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Модель связанных полос в четно-четных ядрах

Предложена модель связанных ротационных полос на основе развития гамильтониана по элементарным операторам переходов, включающая прямую вращательно-колебательную связь с β -, γ - и $K^{\pi} = 1^{+}$фононами. Ее можно рассматривать как обобщение модели пересекающихся полос. Обсужден метод диагонализации в подходяще сконструированном многофононном базисе.

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Model of Coupled Bands in Even-Even Nuclei

A model of coupled rotational bands is proposed on the basis of an expansion of the Hamiltonian in terms of elementary transition operators, including direct rotation-vibrational coupling with β -, γ - and $K^{\pi} = 1^{+}$ phonons. A method for diagonalization in a suitable constructed multi-phonon basis is discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

The intensive research in the field of heavy-ion reactions during the last years allowed to reach high-spin rotational states in a number of nuclei. Various interesting effects (back- and down-bending in the ground state band and even in the β-band,for cranking and band crossing; see for example refs. /1,2/) which were found by "in beam" spectroscopy, stimulated extensive theoretical investigations on the possible mechanisms of these phenomena. Besides the efforts towards a more adequate microscopic description, which have lead as yet at best to a qualitative agreement with the experiment, a plenty of phenomenological models has been proposed. A brief review, covering many of them, can be found in ref. /3/ . These models differ strongly in degree of phenomenology ranging from formal schemes to semi-microscopic calculations.

We shall point out only the band hybridization model /4,5/here, since, as it is generally accepted by many authors, almost any physical mechanism might be expressed in its language. The intersecting bands have been experimentally identified in three back-bending nuclei: ^{154}Gd [6], ^{154}Dy [7] and ^{156}Dy [8] and in other cases only parts of higher bands have been observed. The schematic calculations [4], dealing with two nuclei, indicate the necessity of taking a $K^{\pi} = 1^+$ rotational band into account. This allows a reproduction of the experimental picture of the back- and down-bending effects.

This paper presents a model of coupled rotational bands in even-even nuclei. The model is derived in a natural way from the theory of coupled modes in terms of elementary transition operators, developed in a series of papers [9-11] and generalizes the three-band example, discussed earlier [12,13]. The present model has the advantage of handling a number of bands, which would be difficult in a microscopic theory.

2. MODEL HAMILTONIAN

The description of the nuclear rotation-vibrational motion is based on transition operators $B^+_{\alpha IM}$ [9,10] which are irreducible tensors of rank I with projection M.

$$B^+_{\alpha IM} = \sum_{I_1 M_1 I_2 M_2} \left[(n+1)(2I_1+1)(2I_2+1)\right]^{1/2} \begin{pmatrix} I & I_1 & I_2 \\ K & K_1 & -K_2 \end{pmatrix} \begin{pmatrix} I & I_1 & I_2 \\ M & M_1 & -M_2 \end{pmatrix} (-1)^{M_2-K_2} \times$$
$$\times\ |\nu_2 I_2 M_2\rangle\langle M_1 I_1 \nu_1| , \qquad (2.1)$$

where α denotes the type of the phonon, created by $B^+_{\alpha IM}$, as well as the K-number of the resultant state. The states in eq. (2.1) are supposed to have good angular momentum, definite number of phonons of type α : n in the state "1" and n+1 in the state "2", K-numbers respectively K_1 and K_2 and they do not need further specification (the index ν includes both K and n numbers). In the case n=K=0, $\nu_1 = \nu_2$ the operator does not create any phonon and realizes transitions inside the same rotational band. It is denoted by R^+_{IM}.

