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MICROSCOPIC DESCRIPTION OF COUPLED BANDS, INCLUDING A SUPERBAND, IN EVEN-EVEN NUCLEI

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Микроскопическое описанне связанных полос, включая супер-полосу, в четно-четных ядрах

Ранее предложенный микроскопический подход к связанным модам в четно-четных ядрах обощен включением супер-полосы ($K^\pi = I^+$) в дополнение к основной, β - и γ -полосам. Прямая связь с супер-полосой модифицирует крэнкинг-формулу для момента инерции и RPA уравнения для s -полосы, но оставляет без изменений RPA результаты для β - и γ -полос до первого порядка.

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Microscopic Description of Coupled Bands, Including a Superband, in Even-Even Nuclei

An earliear proposed microscopic approach to coupled modes in even-even nuclei is generalized to the case of a super (s-) band in addition to the ground (g-), β - and γ - bands. The direct coupling with the superband ($K^\pi = I^+$) modifies the cranking model formula for the moment of inertia and the RPA equations for the s - band, but up to first order leaves unchanged the RPA results for the β - and γ -bands.

The investigatigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

Heavy ion reactions during the last years allowed to reach high-spin states in deformed heavy nuclei. A number of interesting effects (back- and down-bending, band crossing, shape transitions and isomerism,etc., see for example /1,2/) were discovered. Various microscopic approaches, models and calculation technics to describe the nuclear rotation have been proposed and most of them are based on cranking model type treatment, e.g., the calculation of the anadiabatic corrections to the transition probabilities in ref. /3/ and the microscopic cranking calculations of the same effects in ref. /4/. The projection method /5, 6/ is another widely used way to derive rotation from the nuclear collective states. But the rotational motion is strongly coupled to the vibrational one. The latter has been treated most successfully by the RPA and its modifications /7,8/. The difference between the theoretical languages, describing these two main kinds of nuclear collective motion gives raise to difficulties to the microscopic theory when trying to account for their coupling.

A method for an unified treatment of nuclear rotations and vibrations was proposed in a series of papers, e.g.,[9,10,11,12]. It is similar to the RPA for vibrations and is based on suitably defined transition operators $B^+_{\alpha IM}$, which have been introduced in ref. [10] (see also ref. [11,12]). In previous works [13,14] the method was applied to describe microscopically the coupling between ground state (g-), β- and γ- bands in even - even nuclei. However, some calculations in the frame of the band hybridization model [15,16], though purely phenomenological, indicate the necessity of taking into account a $K^\pi = 1^+$ rotational band, which allows to reproduce the picture of two or three low-lying intersecting bands, observed experimentally in some back-bending nuclei [17,18,19]. Since in different nuclei the nature of the third band may be different, this band is often referred to as superband, crossing the g- and β-bands.

The purpose of this paper is to extend the method by including the coupling to a fourth 1^+ superband (s-band) and to look for the new first order effects it causes in the frame of the formalism, developed in [13], where more details about the method can be found. This should be a first step towards a microscopic derivation of the coupled band model parameters introduced in a previous paper [20].

2. EXTENDED DENSITY MATRIX AND EQUATION OF MOTION

As it was pointed out elsewhere [11,12], the basis consists of those transition operators with minimal angular momentum $B^+_{\alpha|K|M}$,

$B_{\alpha|K|M}$, R^+_{2M} (R^+_{1M}) and the angular momentum operator $\hat{I}$. The R^+_{IM} labels the roton operator with $K = 0$ inducing transitions inside any rotational band. In the present case of three additional one-phonon modes $\alpha = \beta, s, \gamma$ with K-numbers respectively 0,1,2, any physical quantity in the four-band model (multipole operator acting between the states of these four bands) can be expanded in a power series of the basic operators, and the most general form of the expansion is given in ref. [12]. In order to assure hermicity and time-reversal symmetry, we have introduced the tensor operator $O^{(\pm)}$ [20], which are suitable hermitian combinations of the basic operators $B^+_{\alpha|K|M}$ ($\alpha = \beta, s, \gamma$), $\pm$ ive under time reversal.

The generalized phonon operators $B^+_{\alpha|K|M}$ allow to avoid the search for realistic HFB rotation-vibrational states and to get directly the matrix elements of the operators of interest in terms of the density matrix. It is convenient to handle with "coherent" states [13]:

$$|x\rangle = |x^\pm_r x^\pm_\beta x^\pm_s x^\pm_\gamma\rangle = \exp\Big(i \sum_{\substack{\varkappa = r,\beta,s,\gamma \\ \delta = \pm}} x^\varepsilon_\varkappa \hat{\Gamma}^{(\delta)}_\varkappa\Big)|\;\rangle \,, \tag{2.1}$$

where $|\;\rangle$ is the HFB vacuum and the new basic operators $\hat{\Gamma}_\varkappa$ are combinations of the previous ones; $\pm$ ive under time reversal:

$$\hat{\Gamma}^{(+)}_r = \frac{i}{\sqrt{6}}\left(R^+_{21} + R^+_{2-1}\right) \; ; \; \hat{\Gamma}^{(-)}_r = \frac{1}{\sqrt{6}}\left(R^+_{21} + R^+_{2-1}\right) \; ; \; \hat{\Gamma}^{(\pm)}_\beta = O^{\beta g(\pm)}_{00}$$
$$\hat{\Gamma}^{(\pm)}_s = \pm\left(O^{sg(\pm)}_{11} - O^{sg(\pm)}_{1-1}\right) \; ; \; \hat{\Gamma}^{(\pm)}_\gamma = O^{\gamma g(\pm)}_{22} + O^{\gamma g(\pm)}_{2-2} \tag{2.2}$$

