuclei produced in the primary reaction or the probabilities of electromagnetic radiation emitted by the daughter are increasingly attempted. These, together with the rate measurements, give us a more complete handle to the physics understanding of the primary reaction. The situation here is very similar to the one existing in the field of nuclear β-decay immediately following the discovery of parity violation in the weak interaction. There are now complete theories [2] available for the angular correlations involving the nuclear β-decay and electron capture, and, at the first thought, it might appear that one could readily copy this formalism, particularly for the electron capture, in the problem of nuclear muon capture. Unfortunately, that is not quite the case.

There is an important difference that distinguishes the muon capture problem from that of the electron, the very important rôle of the atomic *hyperfine* (HF) *structure* in the former one. Indeed it is the importance of the HF effects in nuclear capture that makes muon a very valuable probe in the weak interaction aspect. The light nuclear atoms with non-zero nuclear spin I have statistical populations for the HF states of spin $F_{\pm} = I \pm ½$, with very different sensitivity to the weak hadronic form factors as far as the nuclear capture is concerned. In polarized and oriented nuclear targets, these HF populations can also be made non-statistical [3]. Finally, in somewhat heavier elements ($A \geq 19$), the HF conversion takes place due to the ejection of atomic electrons, rapidly on the weak interaction time scale [4]. Again the HF effect becomes mandatary to be considered in the formalism.

Our aim, in this first of two projected papers, is to consider the measurement of average* nuclear polarization of the daughter nuclei produced in the muon capture by a nucleus of *arbitrary* spin, where the daughter may have been produced either in the ground state (g.s.) or in one of its nucleon-stable excited states which decay by the emission of the electromagnetic radiation. The experiments we have in mind are the successors of the one by the Louvain-Saclay collaboration [5] for ^{12}C. While the gamma-neutrino theories have been discussed, at least in special cases in this general context no such work is presently available for the average polarization, except for the special case of spin zero targets [6].

This paper is divided into three principal sections: Direct muon capture transition is given in Sec. 2; Sec. 3 contains cascade transition, i.e., muon capture followed by gamma decay; Sec. 4 contains a combination of direct and cascade sequences.

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*Hereafter the "average" means average over the directions of all unobserved radiations (neutrino, gamma rays).

2. *ONE-STEP NUCLEAR TRANSITION* $I_i \xrightarrow{\mu} I_f$

Let us first consider the direct muon capture transition $I_i \xrightarrow{\mu} I_f$, where I_i and I_f are the spins of the parent and daughter nuclei. The transition amplitude for such a process is given as a matrix element of the weak interaction Hamiltonian $\hat{H}_\mu$:

$$\langle I_f M_f, \vec{\nu} m' | \hat{H}_\mu | I_i M_i, \tfrac{1}{2} m \rangle =$$

$$= \sqrt{\frac{1}{2\pi}} D^{1/2}_{m'h}(\Omega_{\vec{\nu}}) \sum_\mu \overset{*}{D}{}^{1/2}_{m\mu}(\Omega_\nu) \langle I_f M_f, \vec{\nu} h | \hat{H}_\mu | I_i M_i, (\hat{\vec{\nu}}) \tfrac{1}{2} \mu \rangle, \quad (2.1)$$

where on the left-hand side of (2.1) the spin quantization axis for both the leptons and nuclei is arbitrary, while on the right-hand side the spin-quantization axis for the leptons is pointed along the neutrino momentum $\vec{\nu}$, and that for the nucleons is still arbitrary. It will be more convenient in the present case to deal with the latter. The angles $\Omega_{\vec{\nu}}$ represent the rotation needed to bring the arbitrary quantization axis to the direction $\vec{\nu}$. The weak multipole amplitudes T_I can now be defined by the relation [7]

$$\langle M_f \vec{\nu} h | \hat{H} | M_i, (\hat{\vec{\nu}}) \mu \rangle = \frac{1}{\sqrt{3\pi}} \sum_{IM} \begin{pmatrix} I_i & I & I_f \\ M_i & M & M_f \end{pmatrix} \hat{I} i^{-I} T_I^{\mu-h} D^{I}_{M\mu-h}(\Omega_{\vec{\nu}}) \quad (2.2)$$

where we have abbreviated $I_f M_f$ to M_f and so on, $\hat{I}$ means $\sqrt{2I+1}$.

