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STUDY OF BARRIER PENETRATION EFFECTS IN THE RADIOACTIVE EMISSION OF COMPOSITE PARTICLES

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Банг Е., Тобокман В. **E4 - 11825**

Изучение эффектов проникновения через барьер
в радиоактивном испускании сложных ядер

С целью исследования возможностей выбора потенциального барьера и его влияния на волновую функцию испущенной частицы в процессах радиоактивного распада рассматривается трехчастичная модель: две частицы, взаимодействующие друг с другом и с силовым центром. Для решения системы связанных интегральных уравнений, описывающих систему, используется метод итераций. Получено точное выражение для ширины радиоактивного распада. Показано, что ширина радиоактивного распада чувствительна к выбору оптического потенциала и оптический потенциал, искажающий волновую функцию испущенной частицы, необходимо выбрать таким образом, чтобы он с наибольшей возможной точностью аппроксимировал потенциал взаимодействия между испущенной частицей и дочерним ядром.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1978

Bang J., Tobocman W. **E4-11825**

Study of Barrier Penetration Effects in the Radioactive
Emision of Composite Particles

In the expression for the decay width given in two recent approximate analyses there appears to be a lack of uniqueness of the choice of the barrier used to distort the wave function of the emitted particle. This question is studied with a simple three-body model for which an exact expression for the decay width may be found. In that model it is seen that the potential distorting the emitted particle wave function should be the best possible approximation to the true interaction potential acting between the emitted particle and the daughter nucleus.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1978

The width for radioactive decay given in two recent approximate analyses /1,2/ is

$$\Gamma = 2\pi |\langle \psi_{DE} \, | V_{DE} - U_{DE} \, | \Phi_P \rangle |^2 , \tag{1}$$

where Φ_P is the wave function for the parent nucleus, ψ_{DE} is the daughter nucleus + emitted particles wave function distorted by optical potential U_{DE}, and V_{DE} is the true interaction potential between the daughter nucleus and the emitted particle. The derivations of this expression do not appear to provide a criterion for the choice of the optical potential U_{DE}: In this report we analyze a three body model which exhibits radioactive emission of a composite particle. In this model one can deduce the exact expression for the decay width. From this expression one can see that the calculated width will have a sensitivity to the choice of U_{DE} which depends on the extent to which correlation effects are neglected in the parent nucleus wave function Φ_P. The best choice for U_{DE} is then seen to be the one that most nearly approximates V_{DE}.

Our model consists of two equal mass s-wave particles N and P interacting individually with an infinite mass center of force via potentials V_N and V_P. The particles N and P interact with each other via the potential V_{NP} which can support at least one bound state.

The Schroedinger equation for the system is then

$$(E - T - V_N - V_P - V_{NP}) \bar{\Psi} = 0 , \tag{2}$$

where E is the total energy and T is the operator for the total kinetic energy. This equation is rewritten in the form

$$(E-T-V_{NP})\psi_D = V_{NP}(\psi_N+\psi_P), \tag{3a}$$

$$(E-T-V_N)\psi_N = V_N(\psi_P+\psi_D), \tag{3b}$$

$$(E-T-V_P)\psi_P = V_P(\psi_N+\psi_D), \tag{3c}$$

with

$$\bar{\Psi} = \psi_D+\psi_N+\psi_P. \tag{4}$$

Eq. (3) is modified by the introduction of the optical potential U_D.

$$(E-T-V_{NP}-U_D)\psi_D = V_{NP}(\psi_N+\psi_P), \tag{5a}$$

$$(E-T-V_N)\psi_N = V_N(\psi_P+\psi_D)-\frac{1}{2}U_D\psi_D, \tag{5b}$$

$$(E-T-V_P)\psi_P = V_P(\psi_D+\psi_N)-\frac{1}{2}U_D\psi_D. \tag{5c}$$

The optical potential U_D is arbitrary except that it is required to depend only on the value of the center of mass coordinate of particles N and P.