$\hat{\Gamma}^{(+)}_r$, $\hat{\Gamma}^{(\pm)}_\alpha$, $\alpha = \beta, s, \gamma$ being hermitian, $\hat{\Gamma}^{(-)}_r$ - antihermitian. The meaning of the x-coefficients, determining the average increase in angular momentum and phonon numbers, can be seen after expanding the exponent and commuting the transition operators $\hat{r}_\varkappa$, whose algebraic properties are known. Thus one gets:

$$x_r^2 = \langle x|\hat{I}^2|x\rangle - \langle\,|\hat{I}^2|\,\rangle \;;\quad (x_\beta^+)^2 + (x_\beta^-)^2 = \langle x|\hat{n}_\beta|x\rangle - \langle\,|\hat{n}_\beta|\,\rangle \tag{2.3}$$

$$2\left[(x_\alpha^+)^2 + (x_\alpha^-)^2\right] = \langle x|\hat{n}_\alpha|x\rangle - \langle\,|\hat{n}_\alpha|\,\rangle \qquad \text{for } \alpha = s,\gamma \;;\; x_r^- = 0$$

The matrix elements of one-body operators, as well as those of separable two-body interactions, can be expressed by the extended density matrices, allowing to take the pairing into account, and defined here with respect to the states in eq.(2.1):

$$\rho_{ij} = \langle x|\alpha_j^+\alpha_i|x\rangle \qquad p_{ij} = \langle x|\alpha_i^+\alpha_{\bar{j}}^+|x\rangle$$
$$\sigma_{ij} = \langle x|\alpha_i\alpha_j|x\rangle \qquad q_{ij} = \langle x|\alpha_i^+\alpha_j|x\rangle \tag{2.4}$$

The α-s are the Bogolubov quasiparticle operators. Then we expand the density matrices in series of the x-parameters. For example:

$$\rho_{ij} = \sum_{\nu_r^\pm,\nu_\beta^\pm,\nu_s^\pm,\nu_\gamma^\pm} \rho_{ij}^{(\nu_r^\pm\nu_\beta^\pm\nu_s^\pm\nu_\gamma^\pm)} (x_r^+)^{\nu_r^+}(x_r^-)^{\nu_r^-}(x_\beta^+)^{\nu_\beta^+}(x_\beta^-)^{\nu_\beta^-}(x_s^+)^{\nu_s^+}(x_s^-)^{\nu_s^-}(x_\gamma^+)^{\nu_\gamma^+}(x_\gamma^-)^{\nu_\gamma^-} \tag{2.5}$$

where the coefficients in the expansion are of order $\nu = \nu_r + \nu_\beta + \nu_s + \nu_\gamma$, $\nu_\varkappa = \nu_\varkappa^+ + \nu_\varkappa^-$ for $\varkappa = r,\beta,s,\gamma$. The Bogolubov transformation links the four density matrices in the following relations of first order in the operators $\hat{r}_\varkappa$:

$$\begin{aligned} \rho_{ji}^{(1^\tau)} &= V_{ji}^{-\tau}\rho_{ij}^{(1^\tau)} + \tau\, U_{ji}^{\tau}\sigma_{i\bar{j}}^{(1^\tau)} \\ q_{ji} = 0 &= U_{ji}^{\tau}\rho_{ij}^{(1^\tau)} - \tau\, V_{ji}^{-\tau}\sigma_{i\bar{j}}^{(1^\tau)} \end{aligned} \tag{2.6}$$

$$U_{ij}^{\pm} = u_i u_j \pm v_i v_j \quad ; \quad V_{ij}^{\pm} = u_i v_j \pm v_i u_j$$

with the notation $\rho_{ij}^{(1^\tau)}$ for the first order in ν ($\nu = 1$) where $\tau = \pm$ corresponds to $\hat{r}_\varkappa$ $\pm$ive under time reversal. In addition, if $W_{\ell m}^{\tau}$ is a hermitian tensor operator $\pm$ive under time reversal ($\tau = \pm 1$), and $\tau\tau_1 = +1$, then:

$$\mathrm{Tr}\left(W_{\ell m}^{\tau}\rho^{(1^{\tau_1})}\right) = 0. \tag{2.7}$$

The nuclear Hamiltonian is taken to be:

$$\hat{H} = \hat{H}_0 + \hat{H}_p + \hat{H}_Q, \tag{2.8}$$

where $\hat{H}_0$ is the HFB diagonalized part:

$$\hat{H}_0 - \sum_{\sigma=p,n}\lambda_\sigma^{(0000)}\hat{N}_\sigma = \sum_j E_j\alpha_j^+\alpha_j \tag{2.9}$$