We limit our treatment to three types of phonons: β (I=K=0), S (I=K=1) and γ (I=K=2); as pointed out above, the second one was proposed in refs. [4]. It can be proved [11] that in this case the basic operators are:

$$B^+_{\alpha|K|M},\quad B_{\alpha|K|M},\quad R^+_{2M} \quad \text{and} \quad \vec{I}\,, \qquad (2.2)$$

where $\alpha = \beta, S, \gamma$ and $\vec{I}$ is the angular momentum operator. In order to deal with hermitian irreducible tensor operators having time-reversal symmetries ($\pm$ iveness under time reversal), we introduce the following combinations:

$$O^{\beta g(+)}_{00} = B^+_{\beta 00} + B_{\beta 00}\,, \qquad O^{\beta g(-)}_{00} = \frac{1}{i}\left(B^+_{\beta 00} - B_{\beta 00}\right),$$
$$O^{Sg(+)}_{1M} = \frac{1}{i}\left(B^+_{S1M} - (-)^M B_{S1-M}\right), \quad O^{Sg(-)}_{1M} = B^+_{S1M} + (-)^M B_{S1-M}\,,$$
$$O^{\gamma g(+)}_{2M} = B^+_{\gamma 2M} + (-)^M B_{\gamma 2-M}\,, \quad O^{\gamma g(-)}_{2M} = \frac{1}{i}\left(B^+_{\gamma 2M} - (-)^M B_{\gamma 2-M}\right),$$
$$O^{S\beta(+)}_{1M} = \frac{1}{i}\left(B^+_{S1M} B_{\beta 00} - (-)^M B^+_{\beta 00} B_{S1-M}\right), \qquad (2.3)$$
$$O^{S\beta(-)}_{1M} = B^+_{S1M} B_{\beta 00} + (-)^M B^+_{\beta 00} B_{S1-M}\,,$$
$$O^{S\gamma(+)}_{1M} = \frac{1}{i}\left(\underbrace{B^+_{S1} B_{\gamma 2}}_{1M} - \underbrace{B^+_{\gamma 2} B_{S1}}_{1M}\right), \quad O^{S\gamma(-)}_{1M} = \underbrace{B^+_{S1} B_{\gamma 2}}_{1M} + \underbrace{B^+_{\gamma 2} B_{S1}}_{1M}$$
$$O^{\gamma\beta(+)}_{2M} = B^+_{\gamma 2M} B_{\beta 00} + (-)^M B^+_{\beta 00} B_{\gamma 2-M}$$
$$O^{\gamma\beta(-)}_{2M} = \frac{1}{i}\left(B^+_{\gamma 2M} B_{\beta 00} - (-)^M B^+_{\beta 00} B_{\gamma 2-M}\right),$$

where the notation $\underbrace{\quad}_{LM}$ means coupling to angular momentum L by summation with Clebsch-Gordan coefficients.

Any physical quantity, i.e., a multipole operator acting between the states of our model space, which includes states with any number of three types of phonons – β, ς and ℓ, can be expanded in a power series in the basic operators. In particular, the model Hamiltonian is obtained as an irreducible tensor operator of rank zero:

$$\hat{h} = \hat{h}_R + \hat{h}_V + \hat{h}_C \tag{2.4}$$

with:

$$\hat{h}_R = \frac{\vec{I}^2}{2\mathcal{J}}$$

$$\hat{h}_V = \omega_0 \hat{n}_\beta + \omega_1 \hat{n}_\varsigma + \omega_2 \hat{n}_\ell \tag{2.5}$$

$$\hat{h}_C = \chi_0 O_{00}^{\beta g(+)} \vec{I}^2 + \chi_1 \sqrt{\tfrac{3}{2}}\, \underbrace{O_1^{\varsigma g(-)} T_1}_{0} + \chi_2 \sqrt{5}\, \underbrace{O_2^{\ell g(+)} T_2}_{0} + \chi_{10} \sqrt{\tfrac{3}{2}}\, \underbrace{O_1^{\varsigma\beta(-)} T_1}_{0} + \chi_{12} \sqrt{\tfrac{5}{2}}\, \underbrace{O_1^{\varsigma\ell(-)} T_1}_{0} + \chi_{20} \sqrt{5}\, \underbrace{O_2^{\ell\beta(+)} T_2}_{0},$$

where $T_{LM} = \underbrace{\vec{I}\,\vec{I} \dots \vec{I}}_{LM}$ means coupling of L operators $\vec{I}$ to maximum angular momentum L. The phonon number operators are /14/ :