The relation of the multipole amplitudes to those of ref. [8] is:

$$T_I^0 = \frac{1}{2} \sqrt{\frac{3\pi}{2(2I+1)}} \{ \sqrt{I+1} [M_I(-I-1) - iM_I(I+1)] + \sqrt{I} [iM_I(-I) + M_I(I)] \},$$

$$T_I^1 = \frac{1}{2} \sqrt{\frac{3\pi}{2(2I+1)}} \{ \sqrt{I+1} [iM_I(-I) + M_I(I)] - \sqrt{I} [M_I(-I-1) - iM_I(I+1)] \}.$$

The nuclear final state density matrix is symbolically given as $\rho(I_i \xrightarrow{\mu} I_f) = H_\mu \rho_i H_\mu^+$. Explicitly this can be written as

$$<M_f|\hat{\rho}(I_i \xrightarrow{\mu} I_f)|M_f'> = \sum_{M_i M_i' \mu\mu'} \int \frac{d\hat{\vec{\nu}}}{8\pi^2} <M_f, \vec{\nu} h|\hat{H}_\mu|M_i, [\hat{\vec{\nu}}]\mu> \times$$

$$\times <M_i, [\hat{\vec{\nu}}]\mu|\hat{\rho}_i|M_i', [\hat{\vec{\nu}}]\mu'> <M_i', [\hat{\vec{\nu}}]\mu'|\hat{H}_\mu|M_f', \vec{\nu} h>, \qquad (2.3)$$

where ρ_i is the initial density matrix of the muon-nucleus system, discussed in detail in the subsection 2.1. The muon capture rate $\Lambda(I_f)$ and the average polarization of the nucleus $P(I_i \xrightarrow{\mu} I_f)$ are then given by

$$\Lambda(I_f) = q_f^2 32\pi^3 \mathrm{Tr}\,\rho(I_i \xrightarrow{\mu} I_f), \qquad (2.4)$$

$$\vec{P}(I_i \xrightarrow{\mu} I_f) = \sqrt{\frac{I_f + 1}{3I_f}} <\vec{Q}^{(1)}>_f^* , \qquad (2.5)$$

with

$$<Q^{(1)}>_f^* = \mathrm{Tr}\,\rho(I_i \xrightarrow{\mu} I_f)\overset{+}{Q}^{(1)}/\mathrm{Tr}\,\rho(I_i \xrightarrow{\mu} I_f), \qquad (2.6)$$

where the operator $\hat{\vec{Q}}^{(1)}$ is defined as

$$\hat{\vec{Q}}^{(1)} = \sqrt{\frac{3}{I_f(I_f+1)}}\,\hat{\vec{I}} , \qquad (2.7)$$

$\hat{\vec{I}}$ being the angular operator. The reduced matrix element of the operator $\hat{\vec{Q}}^{(\ell)}$ between the states of angular momentum I is simply *

$$<I||\hat{Q}^{(\ell)}||I> = \hat{I}\hat{\ell}. \qquad (2.8)$$

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* In this paper we follow the Wigner-Eckart theorem as defined by Edmonds /9/.

2.1. The Initial Density Matrix

The initial muon-nucleus density operator can be written in the most general case as follows:

$$\hat{\rho}_i = \frac{1}{2(2I_i+1)} \sum_{g m_\mu p m_0} <\hat{Q}_{m_\mu}^{(g)} \hat{Q}_{m_0}^{(p)}>^* \hat{Q}_{m_\mu}^{(g)} \hat{Q}_{m_0}^{(p)} . \qquad (2.9)$$

Its matrix elements can be reexpressed as

$$<M_i m|\hat{\rho}_i|M_i' m'> = \sum_{g m_\mu p m_0} \frac{\widehat{gp}}{2(2I_i+1)} \begin{pmatrix} ½ & g & ½ \\ m' & m_\mu & m \end{pmatrix} \begin{pmatrix} I_i & p & I_i \\ M_i' & m_0 & M_i \end{pmatrix} \times$$

$$\times \begin{pmatrix} g & p & r \\ m_\mu & m_0 & m \end{pmatrix} U_{rm}^{(gp)}, \qquad (2.10)$$

where the quantities $U_{rm}^{(gp)}$, in the hyperfine basis of the muon-nucleus system have the form

$$U_{rm}^{(gp)} = \hat{g}\hat{p}\hat{r}\hat{I}_i \sqrt{2} \sum_{FMM'} <FM|\hat{\rho}_i|FM'> \hat{F} \begin{Bmatrix} ½ & ½ & g \\ I_i & I_i & p \\ F & F & r \end{Bmatrix} \begin{pmatrix} F & r & F \\ M' & m & M \end{pmatrix} \qquad (2.11)$$