$$U_D = U_D\left(\frac{1}{2}|\bar{r}_N+\bar{r}_P|\right). \tag{6}$$

By adding Eqs. (5b) and (5c) and defining

$$\psi_{NP} = \psi_N+\psi_P \tag{7}$$

we find that Eq. (5) may be replaced by

$$(E-T-V_{NP}-U_D)\psi_D = V_{NP}\psi_{NP}, \tag{8a}$$

$$(E-T-V_N-V_P)\psi_{NP} = V_D\psi_D, \tag{8b}$$

where

$$V_D = V_N+V_P-U_D. \tag{9}$$

The coupled differential equations displayed in Eq.(8) may be transformed to a set of coupled integral equations.

$$\psi_D = \phi_D + G_D V_{NP}\psi_{NP}, \tag{10a}$$

$$\psi_{NP} = \phi_{NP} + G_{NP}V_D\psi_D, \tag{10b}$$

where

$$G_D = (E-T-V_{NP}-U_D+i\epsilon)^{-1}, \tag{11a}$$

$$G_{NP} = (E-T-V_N-V_P+i\epsilon)^{-1} \tag{11b}$$

and outgoing wave Green's function operators and ϕ_D and ϕ_{NP} are solutions of the homogeneous equations

$$(E-T-V_{NP}-U_D)\phi_D = 0, \tag{12a}$$

$$(E-T-V_N-V_P)\phi_{NP} = 0. \tag{12b}$$

Eq. (10) can be decoupled by iteration.

$$\psi_D = \phi_D + G_D V_{NP}\phi_{NP} + G_D V_{NP} G_{NP} V_D \psi_D, \tag{13a}$$

$$\psi_{NP} = \phi_{NP} + G_{NP}V_D\phi_D + G_{NP}V_D G_D V_{NP}\psi_{NP}. \tag{13b}$$

The formal solution of these equations is then given by

$$\psi_D = [1-G_D V_{NP} G_{NP} V_D]^{-1}(\phi_D + G_D V_{NP}\phi_{NP}), \tag{14a}$$

$$\psi_{NP} = [1-G_{NP}V_D G_D V_{NP}]^{-1}(\phi_{NP}+G_{NP}V_D\phi_D). \tag{14b}$$

Different choices of ϕ_D and ϕ_{NP} lead to different solutions $\Psi = \psi_D + \psi_{NP}$, ϕ_D and V_D depend on the choice of U_D, but Ψ is independent of this choice.

Now let us choose

$$\phi_D = 0, \quad \phi_{NP} = \phi_{NP}^{(0)} , \tag{15}$$

where $\phi_{NP}^{(0)}$ is a bound state solution of Eq. (12b) with an eigenvalue E located in the continuum of the spectrum of Eq. (12a). Thus V_{NP}, V_N, and V_P must be such that the discrete spectrum of Eq. (12b) overlaps the continuous spectrum of Eq. (12a). Then Eq. (14) may be written as follows.

$$\psi_D^{(0)} = G_D V_{NP} \psi_{NP}^{(0)} , \tag{16a}$$

$$\psi_{NP}^{(0)} = \phi_{NP}^{(0)} + G_{NP} V_D [1 - G_D V_{NP} G_{NP} V_D]^{-1} G_D V_{NP} \phi_{NP}^{(0)} . \tag{16b}$$

From these expressions it is clear that $\psi_D^{(0)}$ describes the asymptotic flux of $N+P=D$ bound states and $\psi_{NP}^{(0)}$ describes the asymptotic flux of free N's and P's.

To construct an explicit expression for G_0 we note that

$$\begin{aligned} G_D &= \left(E + \frac{h^2}{2m}\frac{\partial^2}{\partial r_N^2} + \frac{h^2}{2m}\frac{\partial^2}{\partial r_P^2} - V_{NP} - U_D + i\epsilon\right)^{-1} , \\ &= \left(E + \frac{h^2}{4m}\frac{\partial^2}{\partial R^2} + \frac{h^2}{m}\frac{\partial^2}{\partial r^2} - V_{NP}(r) - U_D(R) + i\epsilon\right)^{-1} , \end{aligned} \tag{17}$$

where

$$r = |\bar{r}_N - \bar{r}_P| , \tag{18a}$$

$$R = \frac{1}{2}|\bar{r}_N + \bar{r}_P| . \tag{18b}$$

Let us write

$$G_D = (E - h - \mathcal{H} + i\epsilon)^{-1} , \tag{19a}$$

$$h = -\frac{\hbar^2}{m}\frac{\partial^2}{\partial r^2} - V_{NP}(\epsilon), \tag{19b}$$

$$\mathcal{H} = -\frac{\hbar^2}{4m}\frac{\partial^2}{\partial R^2} - U_D(R) . \tag{19c}$$