$$\hat{n}_\beta = B^+_{\beta 00} B_{\beta 00}, \quad \hat{n}_\varsigma = -\sqrt{3}\, \underbrace{B^+_{\varsigma 1} B_{\varsigma 1}}_{0}, \quad \hat{n}_\ell = \sqrt{5}\, \underbrace{B^+_{\ell 2} B_{\ell 2}}_{0} \tag{2.6}$$

and they generate the vibrational part $\hat{h}_V$ of $\hat{h}$. The terms in the rotation-vibrational coupling part $\hat{h}_C$ represent the β-g, ς-g, ℓ-g, ς-β, ς-ℓ and ℓ-β coupling terms with their respective coupling strengths. Eq. (2.4) can be obtained as a special case of the general expansion /11/, including lowest order diagonal and non-diagonal terms, acting between the g, β, ς and ℓ bands. It includes terms of second up to fourth order.

We make the following modification of the rotational part:

$$\hat{h}_R = \sum_{\alpha = g,\beta,\varsigma,\ell} \frac{\vec{I}^2 \hat{P}_\alpha}{2\mathcal{J}_\alpha} \tag{2.7}$$

with an intraband projection operator $\hat{P}_g = 1$, $\hat{P}_\alpha = P(n_\alpha)$,
$P(n_\alpha) = \begin{cases} 0, & \text{for } n_\alpha = 0 \\ 1, & \text{for } n_\alpha \neq 0 \end{cases}$, where n_α is the number of phonons of type α and $\mathcal{J}_g$ is the moment of inertia of the ground state band, and $[\mathcal{J}_g^{-1} + \mathcal{J}_\alpha^{-1}]^{-1}$ – of any band with α-phonons only. By means of eq. (2.7) we take into account the differences between the moments of inertia of our four bands in a simple way. We can rewrite the coupling Hamiltonian $\hat{h}_C$ in a form, convenient for further transformations:

$$\hat{h}_C = \chi_0 (B^+_{\beta 00} + B_{\beta 00}) \vec{I}^2 + \chi_1 \sqrt{\tfrac{3}{2}}\, \underbrace{(\underbrace{B^+_{\varsigma 1} + B_{\varsigma 1}}_{1}) T_1}_{0} + \chi_2 \sqrt{5}\, \underbrace{(\underbrace{B^+_{\ell 2} + B_{\ell 2}}_{2}) T_2}_{0} + \chi_{10} \sqrt{\tfrac{3}{2}}\, \underbrace{(\underbrace{B^+_{\beta 0} B_{\varsigma 1} + B_{\beta 0} B^+_{\varsigma 1}}_{1}) T_1}_{0} + \chi_{12} \sqrt{\tfrac{5}{2}}\, \underbrace{(\underbrace{B^+_{\varsigma 1} B_{\ell 2} + B^+_{\ell 2} B_{\varsigma 1}}_{1}) T_1}_{0} + \chi_{20} \sqrt{5}\, \underbrace{(\underbrace{B^+_{\beta 0} B_{\ell 2} + B_{\beta 0} B^+_{\ell 2}}_{2}) T_2}_{0}. \tag{2.8}$$

3. BASIC STATES AND MODEL HAMILTONIAN MATRIX

The basis of the operators (2.2) acting on the ground state is difficult to be handled since the operators have to be coupled by means of Clebsch-Gordan technique to definite angular momenta. To calculate the Hamiltonian matrix elements between such states is a complicated procedure. For this reason we have introduced /14/ the zero rank operators:

$$b_0^+ = B^+_{\beta 0} \vec{I}^2, \quad b_1^+ = \underbrace{B^+_{\varsigma 1} T_1}_{0}, \quad b_2^+ = \underbrace{B^+_{\ell 2} T_2}_{0}. \tag{3.1}$$

Together with their hermitian conjugates they obey definite commutation relations /14/.