This can be simplified further:

$$U_{rm}^{(gp)} = \hat{g}\hat{p}\hat{I}_i \sqrt{2} \sum_F \hat{F} \begin{Bmatrix} I_i & I_i & p \\ ½ & ½ & g \\ F & F & r \end{Bmatrix} \lambda_F \rho_{rm}^F , \qquad (2.11')$$

with

$$\rho_{rm}^F = \hat{r} \sum_{MM'} \begin{pmatrix} F & r & F \\ M' & m & M \end{pmatrix} <FM|\hat{\rho}_F|FM'>, \qquad (2.12)$$

and with the expression for $\hat{\rho}_i$ used

$$\hat{\rho}_i = \sum_F \lambda_F \hat{\rho}_F , \qquad (2.13)$$

where $\mathrm{Tr}(\hat{\rho}_F) = 1$ and $\mathrm{Tr}(\hat{\rho}_i) = \Sigma_F \lambda_F = 1$.

It is now useful to introduce the initial density matrix in the rotated frame of lepton quantization axis

$$<M_i, [\hat{\vec{\nu}}]\mu|\hat{\rho}_i|M_i', [\hat{\vec{\nu}}]\mu'> = \sum_{mm'} \overset{*}{D}{}^{½}_{m\mu}(\Omega_{\vec{\nu}}) <M_i m|\hat{\rho}_i|M_i' m'> D^{½}_{m'\mu'}(\Omega_{\vec{\nu}}). \qquad (2.14)$$

2.2. Expressions for Capture Rate and Average Nuclear Polarization

To proceed further, we substitute (2.14) into (2.3) and obtain the following result:

$$\langle M_f|\hat\rho(I_i \xrightarrow{\mu} I_f)|M_f'\rangle = \sum_{rm} \frac{\hat r}{2I_f+1}\, \rho^r_m(I_i \xrightarrow{\mu} I_f) \begin{pmatrix} I_f & r & I_f \\ M_f' & m & M_f \end{pmatrix}, \qquad (2.15)$$

with $\rho^r_m(I_i \xrightarrow{\mu} I_f)$ as

$$\rho^r_m(I_i \xrightarrow{\mu} I_f) = \frac{(2I_f+1)\hat I_f}{3\pi\sqrt{2}\,\hat I_i} \sum_{II'pg} \frac{\hat p}{\hat g}(-)^{1/2-h+I} B^g_{II'h}\, U^{(gp)}_{rm} \begin{Bmatrix} I_i & I_i & p \\ I' & I & g \\ I_f & I_f & r \end{Bmatrix}; \qquad (2.16)$$

the quantities $B^g_{II'h}$ can be written in terms of the multipole amplitudes T_I:

$$B^g_{II'h} = i^{I'-I}\,\hat I \hat I'\, T_I^{-2h} \overset{*}{T}{}_{I'}^{-2h} \begin{pmatrix} 1/2 & 1/2 & g \\ -h & h & 0 \end{pmatrix} \begin{pmatrix} I & I' & g \\ -2h & 2h & 0 \end{pmatrix} +$$

$$+ (-)^{1+g}\, x_I x_{I'} \sqrt{\frac{II'}{(I+1)(I'+1)}} \begin{pmatrix} I & I' & g \\ 0 & 0 & 0 \end{pmatrix}] + \qquad (2.17)$$

$$+ \begin{pmatrix} 1/2 & 1/2 & g \\ h & h & 2h \end{pmatrix} [x_I \sqrt{\frac{I}{I+1}} \begin{pmatrix} I & I' & g \\ 0 & 2h & 2h \end{pmatrix} + (-)^{1+g} x_{I'} \sqrt{\frac{I'}{I'+1}} \begin{pmatrix} I' & I & g \\ 0 & 2h & 2h \end{pmatrix}]\},$$

with

$$x_I = \sqrt{\frac{I+1}{I}}\, \frac{T_I^0}{T_I^{-2h}}. \qquad (2.18)$$

Two special cases of $B^g_{II'h}$ are particularly interesting:

$$B^0_{IIh} = (-)^{1/2-h+I} \frac{\hat I}{\sqrt{2}} |T_I^{-2h}|^2 \left(1 + \frac{I}{I+1} x_I^2\right); \qquad (2.19a)$$

$$B^1_{IIh} = (-)^{1/2-h+I} \frac{\hat I}{\sqrt{2}} |T_I^{-2h}|^2 \sqrt{\frac{3}{I(I+1)}}\,(1 + 2I x_I). \qquad (2.19b)$$