with quasiparticle energies $E_j = \left[(\varepsilon_j - \lambda_{\sigma_j}^{(0000)})^2 + (\Delta_{\sigma_j}^{(0000)})^2\right]^{1/2}$. The chemical potential λ_σ and the gap parameter Δ_σ are expanded in powers of the x-parameters like the density matrices (e.g., eq. (2.5)). The pairing interaction is:

$$\hat{H}_p = -\frac{1}{4}\sum_{\sigma=p,n} G : \hat{P}^{\sigma+}P^{\sigma} : \qquad P^{\sigma} = \sum_j \delta_{\sigma_j\sigma}\, a_{\bar{j}}a_j \tag{2.10}$$

and the quadrupol interaction:

$$\hat{H}_Q = -\frac{1}{4}\sum_{\mu=-2}^{2}\varkappa_\mu \sum_{\sigma\sigma'=n,p}\eta^{\sigma\sigma'} : \hat{Q}_{2\mu}^{\sigma+} Q_{2\mu}^{\sigma'} : \tag{2.11}$$

As it was shown in a previous paper, the Hamiltonian can be extended in a power series over the basic operators, and with terms up to third power (order) becomes /20/:

$$\begin{aligned} \hat{h} = {} & \sum_{\sigma=p,n}\lambda_\sigma\hat{N}_\sigma + \frac{1}{2J}\hat{I}^2 + \omega_0\hat{n}_\beta + \omega_1\hat{n}_s + \omega_2\hat{n}_\gamma + \\ & + \chi_6\, O_{00}^{\beta g(+)}\hat{I}^2 + \frac{\chi_1}{2}\sqrt{\frac{3}{2}}\{\underbrace{O_1^{sg(-)}, T_1}_{00}\} + \frac{\chi_2}{2}\sqrt{5}\{\underbrace{O_2^{\gamma g(+)}, T_2}_{00}\} + \\ & + \frac{\chi_{10}}{2}\sqrt{\frac{3}{2}}\{\underbrace{O_1^{s\gamma(-)}, T_1}_{00}\} + \frac{\chi_{12}}{2}\sqrt{\frac{5}{2}}\{\underbrace{O_1^{s\gamma(-)}, T_1}_{00}\}, \end{aligned} \tag{2.12}$$

where $T_{\varkappa} = \underbrace{I_1 I_1 \ldots I_1}_{\varkappa}$ and $\hat{n}_{\beta} = B^{+}_{\beta 0} B_{\beta 0}$; $\hat{n}_s = -\sqrt{3}\, \underbrace{B^{+}_{s1} B_{s1}}_{00}$; $\hat{n}_{\gamma} = \sqrt{5}\, \underbrace{B^{+}_{\gamma 2} B_{\gamma 2}}_{00}$ are the phonon number operators for β, s and γ phonons, respectively. The terms with χ_0, χ_1 , χ_2 , χ_{10} and χ_{12} are the "direct" coupling terms between g-β, g-s; g-γ, s-β and s-γ bands, respectively.

The equations of motion [13]:

$$\langle x | [\hat{A} , \hat{H} - \hat{\lambda}] | x \rangle = 0 \tag{2.13}$$

together with relations (2.6), (2.3) allow one, as we shall see later, to solve our problem. In (2.13): $\hat{A} = \alpha^{+}_{i} \alpha^{+}_{j}$, $\hat{H}$ and $\hat{\lambda}$ are given by (2.8) and (2.12). For the sake of simplicity, in the present work we solve the equations of motion after expanding $|x\rangle$ from (2.1) up to first power in x in the vicinity of $x=0$ ($x = \{ x^{\varrho}_{\varkappa} \}$), i.e., not far from the ground state. Neverthless, the inclusion of the superband with $K^{\pi} = 1^{+}$ influences, as we shall see in the next section, even the first order effects in our theory such as: corrections to the cranking model formula for the moment of inertia, RPA equations for $K^{\pi} = 1^{+}$ state.

3. TRANSITION RATES

Following [12], the expansion of any hermitian multipole operator $\hat{F}_{LM}$, even under time reversal, with even L, truncated up to third - power terms, is:

$$\hat{F}_{LM} = a_0 R^{+}_{LM} + \frac{a_2}{2} \{ R^{+}_{LM}, I^2 \} + \frac{a_4}{2} \{ \underbrace{R^{+}_{L-2} , T_2}_{LM} \} +$$

$$+ b_0\, O^{\beta g (+)}_{00} R^{+}_{LM} + \frac{b_1}{2i} O^{\beta g (-)}_{00} [R^{+}_{LM} , I^2] +$$

$$+ c_0\, \underbrace{O^{sg(-)} R^{+}_{L-1}}_{LM} + \frac{c_1}{2i} [\underbrace{O^{sg(+)}_{1} R^{+}_{L-1}}_{LM} , I^2] + \frac{c_3}{2} \{ \underbrace{\underbrace{O^{sg(-)}_{1} R^{+}_{L-2}}_{L-1} , T_1}_{LM} \} + \tag{3.1}$$