The zero rank operators (3.1) can not bring any angular momentum into the state they create, so we make them acting not only on the ground state, but also on each state of the ground-state rotational band with appropriate values of I and M: $|0IM\rangle$. Such procedure is very convenient since the model Hamiltonian $\hat{h}$ is invariant with respect to rotations, and thus it does not mix different values of I and M. Therefore the problem can be solved for each IM separately. Thus our basic states are:

$$|\alpha IM\rangle = |n_0 n_1 n_2 IM\rangle = N_\alpha \left(b_0^+\right)^{n_0} \left(b_1^+\right)^{n_1} \left(b_2^+\right)^{n_2} |0IM\rangle , \qquad (3.2)$$

where n_0, n_1, n_2 are the numbers of β, S, γ phonons (the eigenvalues of $\hat{n}_\beta$, $\hat{n}_S$, $\hat{n}_\gamma$), respectively.

The calculation of the normalizing factor N_α, performed elsewhere /14/, requires all the commutators of the operators in eqs. (3.1)/14/ and it is useful to introduce the K-number operator /9/:

$$\hat{K} = \sqrt{3}\, \left[R_1^+ \vec{I}_1\right]_0 \qquad (3.3)$$

which gives the K-number of the state when acting on the state, defined by eq. (3.2). It is easily seen indeed that its K-number eigenvalue is $K = n_1 + 2n_2$. Another operator sequence in eq.(3.2), namely with the operators of type "2" preceeding these of type "1", yields the same state, but different expressions for N_α. Thus one gets /14/ two expressions:

$$N_\alpha = \left[\prod_{\nu_0=1}^{n_0} \nu_0 D^2(I,0) \prod_{\nu_1=1}^{n_1} \frac{\nu_1}{6} D(I,K_{\nu_1 n_2}-1) \prod_{\nu_2=1}^{n_2} \frac{\nu_2}{20} D(I,K_{0\nu_2}-1) D(I,K_{0\nu_2}-2)\right]^{-1/2} =$$

$$= \left[\prod_{\nu_0=1}^{n_0} \nu_0 D^2(I,0) \prod_{\nu_1=1}^{n_1} \frac{\nu_1}{6} D(I,K_{\nu_1 0}-1) \prod_{\nu_2=1}^{n_2} \frac{\nu_2}{20} D(I,K_{n_1\nu_2}-1) D(I,K_{n_1\nu_2}-2)\right]^{-1/2}, \qquad (3.4)$$

where $D(I,K) = I(I+1) - K(K+1)$ and $K_{n_1 n_2} = n_1 + 2n_2$. The states (3.2) are orthonormal. The D-coefficients vanish at each given value of the spin I for $K = I + 1$. This reflects the fact that a band, built up on a phonon state with definite K-number does not contain rotational states with spin $I < K$.

Before calculating the matrix elements of $\hat{h}$, it is more convenient if we express the Hamiltonian (2.8) in terms of the new operators (3.1). It is only the term with χ_{12} in eq. (2.8) which needs special algebraic transformations - the other ones being obtained directly using eqs. (3.1). From ref. /14/ we have:

$$\left[\left[B_{S1}^+ B_{\gamma 2}\right]_1 \vec{I}_1\right]_0 = \frac{2\sqrt{3}}{\hat{K}^2 - \hat{K} - \vec{I}^2}\, b_1^+ b_2 \qquad \left[\left[B_{\gamma 2}^+ B_{S1}\right]_1 \vec{I}_1\right]_0 = 2\sqrt{3}\, b_2^+ b_1 \frac{1}{\hat{K}^2 - \hat{K} - \vec{I}^2} \qquad (3.5)$$

and finally eq. (2.8) becomes:

$$\hat{h}_c = \chi_0 (b_0^+ + b_0) + \chi_1 \sqrt{\tfrac{3}{2}} (b_1^+ + b_1) + \chi_2 \sqrt{5} (b_2^+ + b_2) +$$

$$+ \chi_{10} \sqrt{\tfrac{3}{2}} \frac{1}{\vec{I}^2} (b_0^+ b_1 + b_1^+ b_0) + \chi_{12} \sqrt{\tfrac{5}{2}}\, 2\sqrt{3} \left(\frac{1}{\hat{K}^2 - \hat{K} - \vec{I}^2} b_1^+ b_2 + b_2^+ b_1 \frac{1}{\hat{K}^2 - \hat{K} - \vec{I}^2}\right) +$$

$$+ \chi_{20} \sqrt{5} \frac{1}{\vec{I}^2} (b_0^+ b_2 + b_2^+ b_0). \qquad (3.6)$$