Now, the quantities $\mathrm{Tr}\,\rho(I_i \xrightarrow{\mu} I_f)$ and $\mathrm{Tr}\,\rho(I_i \xrightarrow{\mu} I_f)\overset{+}{Q}{}^{(1)}$ can be computed in the normal way, viz.,

$$\mathrm{Tr}\,\rho(I_i \xrightarrow{\mu} I_f) = \Sigma_{M_f} \langle M_f|\hat\rho(I_i \xrightarrow{\mu} I_f)|M_f\rangle,$$

$$\mathrm{Tr}\rho(I_i \xrightarrow{\mu} I_f)\overset{+}{Q}{}^{(1)}_m = \sum_{M_f M_f'} \langle M_f'|\overset{+}{Q}{}^{(1)}_m|M_f\rangle \langle M_f|\hat\rho(I_i \xrightarrow{\mu} I_f)|M_f'\rangle.$$

These are

$$\mathrm{Tr}\,\rho(I_i \xrightarrow{\mu} I_f) = \frac{(2I_f+1)\hat I_f}{3\pi\sqrt{2}\,\hat I_i} \sum_{II'g} (-)^{1/2-h+I} B^g_{II'h} U_0^{(gg)} \begin{Bmatrix} I_i & I_i & g \\ I' & I & g \\ I_f & I_f & 0 \end{Bmatrix} \qquad (2.20)$$

$$\mathrm{Tr}\,\rho(I_i \xrightarrow{\mu} I_f)\overset{+}{Q}{}^{(1)}_m =$$

$$= \frac{(2I_f+1)\hat I_f}{3\pi\sqrt{2}\,\hat I_i} \sum_{II'pg} \frac{\hat p}{\hat g} (-)^{1/2-h+I} B^g_{II'h} U^{(gp)}_{1m} \begin{Bmatrix} I_i & I_i & p \\ I' & I & g \\ I_f & I_f & 1 \end{Bmatrix}. \qquad (2.21)$$

Then using (2.20) and (2.21) and the definition of $\vec P(I_i \xrightarrow{\mu} I_f)$, eq. (2.5), we obtain

$$\vec P(I_i \xrightarrow{\mu} I_f) = \sqrt{\frac{I_f+1}{3I_f}}\; \frac{\Sigma_F \lambda_F B_F^{(1)} \vec\rho^{\,F}_1}{\Sigma_{F'} \lambda_{F'} B_{F'}^{(0)} \rho^{F'}_0}, \qquad (2.22)$$

where $B_F^{(r)}$ are given by

$$B_F^{(r)} = \hat{I}_i \sqrt{2}\, \hat{F} \sum_{II'g} (-)^{1/2-h+I}\, B^g_{II'h}\, C^r_F(I,I',g), \tag{2.23}$$

with

$$C^r_F(I,I',g) = \sum_p (2p+1) \begin{Bmatrix} I_i & I_i & p \\ I' & I & g \\ I_f & I_f & r \end{Bmatrix} \begin{Bmatrix} I_i & I_i & p \\ 1/2 & 1/2 & g \\ F & F & r \end{Bmatrix}. \tag{2.23'}$$

It is also useful to express the average polarization due to capture from individual HF states. Calling these quantities $\vec{P}_F(I_i \xrightarrow{\mu} I_f)$, we have

$$\vec{P}_F(I_i \xrightarrow{\mu} I_f) = \sqrt{\frac{I_f+1}{3I_f}}\, \frac{B_F^{(1)}\vec{\rho}^{\,F}_1}{B_F^{(0)}\rho_0^F}. \tag{2.24}$$