$$+ d_0\, \underbrace{O^{\gamma g (+)}_{2} R^{+}_{L-2}}_{LM} + \frac{d_1}{2i} [\underbrace{O^{\gamma g (-)}_{2} R^{+}_{L-2}}_{LM} , I^2] +$$

$$+ b\, B^{+}_{\beta 0} B_{\beta 0} R^{+}_{LM} + c\, \underbrace{B^{+}_{s1} B_{s1}}_{00} R^{+}_{LM} + d\, \underbrace{B^{+}_{\gamma 2} B_{\gamma 2}}_{00} R^{+}_{LM} +$$

$$+ f_0\, \underbrace{C^{s\beta(-)}_{1} R^{+}_{L-1}}_{LM} + g_0\, \underbrace{C^{s\gamma(-)}_{1} R^{+}_{L-1}}_{LM} + h_0\, \underbrace{C^{\gamma\beta(+)}_{2} R^{+}_{L-2}}_{LM} ,$$

where $O^{(\pm)}$ are given in ref. [20].

The components of the density matrix ρ enable us to evaluate the coefficients in (3.1). Relations between these quantities can be found by making up to second commutators of eq.(3.1) with the seven hermitian operators $\hat{F}_{\varkappa}$ and avereging over $| \ \rangle$. In the left-hand side the commutators of $\alpha^{+}_{i} \alpha_{j}$ with $\hat{F}$, averaged over $| \ \rangle$, give $\rho^{(\nu^{\pm}_{i} \nu^{\pm}_{\beta} \nu^{\pm}_{s} \nu^{\pm}_{\gamma})}_{ji}$, and the right-hand side is calculated by using known algebraic properties. Thus we get relations:

$$\begin{aligned}
a_0 &= Tr (F_{L0}\, \rho^{(0000)}) \\
a_2 &= Tr (F_{L0}\, \rho^{(2000)}) + 2 \left[\frac{L(L-1)}{(L+1)(L+2)} \right]^{1/2} Tr (F_{L2}\, \rho^{(2000)}) \\
a_4 &= 4 \left[\frac{(2L-1)(2L-3)}{(L+1)(L+2)} \right]^{1/2} Tr (F_{L2}\, \rho^{(2000)}) \\
b_0 &= \frac{1}{2} Tr (F_{L0}\, \rho^{(01^{+}00)}) \quad , \quad b_1 = \left[\frac{1}{L(L+1)} \right]^{1/2} \frac{1}{i} Tr (F_{L1}\, \rho^{(1^{+}1^{+}00)}) \\
c_0 &= \left[\frac{2L-1}{2(L+1)} \right]^{1/2} \frac{1}{i} Tr (F_{L1}\, \rho^{(001^{-}0)}) \\
c_1 &= - \frac{1}{2L} \left\{ \left[\frac{2L-1}{2L} \right]^{1/2} Tr (F_{L0}\, \rho^{(1^{+}01^{+}0)}) + \left[\frac{(2L-2)(2L-1)}{(L+1)(L+2)} \right]^{1/2} Tr (F_{L2}\, \rho^{(1^{+}01^{+}0)}) \right\} \\
c_3 &= - \frac{1}{2L} [(2L-1)(2L-3)]^{1/2} \left\{ \left[\frac{L-1}{2L} \right]^{1/2} Tr (F_{L0}\, \rho^{(1^{+}01^{+}0)}) - \left[\frac{2L+2}{L+2} \right]^{1/2} Tr (F_{L2}\, \rho^{(1^{+}01^{+}0)}) \right\} \\
d_0 &= \left[\frac{(2L-1)(2L-3)}{(L+1)(L+2)} \right]^{1/2} Tr (F_{L2}\, \rho^{(0001^{-})}) \\
d_1 &= \frac{2}{L+2} \left[\frac{(2L-1)(2L-3)}{(L-1)(L+1)} \right]^{1/2} \frac{1}{i} Tr (F_{L1}\, \rho^{(1^{+}001^{+})}) \\
b &= \frac{1}{2} Tr (F_{L0}\, \rho^{(02000)}) \qquad c = - \frac{\sqrt{3}}{4} Tr (F_{L0}\, \rho^{(00200)}) \qquad d = \frac{\sqrt{5}}{4} Tr (F_{L0}\, \rho^{(00020)})
\end{aligned} \tag{3.2}$$

$$f_0 = \frac{1}{2L}\left[\frac{2L-1}{2L+2}\right]^{1/2} \mathrm{Tr}\left(F_{L1}\,\rho^{(0110)}\right) \qquad g_0 = \frac{1}{2L}\sqrt{\frac{5}{6}}\left[\frac{2L-1}{2L}\right]^{1/2} \mathrm{Tr}\left(F_{L1}\,\rho^{(0011)}\right)$$

$$h_0 = \frac{1}{2}\left[\frac{(2L-1)(2L-3)}{(L+1)(L+2)}\right]^{1/2} \mathrm{Tr}\left(F_{L2}\,\rho^{(0101)}\right).$$