In this representation it is easy to calculate the Hamiltonian matrix elements with the help of eqs. (3.4) - the first of them is suitable for some of the terms in eq. (3.6), while the second expression is required by the other ones. Finally, we shall give an expression, ready for the calculation of the Hamiltonian matrix elements between the orthonormal states (3.2):

$$\hat{h} |n_0 n_1 n_2\rangle = \left[\left(\frac{1}{2\mathcal{J}_g} + \sum_{\alpha=\beta,S,\gamma} \frac{P(n_\alpha)}{2\mathcal{J}_\alpha}\right) D(I,0) + \sum_{\alpha=\beta,S,\gamma} \omega_\alpha n_\alpha\right] |n_0 n_1 n_2\rangle +$$

$$
\begin{aligned}
&+ \chi_0 D(I,0)\left[n_0^{1/2}|n_0-1\; n_1 n_2> + (n_0+1)^{1/2}|n_0+1\; n_1 n_2>\right] + \\
&+ \frac{\chi_1}{2}\Big\{\left[n_1 D(I,K-1)\right]^{1/2}|n_0\; n_1-1\; n_2> + \\
&\qquad + \left[(n_1+1) D(I,K)\right]^{1/2}|n_0\; n_1+1\; n_2>\Big\} + \\
&+ \frac{\chi_2}{2}\Big\{\left[n_2 D(I,K-1) D(I,K-2)\right]^{1/2}|n_0 n_1\; n_2-1> + \\
&\qquad + \left[(n_2+1) D(I,K+1) D(I,K)\right]^{1/2}|n_0 n_1 n_2+1>\Big\} + \\
&+ \frac{\chi_{10}}{2}\Big\{\left[(n_0+1) n_1 D(I,K-1)\right]^{1/2}|n_0+1\; n_1-1\; n_2> + \\
&\qquad + \left[n_0(n_1+1) D(I,K)\right]^{1/2}|n_0-1\; n_1+1\; n_2> - \\
&- \frac{\chi_{12}}{2}\Big\{\left[(n_1+1) n_2 D(I,K-1)\right]^{1/2}|n_0\; n_1+1\; n_2-1> + \qquad (3.7) \\
&\qquad + \left[n_1(n_2+1) D(I,K)\right]^{1/2}|n_0\; n_1-1\; n_2+1>\Big\} + \\
&+ \frac{\chi_{20}}{2}\Big\{\left[(n_0+1) n_2 D(I,K-1) D(I,K-2)\right]^{1/2}|n_0+1\; n_1\; n_2-1> + \\
&\qquad + \left[n_0(n_2+1) D(I,K+1) D(I,K)\right]^{1/2}|n_0-1\; n_1\; n_2+1>\Big\}.
\end{aligned}
$$

After truncating the basis by the inclusion of a resonable number of low lying phonon states of the type (3.2), one may evaluate the matrix elements of the model Hamiltonian (2.4) by means of eq. (3.7) and diagonalize it by standard numerical methods to fit the experimental spectrum with an optimal choice of the parameters $\mathcal{J}_\alpha$, ω_α and χ_μ.

4. SEPARABLE SOLUTION APPROXIMATION

In order to simplify the problem, in this section we are going to consider only two types of phonons – β and γ, or β and S phonons. The K-number of the state is now simply $K_{n_1 0} = n_1$, or $K_{0 n_2} = 2n_2$, and eq. (3.4) reduces to:

$$
\begin{aligned}
&N_1(n_0 n_1) = \left[\prod_{\nu_0=1}^{n_0} \nu_0 D^2(I,0) \prod_{\nu_1=1}^{n_1} \frac{\nu_1}{6} D(I,K_{\nu_1 0}-1)\right]^{-1/2} \quad \text{for } S \text{ phonons or:} \\
&N_2(n_0 n_2) = \left[\prod_{\nu_0=1}^{n_0} \nu_0 D^2(I,0) \prod_{\nu_2=1}^{n_2} \frac{\nu_2}{20} D(I,K_{0\nu_2}-1) D(I,K_{0\nu_2}-2)\right]^{-1/2} \quad \text{for } \gamma \text{ phonons.}
\end{aligned}
\qquad (4.1)
$$