We can now express $\vec{P}$ in terms of $\vec{P}_F$:

$$\vec{P}(I_i \xrightarrow{\mu} I_f) = \sum_F \omega_F \vec{P}_F(I_i \xrightarrow{\mu} I_f), \tag{2.25}$$

where

$$\omega_F = \frac{\lambda_F B_F^{(0)}\rho_0^F}{\sum_{F'} \lambda_{F'} B_{F'}^{(0)} \rho_0^{F'}}. \tag{2.25'}$$

2.3. Some Illustrative Examples

To apply the above obtained results (2.22, 2.24), we consider the allowed transitions in 1p-shell nuclei [10] and assume the dominance of the Gamow-Teller matrix elements, which is a reasonable approximation [11] for many transitions in computing the transition rates. This approximation is made here for illustration only and, in more accurate calculations, it can be relaxed with more careful attention to the nuclear physics. The approximation makes the average polarization independent of the nuclear structure effects, and it allows us to display the rôles of the weak interaction form factors explicitly. With this, $B^g_{II'h}$ becomes

$$B^g_{II'h} = \delta_{I1}\delta_{I'1} B^g_{11h}, \tag{2.26}$$

and we can make use of the special cases of B^g_{IIh} given in eqs. (2.19). It is convenient to define the quantity β

$$\beta = \frac{1+2x}{2+x^2}, \tag{2.27}$$

where x is simply

$$x = \sqrt{2}\,\frac{T_1^0}{T_1^1} \simeq 1 - \frac{G_P}{G_A}. \tag{2.28}$$

Particularly simple is the case where the initial nuclear spin is zero. Thus for the transition $0 \xrightarrow{\mu} 1$, discussed frequently in the literature, within the context of ^{12}C nucleus,

$$|\vec{P}(0 \xrightarrow{\mu} 1)| = \frac{2}{3}\beta. \tag{2.29}$$

Other interesting special cases: (1) $\frac{1}{2} \to \frac{1}{2}$, (2) $\frac{1}{2} \to \frac{3}{2}$, (3) $1 \to 1$, , (4) $1 \to 2$, (5) $\frac{3}{2} \to \frac{1}{2}$, (6) $\frac{3}{2} \to \frac{3}{2}$, (7) $\frac{3}{2} \to \frac{3}{2}$.

We show their results in the *Table.*

Within the above approximation, some interesting results are apparent. Taking the example of the $1 \to 2$ transition, the average polarization from the upper hyperfine state only depends on the weak interaction dynamics, while the corresponding result for the lower one does not. In the $\frac{3}{2} \to \frac{3}{2}$ transition, this is the opposite. These are, of course, not rigorously true due to our neglect of all nuclear matrix elements other than the GT one. Nevertheless, they give a feeling as to what HF processes are more interesting in polarization measurement from the weak interaction point of view.

An interesting example of the Fermi and GT matrix elements both playing important rôles is the superallowed transition $\frac{1}{2} \to \frac{1}{2}$ in the same isomultiplet ($p \to n$, $^3He \to {}^3H$).

Table

Average nuclear polarization coefficients $B^{(1)}_F/B^{(0)}_F$ for capture from hyperfine states in $I_i \neq 0$ nuclear targets in the allowed GT approximation. The quantity β is defined in the text.

Transition	$B^{(1)}_+/B^{(0)}_+$	$B^{(1)}_-/B^{(0)}_-$
$\frac{1}{2} \to \frac{1}{2}$	$-\sqrt{\frac{2}{3}}\,\frac{1-2\beta}{3-2\beta}$	
$\frac{1}{2} \to \frac{3}{2}$	$\sqrt{\frac{10}{3}}\,\frac{1+\beta}{3+\beta}$	
$1 \to 1$	$\frac{1}{3}\sqrt{\frac{5}{6}}\,\frac{3+\beta}{2-\beta}$	$\sqrt{\frac{1}{6}}\,\frac{1+\frac{4}{3}\beta}{1+\beta}$
$1 \to 2$	$\sqrt{\frac{1}{10}}\,\frac{5+4\beta}{2+\beta}$	$\sqrt{\frac{1}{2}}$
$\frac{3}{2} \to \frac{1}{2}$	$\sqrt{\frac{1}{2}}$	$\frac{5}{3}\sqrt{\frac{1}{6}}\,\frac{1+\frac{11}{5}\beta}{1+\frac{5}{3}\beta}$
$\frac{3}{2} \to \frac{3}{2}$	$\sqrt{\frac{1}{10}}\,\frac{11-2\beta}{5-2\beta}$	$\sqrt{\frac{1}{30}}\,\frac{11+14\beta}{3+2\beta}$
$\frac{3}{2} \to \frac{5}{2}$	$\frac{3}{10}\sqrt{\frac{21}{5}}\,\frac{1+\frac{3}{5}\beta}{1+\frac{7}{9}\beta}$	$\sqrt{\frac{7}{10}}$