The expansion in rel. (3.1) allows us to calculate the F_{LM} reduced matrix elements between $|\alpha IM\rangle$ without knowing these states directly, by using the algebraic properties of $B^+_{\alpha IM}$ and $\hat{I}$. Thus one obtains Table 1. The rotational motion kinematic separates from the dynamics, the latter remaining in the $a, b, c, \ldots$ coefficients, which have to be determined microscopically. In the case $F_{LM} = Q_{2M}$ (the E2 moment) these coefficients are related to a series of adiabatic and non-adiabatic effects, obtained experimentally. The coefficients a_0, b_0, c_0, d_0, b, c, d, f_0, g_0, h_0 give the Alaga transition rules in and between g, β, s and γ, bands, and a_2, a_4, b_1, c_1, c_3, d_1 give I-dependent corrections to them. The coefficient c_0 gives in first s-order the Alaga rules for the reduced probability of g-s transition; c_1 and c_3 determine an I-dependent correction to the g-s probabilities in second order (first in r and first in s); c indicates what is the deviation of the intrinsic E2 moment of the s-band with respect to the ground band in second s-order; f_0 (resp. g_0) gives in second order (first in s and first in β (resp. γ)) the rules for β-s (resp. γ-s) transitions. This meaning of the coefficients is easily seen from Table 1 and rel. (3.2). Details for the other coefficients can be found in ref. /13/.

4. MOMENT OF INERTIA, SUPERBAND ENERGY AND g-s TRANSITION RATES

The inclusion of $K^\pi = 1^+$ s-band modifies the first order density matrices with respect to ref. /13/. Following the same approximations one can solve the equations of motion (2.13). To find the first order density matrices we expand the state $|X\rangle$ from eq. (2.13) keeping terms up to first power in x. We obtain first commutators of $\alpha^+\alpha^+$, $\alpha^+\alpha$ and $\alpha\alpha$ with $\hat{F}$ operators averaged over $|\ \rangle$. They give us $\rho_{ij}^{(\nu_t^\pm \nu_\beta^\pm \nu_s^\pm \nu_\gamma^\pm)}$ in first ($\nu = 1$) order, given in Appendix. So the usual RPA equations for β- and γ- vibrations and for separating the spurious state $K^\pi = 1^+$ with $\omega = 0$ remain unchanged /13/.

We introduce the notations:

$$X^\sigma_\mu(\omega) = \sum_{ij} \delta_{\sigma_i \sigma} \frac{(E_i+E_j)\,(V^+_{ij})^2}{(E_i+E_j)^2 - \omega^2} \left|Q^{(2\mu)}_{ij}\right|^2 \qquad J(\omega) = \sum_{ij} \frac{(E_i+E_j)(V^-_{ij})^2}{(E_i+E_j)^2-\omega^2} \left|I^{(x)}_{ij}\right|^2$$

$$Y^\sigma_\mu(\omega) = \sum_{\sigma'} \eta^{\sigma\sigma'}_\mu \mathrm{Tr}\left(Q^{\sigma'}_{2\mu}\,\rho^{(01^-_\mu)}\right) \qquad P^\sigma_\mu(\omega) = \sum_{ij} \delta_{\sigma_i \sigma} \frac{(E_i+E_j)\,V^-_{ij}\,V^+_{ij}}{(E_i+E_j)^2 - \omega^2}\, \bar{Q}^{(2\mu)}_{ji} \tag{4.1}$$

$$Z^\sigma_\mu(\omega) = \sum_{ij} \delta_{\sigma_i \sigma} \frac{(V^+_{ij})^2 \left|Q^{(2\mu)}_{ij}\right|^2}{(E_i+E_j)^2-\omega^2} \qquad Q^\sigma_\mu(\omega) = \sum_{ij} \delta_{\sigma_i \sigma} \frac{V^-_{ij}\,V^+_{ij}\,I^{(x)}_{ij}\,\bar{Q}^{(2\mu)}_{ji}}{(E_i+E_j)^2 - \omega^2}.$$

Now one can obtain an equation, determining microscopically the strength χ_1 of the direct g-s coupling. For this purpose we use an additional kinematic relation

$$\mathrm{Tr}\left(I_x\,\rho^{(01^+_1)}\right) = 0 \tag{4.2}$$

following from the commutation relation of the transition operators with $\hat{I}$. Replacing the ρ matrix from (A.3) into eq. (4.2) we get:

$$0 = \omega_1 \varkappa_1 \sum_\sigma Q^\sigma_1(\omega_1)\, Y^\sigma_1 + \chi_1 J(\omega_1). \tag{4.3}$$

We notice the physical meaning of the coefficient $J(\omega)$:

$$\mathcal{J}(0) = \sum_{ij} \frac{(V_{ij}^-)^2 |I_{ij}^{(x)}|^2}{E_i + E_j} = \bar{\mathcal{J}} . \qquad (4.4)$$

$\mathcal{J}(0) = \bar{\mathcal{J}}$ is just the moment of inertia from the cranking model formula with pairing, obtained in ref. /13/, and $\mathcal{J}(\omega)$ means anadiabatic generalisation of this formula for $\omega \neq 0$.