Then we can set

$$\begin{aligned} G_D = -\frac{4m}{\hbar^2} \mathop{\displaystyle\sum\!\!\!\!\!\!\int}_n \int_0^\infty dR' \int_0^\infty dR'' |\zeta_n(r)\,\delta(R-R')\rangle \times \\ \times k_n^{-1} f(k_n R_<) g(k_n R_>) \langle \zeta_n(r)\,\delta(R-R'')| , \end{aligned} \tag{20}$$

where the sum and integral is over the spectrum of eigenvalues ϵ_n of h,

$$(\epsilon_n - h)\,\zeta_n(r) = 0 , \tag{21a}$$

$$\left(\frac{\hbar^2 k_n^2}{4m} - \mathcal{H}\right) \left\{ \begin{matrix} f \\ g \end{matrix} \right\} = 0 , \tag{21b}$$

and

$$\frac{h^2 k_n^2}{4m} = E - \epsilon_n . \tag{21c}$$

f is the regular solution of Eq. (21b) which has the asymptotic form

$$f \to \sin(k_n R + \delta_n) , \tag{22a}$$

and g is the irregular solution of Eq. (21b) which has the asymptotic form

$$g \to \exp i(k_n R + \delta_n) . \tag{22b}$$

We return to the wave function given by Eq. (16)

$$\Psi^{(0)} = \phi^{(0)}_{NP} + G_D V_{NP} \psi^{(0)}_{NP} + G_{NP} V_D \chi^{(0)}_D \ . \tag{23}$$

This is the wave function for a situation where some external agency is acting as a source of N and P particles so as to maintain the amplitude of the term $\phi^{(0)}_{NP}$ while feeding flux to the open channels described by the asymptotic parts of the other two terms.

Let $\zeta_0(n)$ be the internal motion wave function for one of the open D channels. Then the flux into that channel is

$$J_0 = \frac{\hbar}{4mi}\left[g(k_0R)^* \frac{\partial}{\partial R} g(k_0R) - g(k_0R)\frac{\partial}{\partial R} g(k_0R)\right] \times$$

$$\times \left(\frac{8m}{\hbar^2 k_0}\right)^2 |<\zeta_0(r)\, f(k_0R)|V_{NP}|\psi^{(0)}_{NP}>|^2 =$$

$$= \frac{8m}{\hbar^3 k_0}|<\zeta_0 f|V_{NP}|\psi^{(0)}_{NP}>|^2 \ . \tag{24}$$

Now we suppose that $\phi^{(0)}_{NP}$ is normalized to unit probability.

$$1 = \int_0^\infty dr_N \int_0^\infty dr_P (\phi^{(0)}_{NP})^2 \ . \tag{25}$$

Then the partial width for decay into the ζ_0 channel is

$$\Gamma_0 = \hbar J_0 = \frac{8m}{h^2 k_0}|<\zeta_0 f|V_{NP}|\psi^{(0)}_{NP}>|^2 \tag{26}$$

since the mean lifetime for decay into that channel is $J_0^{-1} = \hbar/\Gamma_0$. To introduce the conventional normalization we define

$$\Phi^{(0)}_D = \zeta_0(r)\, f(k_0R) \ , \tag{27a}$$

$$f(k_0R) = \left(\frac{4m}{\pi\hbar^2 k_0}\right)^{1/2} f(k_0R) \ , \tag{27b}$$

so that

$$\delta(\mathcal{E}-\mathcal{E}') = \int_0^\infty dR\, f(kR)\, f(k'R) \ , \tag{28a}$$

$$\mathcal{E} = \frac{\hbar^2 k^2}{4m} \ . \tag{28b}$$

Then the decay width is just

$$\Gamma_0 = 2\pi|<\Phi^{(0)}_D|V_{NP}|\psi^{(0)}_{NP}>|^2 =$$

$$= 2\pi|<\Phi^{(0)}_D|V_{NP}[1-G_{NP}V_D G_D V_{NP}]^{-1}|\Phi^{(0)}_{NP}>|^2 , \tag{29}$$

where we have made use of Eqs. (14b) and (15).