Thus eq. (3.8) becomes:

$$
\begin{aligned}
\hat{h}|n_0 n_\mu> &= \left[E_0(n_0) + E_\mu(n_\mu)\right]|n_0 n_\mu> + \qquad (4.2) \\
&+ h_0(n_0)|n_0-1\; n_\mu> + h_0(n_0+1)|n_0+1\; n_\mu> + h_\mu(n_\mu)|n_0\; n_\mu-1> + h_\mu(n_\mu+1)|n_0\; n_\mu+1>
\end{aligned}
$$

with the notations:

$$
\begin{aligned}
&E_0(n_0) = \left[\frac{1}{2\mathcal{J}_3} + \frac{P(n_0)}{2\mathcal{J}_0}\right] D(I,0) + \omega_0 n_0 , \\
&E_\mu(n_\mu) = \frac{P(n_\mu)}{2\mathcal{J}_\mu} D(I,0) + \omega_\mu n_\mu , \qquad (4.3) \\
&h_0(n_0) = \chi_0\, n_0^{1/2} D(I,0) , \qquad h_1(n_1) = \frac{\chi_1}{2}\left[n_1 D(I, n_1-1)\right]^{1/2} , \\
&h_2(n_2) = \frac{\chi_2}{2}\left[n_2 D(I,2n_2-1) D(I,2n_2-2)\right]^{1/2} ,
\end{aligned}
$$

where μ = 1 or 2 refers to the case of S or γ phonons.

For a given spin value I, the basis is composed of the states

$|n_0 n_\mu\rangle$ with all the possible combinations of $n_0 = 0, 1, 2, \ldots$ and $2n_2 \leq$ I. The Schrödinger equation is: $\hat{h}|\Psi\rangle = E|\Psi\rangle$, where $|\Psi\rangle = \sum_{n_0 n_\mu} a(n_0 n_\mu)|n_0 n_\mu\rangle$. It is useful to write this matrice equation as an infinite system of equations, since $\hat{h}$ has many zero matrix elements. Then the row, labelled by $n_0 n_\mu$, is:

$$\left[E_0(n_0) + E_\mu(n_\mu) - E\right] a(n_0 n_\mu) = h_0(n_0)\, a(n_0 - 1, n_\mu) + $$
$$+ h_0(n_0+1)\, a(n_0+1, n_\mu) + h_\mu(n_\mu)\, a(n_0, n_\mu - 1) + h_\mu(n_\mu+1)\, a(n_0, n_\mu+1). \tag{4.4}$$

By inserting $a(n_0 n_\mu) = a_0(n_0) a_\mu(n_\mu)$, $E = E_0 + E_\mu$, eqs. (4.4) can be separated in two parts: an infinite sub-system for the β-phonons and a finite one (remember that D(I,K) vanishes for K = I) for the μ-phonons (μ = 1 or 2). After separation the phonon number is a natural index, ordering the system, so that the Kth equation is:

$$\left[E_\nu - E_\nu(K)\right] a_\nu(K) = h_\nu(K)\, a_\nu(K-1) + h_\nu(K+1)\, a_\nu(K+1), \tag{4.5}$$

where $\nu = 0, \mu$, i.e., $\nu = 0, 1$ or $\nu = 0, 2$. The g.s.b. rotational energy is: $E_0(0) = I(I+1)/2\mathcal{J}_g$. Defining:

$$\Psi_\nu(K) = a_\nu(K)/a_\nu(K-1)$$

one gets a recurrence relation for $\Psi_\nu(K)$:

$$h_\nu(K)/\Psi_\nu(K) + h_\nu(K+1)\Psi_\nu(K+1) = E_\nu - E_\nu(K) \tag{4.6}$$

and an equation for $\Psi_\nu(1)$:

$$h_\nu(1)\Psi_\nu(1) = E_\nu - E_\nu(0), \tag{4.7}$$

which allows to obtain the following eigen-value infinite fraction equation:

$$E_\nu - E_\nu(0) = \cfrac{h_\nu^2(1)}{E_\nu - E_\nu(1) - \cfrac{h_\nu^2(2)}{E_\nu - E_\nu(2) - \ldots}} \tag{4.8}$$