In ref. /13/ the cranking model formula with pairing for the moment of inertia has been obtained from the kinematic relation which is still valid:

$$\mathrm{Tr}(I_x \, \rho^{(1^+0)}) = -1 . \qquad (4.5)$$

But in our new case, when we include coupling with a superband, $\rho^{(1^+0)}$ is modified. Inserting $\rho^{(1^+0)}$ from (A.4) into eq. (4.5) and using eq. (4.3) we obtain:

$$\bar{\mathcal{J}}^{-1} = \mathcal{J}^{-1}(0) = \mathcal{J}^{-1} - \frac{\chi_1^2}{\omega_1} . \qquad (4.6)$$

This means that the moment of inertia $\mathcal{J}$ from eq. (4.6) is renormalized with respect to its cranking value $\bar{\mathcal{J}}$ from (4.4): $\mathcal{J}$ is less than $\bar{\mathcal{J}}$.

One cane find also dispersion equations for the energy ω_1. We replace the $\rho^{(1^-0)}$ and $\rho^{(01_1^-)}$ matrices from (A.5) and (A.6) in the traces. We use the relation:

$$\rho_{ij}^{(1^+0)} = \frac{V_{ij}^-}{V_{ij}^+} \, i \rho_{ij}^{(1^-0)} \qquad (4.7)$$

and eq. (4.3). After elimination of $\mathrm{Tr}(Q_{21}^{\sigma} \rho^{(1^-0)})$ and transformations we get:

$$X_1^{\sigma}(\bar{\omega}_1) \sum_{\sigma'} \bar{\varkappa}^{\sigma\sigma'}(\bar{\omega}_1) \, i \, \mathrm{Tr}(Q_{21}^{\sigma'} \rho^{(01_1^-)}) = i \, \mathrm{Tr}(Q_{21}^{\sigma} \rho^{(01_1^-)}) . \qquad (4.8)$$

Here ω_1 is reduced by a factor $\alpha = (\mathcal{J}/\bar{\mathcal{J}})^{1/2}$ to $\bar{\omega}_1 = \alpha \omega_1$.

These equations are analogous to the usual RPA ones, but more complicated because of the renormalized strength $\bar{\varkappa}^{\sigma\sigma'}$:

$$\bar{\varkappa}^{\sigma\sigma'}(\omega) = \varkappa_1 \eta_1^{\sigma\sigma'} - \frac{\omega^2}{X_1^{\sigma}(0)} \, \frac{Z_1^{\sigma}(\omega) \, \bar{Q}_1^{\sigma'}(\omega)}{X_1^{\sigma}(\omega) \, \bar{P}_1^{\sigma}(\omega)}$$

$$\bar{P}_1^{\sigma}(\omega) = \frac{P_1^{\sigma}(\omega)}{\varkappa_1 X_1^{\sigma}(0)} + \frac{\eta_1^{\bar{\sigma}\sigma} \, P_1^{\bar{\sigma}}(\omega)}{1 - \varkappa_1 \eta_1^{\bar{\sigma}\bar{\sigma}} X_1^{\bar{\sigma}}(0)} \qquad \bar{Q}_1^{\sigma}(\omega) = \sum_{\sigma'} \eta_1^{\sigma\sigma'} Q_1^{\sigma'}(\omega) \qquad (4.9)$$

If $\sigma = n, p$, $\bar{\sigma} = p, n$.

One can see that $\bar{\varkappa}^{\sigma\sigma'}(\bar{\omega}_1)$ depends on energy $\bar{\omega}_1$, which modifies the RPA equations, as a consequence of including the "direct" g-s coupling into the theory.

Finding the moment of inertia from eq. (4.6), and the vibrational energies; for the s-band ω_1 from eq. (4.8), for the β- and γ-bands respectively ω_0 and ω_2 /13/, we solve the problem for energies of vibrational and rotational motion on the same footing.

To obtain transition probabilities one needs $\mathrm{Tr}(Q_{21}^{\sigma} \rho^{(01_1^-)})$. We use phonon normalization equation for s-phonons:

$$\sum_{ij} \frac{\rho_{ij}^{(01_1^-)} \rho_{ji}^{(01_1^+)}}{V_{ij}^+ V_{ji}^-} = 4i \qquad (4.10)$$

to normalize both traces $\mathrm{Tr}(Q_{21}^{\sigma} \rho^{(01_1^{\pm})})$. In eq. (4.10) we replace $\rho^{(01_1^{\pm})}$ from (A.6). We exclude $i \, \mathrm{Tr}(Q_{21}^{\sigma} \rho^{(1^+0)})$, and after transformations we get:

$$\frac{1}{2} \alpha \bar{\omega}_1 \sum_{ij} \frac{(E_i + E_j)(V_{ij}^+)^2}{[(E_i + E_j)^2 - \bar{\omega}_1^2]^2} \, |Q_{ij}^{(21)}|^2 \, \Big| \sum_{\sigma} \bar{\bar{\varkappa}}_{ij}^{\sigma\sigma}(\bar{\omega}_1) \, i \, \mathrm{Tr}(Q_{21}^{\sigma} \rho^{(01_1^-)}) \Big|^2 = 1, \qquad (4.11)$$

where

$$\bar{\bar{\varkappa}}_{ij}^{\sigma\sigma'}(\omega) = \varkappa_1 \eta_1^{\sigma\sigma'} - \frac{\omega^2}{(E_i+E_j)\,\chi_1^{\sigma}(0)} \; \frac{\bar{G}_1^{\sigma'}(\omega)}{\bar{P}_1^{\sigma}(\omega)} \; . \tag{4.12}$$