Eq. (29) is an exact expression for the decay width. The first order approximation to it is

$$\Gamma^{(1)}_0 = 2\pi|<\Phi^{(0)}_D|V_{NP}|\Phi^{(0)}_{NP}>|^2 \tag{30}$$

$\Phi^{(0)}_{NP}$ is a bound state wave function, and V_{NP} is a short range operator. Thus $V_{NP}\Phi^{(0)}_{NP}$ will be nonvanishing only in a small region enclosing the center of force causing V_N and V_P. The value of $\Gamma^{(1)}_0$ is therefore very sensitive to the value of $\Phi^{(0)}_D$ in the vicinity of the center of force. This quantity in turn is very sensitive to the potential barrier contained in the optical potential U_D. Thus the first order approximation to the decay width depends very strongly on the choice of the optical potential.

The decay width Γ_0 is, of course, independent of the optical potential U_D. Thus the dependence on U_D of $\Phi^{(0)}_D$ and V_D in Eq. (29) must cancel each other. On the other hand, the first order approximation $\Gamma^{(1)}_0$ depends strongly on U_D. Clearly, the best choice for U_D in the evaluation of $\Gamma^{(1)}_0$ is that value which renders $\Gamma^{(1)}_0$ the best possible approximation to Γ_0. This is the choice which minimizes the contribution of the higher order terms in Eq. (29), which may be written

$$\Gamma_0 = 2\pi \left| \sum_{n=0}^{\infty} \langle \Phi_D^{(0)} | V_{NP} (G_{NP} V_D G_D V_{NP})^n | \Phi_{NP}^{(0)} \rangle \right|^2 . \qquad (31)$$

The action of the factors $G_{NP} V_D G_D V_{NP}$ in Eq. (31) is to generate the correlations between N and P which are present in $\psi_{NP}^{(0)}$ but absent in $\Phi_{NP}^{(0)}$. The importance of the higher order terms in Eq. (31) can be minimized by choosing U_D so that V_D is effectively as small as possible. For instance, the choice

$$U_D(R) = V_N(R) + V_P(R) \qquad (32)$$

which gives

$$V_D = V_N(|\bar{R} + \frac{1}{2}\bar{r}|) + V_P(|\bar{R} - \frac{1}{2}\bar{r}|) - V_N(R) - V_P(R) \qquad (33)$$

might be a good one. Minimizing the importance of the higher order terms means that correlation effects are less important and the first order term is a better approximation to the exact result.

Our first order approximation to the width, Eq. (30), differs from the expression given in Eq. (1), but the two are equivalent by virtue of post-prior equivalence in Born approximation. By virtue of post-prior equivalence Eq. (30) may be rewritten to read

$$\Gamma_0^{(1)} = 2\pi | \langle \Phi_D^{(0)} | V_D | \Phi_{NP}^{(0)} \rangle |^2 \qquad (34)$$

which is equivalent to Eq. (1).

With the numerical techniques presently available /3/ it is possible to solve this model problem with any desired accuracy. The result of such a calculation could then be used to provide an exact value of the decay width Γ_0. Values for the first order width $\Gamma_0^{(1)}$ calculated with different choices for the optical potential U_D could then be compared with the exact width. In this way it would be possible to test the validity of our proposal concerning the best choice for U_D.

Another means of studying this question is the numerical evaluation of the higher order contributions to the width and the comparison of these with the first order contribution as a function of U_D.

The sort of correlation effects manifested by our model are of course rather different from that which occurs in complex nuclei. A crude approximation to reflect one aspect of these many-body effects could be introduced into our model by adding a short range R dependence into V_{NP} causing it to be weaker when the N+P=D system is close to the center of force producing V_N and V_P.

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