Both systems ($\nu = 0, \mu$) can be solved separately. Let E_ν be a solution of eq. (4.8). Then one can get $\Psi_\nu(K)$ for any K, using eq. (4.6) with $\Psi_\nu(1)$ from eq. (4.7). This means that the system (4.6), K= 1,2,... is satisfied. In calculations one may cut off the infinite chain fraction for β-phonons ($\nu = 0$) at a fixed number, say 30 phonons, and include in it, in the case of μ-phonons ($\nu = \mu$, finite fraction) all the allowed states. Eq. (4.8) can be solved only numerically by evaluating its right-hand side for each value of E_ν. Then one gets $\Psi_\nu(1)$ from eq. (4.7) and one may find each $\Psi_\nu(K)$ from eqs. (4.6). They give the eigenvector components $a_\nu(K)$, K= 0,1,2,... within a constant factor, which may be used to normalize the vector.

An exact solution can be found in the case $\nu = 0$ (β-phonons) if we take:

$$\Psi_0(K) = -\chi_0 D(I,0)/\omega_0\sqrt{K}, \quad \mathcal{J}^{-1} = 0. \tag{4.9}$$

Then the equation set (4.6) is fulfilled since all the right-hand sides become equal and depend no more on the index K. The ground-state band energies become:

$$E_I = \frac{I(I+1)}{2\mathcal{J}_g} - \frac{\chi_0^2}{\omega_0}\left[I(I+1)\right]^2. \tag{4.10}$$

If we consider the ground-state and β-phonon states, we can get a simple solution not only for the energies, but also for the eigenvector. After inserting $a_0 = 1$ we obtain the correlated ground-state band:

$$|IM\rangle = \sum_{n=0}^{\infty} \frac{1}{\sqrt{n!}} \left[- \frac{\chi_0 I(I+1)}{\omega_0} \right]^n |nIM\rangle . \qquad (4.11)$$

where the n^{th} term represents the admixture of the band built on the state with n β-phonons.

The physical meaning of the solution can be easily seen in a simplified treatment when one considers the ground-state band and a second band built on a state with one phonon of the type ν ($\nu = \beta$ or ς, or $\not{\varepsilon}$). Then from eqs. (4.3) $h_\nu(1) \neq 0$, $h_\nu(k)=0$ for $k \geqslant 2$ and eq. (4.8) reduces to a quadratic equation with the solutions:

$$E_\nu^{I,II} = \frac{E_\nu(0) + E_\nu(1)}{2} \pm \left\{ \left[\frac{E_\nu(0) - E_\nu(1)}{2} \right]^2 + h_\nu^2(1) \right\}^{1/2} \qquad (4.12)$$

the second one corresponding to the yrast band. If the two bands intersect at spin $I = I_0$, for low spins $I \ll I_0$ the energy is $E^{II} \approx E_\nu(0)$, i.e., near the g.s.b. energy and for $I \gg I_0$ $E^{II} \approx E_\nu(1)$, i.e., the well-known picture of band hybridization as a special case.

5. CONCLUSION

Calculations on realistic cases of band crossing are in progress. The parameters of the model Hamiltonian in these calculations are extracted by a fit to experimental level energies. Such calculations must show on the first place how far in the high spin region the experiment might be reproduced phenomenologically. Secondly, they can give some rude estimates of the role of many phonon states in the band hybridization picture since this picture, as shown at the end of sec. 4, can be viewed upon as a special case of our model when we take only the ground and one-phonon bands into account. Thirdly, they can give experimental values of the model Hamiltonian parameters, in particular - of the different type coupling strengths. And finally, we hope to be able to reproduce the model Hamiltonian parameters microscopically /15/ by a method, similar to the one, developed for the low-spin region /13,16/ and thus - to obtain a simple microscopic description of band crossing and in particular - of the back-bending.

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