This is analogous to the equations obtained for β- and γ-traces in /13/. But the strength of quadrupol-quadrupol interaction $\bar{\bar{\varkappa}}_{ij}^{\sigma\sigma'}$ is renormalized even more than for the dispersion equation, and depends not only on ω_1, but also on quasiparticle energies. In this way one finds $Tr(G_{21}^{\sigma} \rho^{(01_1^-)})$ and, by eq. (3.2), the coefficient c_i defining the g-s transition probability according to eq.(3.1) and Table 1.

5. CONCLUSION

This paper shows that an inclusion of the fourth band with $K^{\pi} = 1^+$ in our model gives modifications of the main quantities characterizing nuclear rotations, vibrations and their coupling even in first order: the cranking model formula for the moment of inertia, the RPA results for the vibrational energy ω_1, and defines the g-s transition probability. Second order effects will be studied in a next paper.

The importance of the "direct" coupling terms with χ-coefficients in the Hamiltonian eq. (2.12), being responsible for the renormalization, is going to be investigated numerically: its physical meaning is to modify usual cranking model rotation-quasiparticle and RPA vibration-quasiparticle coupling by taking higher-order "direct" band coupling effects into account. It unifies microscopic cranking model and RPA type "smooth" coupling effects with semi-phenomenological Coriolis band coupling effects.

The inclusion of "direct" coupling with a super band would be of particular importance in high spin microscopic calculations, which are in project.

APPENDIX

NOTATIONS AND FIRST ORDER DENSITY MATRICES

We introduce short notations:

$$\bar{Q}_{ij}^{(2\mu)} = (-)^{\mu} \frac{Q_{ij}^{(2\mu)} + Q_{ij}^{(2-\mu)}}{1+\delta_{\mu 0}} \tag{A.1}$$

$$\rho^{(\nu_i^{\pm}000)} = \rho^{(\nu_i^{\pm}0)} \; ; \qquad \rho^{(\nu_i^{\pm}\nu_{\beta}^{\pm}00)} = \rho^{(\nu_i^{\pm}\nu_{\beta 0}^{\pm})}$$
$$\rho^{(\nu_i^{\pm}0\nu_s^{\pm}0)} = \rho^{(\nu_i^{\pm}\nu_{s1}^{\pm})} \; ; \qquad \rho^{(\nu_i^{\pm}00\nu_{\gamma}^{\pm})} = \rho^{(\nu_i^{\pm}\nu_{\gamma 2}^{\pm})} \tag{A.2}$$

For β-, s- and γ- vibrations we have ($\mu = 0,1,2$ resp.) first order density matrices:

$$\rho_{ij}^{(01_{\mu}^-)} = \varkappa_{\mu} Y_{\mu}^{\sigma_i} \frac{(E_i+E_j)V_{ij}^+}{(E_i+E_j)^2-\omega_{\mu}^2} \left[\bar{Q}_{ij}^{(2\mu)} V_{ij}^+ + \delta_{\mu 0}\delta_{ij}(\lambda'_{\sigma_i} V_{ii}^+ + \Delta'_{\sigma_i} U_{ii}^-)\right] + $$
$$+ \frac{2}{i} \chi_1 \omega_1 \delta_{\mu 1} \frac{V_{ij}^+ V_{ij}^- \, I_{ij}^{(x)}}{(E_i+E_j)^2 - \omega_1^2} \tag{A.3}$$

$$\rho_{ij}^{(01_{\mu}^+)} = \frac{i\,\omega_{\mu}}{E_i+E_j} \frac{V_{ij}^-}{V_{ij}^+} \rho_{ij}^{(01_{\mu}^-)} + 2\chi_1 \delta_{\mu 1} \frac{(V_{ij}^-)^2 I_{ij}^{(x)}}{E_i+E_j} \; .$$

For rotations we get:

$$\rho_{ij}^{(1^+0)} = -\frac{1}{J} \frac{(V_{ij}^-)^2 |I_{ij}^{(x)}|^2}{E_i+E_j} - \frac{1}{2}\chi_1 \frac{V_{ij}^-}{V_{ij}^+} \frac{i\rho_{ij}^{(01_1^-)}}{E_i+E_j} \; ; \qquad \rho_{ij}^{(1^-0)} = i \frac{V_{ij}^-}{V_{ij}^+} \rho_{ij}^{(1^+0)} \tag{A.4}$$

or:

$$\rho_{ij}^{(1^-0)} = \varkappa_1 \sum_{\sigma} \eta_1^{\sigma_i\sigma} \, Tr(Q_{21}^{\sigma} \rho^{(1^-0)}) \frac{(V_{ij}^+)^2 \bar{Q}_{ij}^{(21)}}{E_i+E_j} \tag{A.5}$$