In this case, the results are as follows:

$$\vec{P}_+\left(\tfrac{1}{2} \xrightarrow{\mu} \tfrac{1}{2}\right) = \sqrt{\frac{2}{3}}\,\frac{(1+2\xi)\,\vec{\rho}^{\,+}_1}{(2+\xi^2)\rho^+_0}, \qquad (2.30)$$

where ξ is

$$\xi = \frac{G_V\sqrt{3}\int 1 + (G_A + G_P)\frac{1}{\sqrt{3}}\int\sigma}{G_V\sqrt{3}\int 1 + (G_A - G_P)\frac{1}{\sqrt{3}}\int\sigma} \qquad (2.30')$$

3. *TWO-STEP NUCLEAR TRANSITION* $I_i \xrightarrow{\mu} I_k \xrightarrow{\gamma} I_f$

Let us now consider a more complicated problem in which the muon capture transition leads to a nuclear state of angular momentum I_k which decays by gamma ray emission to the state I_f. In this case, the final state density matrix is given by

$$\langle M_f|\hat{\rho}(I_i \xrightarrow{\mu} I_k \xrightarrow{\gamma} I_f)|M'_f\rangle =$$

$$= \sum_{M_k M'_k} \int \frac{d\hat{\vec{k}}_\gamma}{4\pi} \langle M_f \gamma_k|\hat{H}_\gamma|M_k\rangle\langle M_k|\hat{\rho}(I_i \xrightarrow{\mu} I_k)|M'_k\rangle \times$$

$$\times \langle M'_k|\hat{H}_\gamma|\gamma_k M'_f\rangle . \qquad (3.1)$$

Here the electromagnetic transition amplitude is simply

$$\langle M_f\,\gamma_k|\hat{H}_\gamma|M_k\rangle = \sum_{L M_L} \begin{pmatrix} L & I_f & I_k \\ M_L & M_f & M_k \end{pmatrix} b_{L\eta}\, A_L\, \overset{*}{D}{}^L_{M_L\eta}(\Omega_\gamma), \qquad (3.2)$$

where $b_{L\eta} = \hat{L}$ (electric), $\eta\hat{L}$ (magnetic L-pole), $\eta = \pm 1$ for right and left circularly polarized γ-quanta.

Following the same procedure as in Section 2, we can calculate the resultant average nuclear polarization due to this sequence:

$$\vec{P}(I_i \xrightarrow{\mu} I_k \xrightarrow{\gamma} I_f) = \sqrt{\frac{I_f+1}{3I_f}}\,\langle \vec{Q}^{(1)}\rangle^*_{kf}, \qquad (3.3)$$

where

$$< \vec{Q}^{(1)} >^{*}_{kf} = \mathrm{Tr}\,\rho(I_i \xrightarrow{\mu} I_k \xrightarrow{\gamma} I_f)\vec{Q}^{+(1)} / \mathrm{Tr}\,\rho(I_i \xrightarrow{\mu} I_k \xrightarrow{\gamma} I_f). \tag{3.4}$$

Calculation of the trace in (3.4) proceeds along the same lines as in Section 2 and we obtain the result:

$$\vec{P}(I_i \xrightarrow{\mu} I_k \xrightarrow{\gamma} I_f) = d(I_k \xrightarrow{\gamma} I_f)\vec{P}(I_i \xrightarrow{\mu} I_k), \tag{3.5}$$

where

$$d(I_k \xrightarrow{\gamma} I_f) = \sqrt{\frac{(I_f+1)I_k}{I_f(I_k+1)}}\,\hat{L}_k \hat{I}_f \sum_L (-)^{L+I_f+I_k+1} \begin{Bmatrix} L & I_f & I_k \\ 1 & I_k & I_f \end{Bmatrix} |A_L|^2 / \sum_{L'} |A_{L'}|^2, \tag{3.6}$$

and $\vec{P}(I_i \xrightarrow{\mu} I_k)$ can be obtained from Eq. (2.22) by replacing I_f with I_k. Thus, the electromagnetic and weak contributions in (3.5) factorize.