Using the relation between $\rho^{(1^+0)}$ and $\rho^{(1^-0)}$ eq. (A.2), and also (A.3), (A.4), (A.5), we get:

$$\rho_{ij}^{(01_1^-)} = \frac{1}{i}\varkappa_1 \frac{(V_{ij}^+)^2\ \bar{Q}_{ij}^{(21)}}{(E_i+E_j)^2-\bar{\omega}_1^2}\left[(E_i+E_j)\sum_\sigma \eta_1^{\zeta\sigma}\, i\,Tr(Q_{21}^\sigma \rho^{(01_1^-)}) - 2\frac{\bar{\omega}_1}{\alpha}\chi_1 J \sum_\sigma \eta_1^{\zeta\sigma}\, i\,Tr(Q_{21}^\sigma \rho^{(1^-0)})\right]$$

$$\rho_{ij}^{(01_1^+)} = \varkappa_1 \frac{V_{ij}^+ V_{ij}^-\ \bar{Q}_{ij}^{(21)}}{(E_i+E_j)^2-\bar{\omega}_1^2}\left[\alpha\bar{\omega}_1\sum_\sigma \eta_1^{\zeta\sigma}\, i\,Tr(Q_{21}^\sigma \rho^{(01_1^-)}) - 2\chi_1 J (E_i+E_j)\sum_\sigma \eta_1^{\zeta\sigma}\, i\,Tr(Q_{21}^\sigma \rho^{(1^-0)})\right] \tag{A.6}$$

After elimination of $Tr(Q_{21}^\sigma \rho^{(1^-0)})$ we obtain:

$$\rho_{ij}^{(01_1^-)} = \frac{1}{i}\frac{(E_i+E_j)(V_{ij}^+)^2}{(E_i+E_j)^2-\bar{\omega}_1^2}\bar{Q}_{ij}^{(21)}\sum_\sigma \overline{\overline{\varkappa}}_{ij}^{\zeta\sigma}(\bar{\omega}_1)\, i\,Tr(Q_{21}^\sigma \rho^{(01_1^-)})$$

$$\rho_{ij}^{(01_1^+)} = \alpha\bar{\omega}_1\frac{V_{ij}^+ V_{ij}^-}{(E_i+E_j)^2-\bar{\omega}_1^2}\bar{Q}_{ij}^{(21)}\sum_\sigma \widetilde{\widetilde{\varkappa}}_{ij}^{\zeta\sigma}(\bar{\omega}_1)\, i\,Tr(Q_{21}^\sigma \rho^{(01_1^-)}) \tag{A.7}$$

where $\overline{\overline{\varkappa}}_{ij}^{\sigma\sigma'}(\omega)$ has been introduced by eq. (4.12), and:

$$\widetilde{\widetilde{\varkappa}}_{ij}^{\sigma\sigma'}(\omega) = \varkappa_1 \eta_1^{\sigma\sigma'} - \frac{(E_i+E_j)\ \bar{Q}_1^{\sigma'}(\omega)}{X_1^\sigma(0)\ \bar{P}_1^\sigma(\omega)}.$$

Table 1.

Transition		Reduced matrix element $\langle \alpha_f I_f \Vert F_{L=2} \Vert \alpha_i I_i \rangle$		
α_f	α_i	0^{th} order	1^{st} order	2^{nd} order
g	g	a_0		$\frac{1}{2}a_2[I_f(I_f+1)+I_i(I_i+1)] - 2\sqrt{\frac{2}{3}}a_4[I(I+1)-\frac{3}{4}]\delta_{if}$
β	β	a_0		$b+\frac{1}{2}a_2[I_f(I_f+1)+I_i(I_i+1)] - 2\sqrt{\frac{2}{3}}a_4[I(I+1)-\frac{3}{4}]\delta_{if}$
s	s	a_0		$-\sqrt{\frac{1}{3}}c + \frac{1}{2}a_2[I_f(I_f+1)+I_i(I_i+1)] - 2\sqrt{\frac{2}{3}}[I(I+1)-\frac{3}{4}]\frac{I(I+1)}{I(I+1)-3.1^2}\delta_{if}$
γ	γ	a_0		$\sqrt{\frac{1}{5}}d + \frac{1}{2}a_2[I_f(I_f+1)+I_i(I_i+1)] - 2\sqrt{\frac{2}{3}}[I(I+1)-\frac{3}{4}]\frac{I(I+1)}{I(I+1)-3.2^2}\delta_{if}$
g	β		b_0	$-\frac{1}{2}b_1[I_f(I_f+1)-I_i(I_i+1)]$
g	s		$-\sqrt{\frac{1}{2}}c_0$	$\frac{1}{2}\sqrt{\frac{1}{2}}c_1[I_f(I_f+1)-I_i(I_i+1)] - \sqrt{\frac{1}{2}}c_3[I_i\delta_{I_f,I_i+1}-(I_i+1)\delta_{I_f,I_i-1}+\frac{(2I-1)(2I+3)}{3}\delta_{if}]$
g	γ		d_0	$-\frac{1}{2}d_1[I_f(I_f+1)-I_i(I_i+1)]$
s	β			$\sqrt{\frac{1}{2}}f_0$
s	γ			$\sqrt{\frac{3}{10}}g_0$
β	γ			h_0

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