It is straightforward to check the ineequality

$$| d(I_k \xrightarrow{\gamma} I_f)| \le 1. \tag{3.7}$$

To do this it is enough to check this for the maximum value of $d(I_k \xrightarrow{\gamma} I_f)$ which is

$$| d(I_k \xrightarrow{\gamma} I_f)|_{max} = \left| \frac{I_f(I_f+1) + I_k(I_k+1) - L_{min}(L_{min}+1)}{2I_f(I_k+1)} \right|. \tag{3.8}$$

For the cases (1) $I_f = I_k$, or (2) $I_f < I_k$, this quantity is unity, while, for $I_k < I_f$, it is less than unity. Hence, (3.7) follows.

4. *COMBINATION OF THE DIRECT (ONE-STEP) AND CASCADE (TWO-STEP) TRANSITIONS*

Let us suppose we have the processes (1) $I_i \xrightarrow{\mu} I_f$ and (2) $I_i \xrightarrow{\mu} I_k \xrightarrow{\gamma} I_f$, both taking place, and the experiment

determines the average nuclear polarization in the daughter state I_f only. For $I_i = 0$, this is what was done in the Saclay-Louvain experiment /5, 12/. In this case the final nuclear density operator

$$\hat{\rho}_f = \omega(I_f)\hat{\rho}(I_i \xrightarrow{\mu} I_f) + \omega(I_k)\hat{\rho}(I_i \xrightarrow{\mu} I_k \xrightarrow{\gamma} I_f), \tag{4.1}$$

where

$$\omega(a) = \Lambda(a)/(\Lambda(I_f) + \Lambda(I_k)), \tag{4.2}$$

Λ's being the appropriate muon capture rate. Then the average nuclear polarization, measured in such an experiment, is given by

$$\vec{P} = \omega(I_f)\vec{P}(I_i \xrightarrow{\mu} I_f) + \omega(I_k)\vec{P}(I_i \xrightarrow{\mu} I_k \xrightarrow{\gamma} I_f), \tag{4.3}$$

where ω's are given by (4.2) and $\vec{P}$'s have already been given in Sections 2 and 3.

As an illustrative example, let us take the cases

1) $^{9}\mathrm{Be}(\frac{3}{2}^-, \mathrm{g.s.}) \xrightarrow{\mu^-} {}^{9}\mathrm{Li}(\frac{3}{2}^-, \mathrm{g.s.})$

and

2) $^{9}\mathrm{Be}(\frac{3}{2}^-, \mathrm{g.s.}) \xrightarrow{\mu^-} {}^{9}\mathrm{Li}(\frac{1}{2}^-, 2.691) \xrightarrow{\gamma} {}^{9}\mathrm{Li}(\frac{3}{2}^-, \mathrm{g.s.})$.

Here

$$\Lambda(\frac{3}{2}^-, \mathrm{g.s.}) \cong 200\ \mathrm{s}^{-1}, \quad \Lambda(\frac{1}{2}^-, 2.691) < 20\ \mathrm{s}^{-1}.$$

Taking

$$\Lambda(\frac{1}{2}^-) \cong 20\ \mathrm{s}^{-1},$$

we get

$$\vec{P} = 0.91\vec{P}(\frac{3}{2}^- \xrightarrow{\mu} \frac{3}{2}^-) + 0.05\vec{P}(\frac{3}{2}^- \xrightarrow{\mu} \frac{1}{2}^-), \tag{4.4}$$

where the approximation (3.8) has been used for the value of $d(\frac{1}{2} \xrightarrow{\gamma} \frac{3}{2})$, in which $d(\frac{1}{2} \xrightarrow{\gamma} \frac{3}{2}) \simeq \frac{5}{9}$.

5. CONCLUSION

We have presented a formalism for the calculation of average nuclear polarization in muon capture by nuclei with arbitrary spin either leading to the ground state or to the excited states which can decay by electromagnetic radiation, or a combination of both. The illustrative examples under the assumption of a dominant matrix element show the sensitivity of this observable to the weak interaction physics or the lack of it. It should now be possible to use sophisticated nuclear physics inputs to compute this observable reliably in the cases of practical interest some of which are demonstrated by our examples. Our formalism is adequate also to the consideration of polarized target nuclei and to nuclei where the HF states relax as a function of time.

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