СООБЩЕНИЯ
ОБЪЕДИНЕННОГО
ИНСТИТУТА
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

E4 - 11682

J.Žofka, M.Sotona

ON THE COLLECTIVE PHENOMENA IN HYPERNUCLEAR GROUND STATES

1978

E4 - 11682

J.Žofka,[1] M.Sotona[2]

ON THE COLLECTIVE PHENOMENA IN HYPERNUCLEAR GROUND STATES

[1] Permanent address Nuclear Physics Institute, 250 68 Řež, Czechoslovakia.

[2] Nuclear Physics Institute, Czechoslovak Academy of Sciences, Řež, Czechoslovakia.

Жофка Я., Сотона М. E4 - 11682

О коллективных явлениях в основном состоянии гиперядер

В минимальном приближении метода К-гармоник, описывающем радиальные осцилляции многонуклонной системы, исследуется влияние добавочного Λ-гиперона на структуру основного состояния гиперядра и на структуру соответствующего дваждымагического ядерного остова. Показано, что по значениям энергии связи Λ-частицы и по изменениям радиусов и кулоновских энергий остова можно получить некоторую информацию о Λ-N взаимодействии. Подробно анализируется связь этих данных со сжимаемостью ядерного вещества.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1978

Žofka J., Sotona M. E4 - 11682

On the Collective Phenomena in Hypernuclear Ground States

In the hyperspherical scheme, pertinent to radial oscillations, the influence of added Λ-hyperon on the structure of hypernuclear ground states of corresponding doubly magic nuclear cores has been studied. Possible consequences of values of Λ-binding energies, radii and Coulomb energies on Λ-N interaction have been shown. The relation to the nuclear compressibility has been analyzed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1978

Substantial increase of interest in hypernuclear theory encountered in the last few years may be ascribed not only to rapidly growing experimental data collection, but to special properties of strange particles embedded in matter of nucleons above all. The simplicity due to non-necessity of antisymmetrization between Λ and N's and the location of Λ in the lowest orbital for the ground state (GS) makes the problem attractive. At present models of hypernuclear structure serve to establish a form of Λ - N low energy potential [1 ÷ 3] and to gain new information on the nuclear system itself. First, the center-of-mass motion (CMM) is to be treated as properly as possible in view of treating preferentially light systems (A $\lesssim$ 40). CMM here may influence density distribution which distorts the Λ -nucleus potential in turn. Second, the nuclear core should be allowed to polarize (to breathe) in the presence of Λ -particle. In GS, Λ particle in the $1S_{1/2}$ orbit causes radial polarization of the core only. It seems natural, when GS hypernuclear description is contemplated, to start with a model, which is microscopic, but which enables to pick-up a particular degree of freedom as special (e.g. pertinent to radial polarization).

It is suggestive to employ here the model of hyperspherical harmonics (HHM) [4], which appeared useful in the description of nuclear breathing and nuclear compressibility [5]. Even when severely truncated, HHM should yield main features of the Λ-nucleus potential, of the hypernuclear radial polarization and their relation with Λ-N interaction properly. Moreover, when compared with approaches [1 ÷ 3] it does not suffer from CMM uncertainty and it automatically folds wave functions of Λ and N's. It is the aim of this study to demonstrate microscopically, how the N-N potential and especially the compressibility are related to Λ binding energy B_Λ and to changes of radii or Coulomb energies, when Λ-particle is added.

HHM for nucleons has been already often described before [4, 6]. When adding Λ [7] the only change is the use of Jacobi coordinates for unequal masses (system of (A-1) nucleons + Λ):

$$\vec{\xi}_i = \frac{1}{\sqrt{i(i+1)}}\left(\sum_{j=1}^{i}\vec{x}_j - i\,\vec{x}_{i+1}\right), \quad i = 1 \dots A-2,$$
$$\vec{\xi}_{A-1} = \frac{1}{\sqrt{(A-1)(1-D+AD)}}\left(\sum_{j=1}^{A-1}\vec{x}_j - (A-1)\,\vec{x}_\Lambda\right), \qquad (1)$$
$$\vec{\xi}_A \equiv \vec{R}_A = \frac{1}{A-1+D^{-1}}\left(\sum_{i=1}^{A-1}\vec{x}_i + D^{-1}\vec{x}_\Lambda\right).$$

In eq. (1), $D = \frac{m_N}{m_\Lambda}$ and the intrinsic kinetic energy is homogeneous in Δ_{ξ_i}, i = 1, ... A-1, the hyperradius $\rho^2 = \sum_{i=1}^{A-1}\xi_i^2$ has a simple form in x_i^2 and R_A^2 and all technical hints of ref. [4] are legal. Rewriting total

intrinsic hypernuclear Hamiltonian, it gives just ordinary HHM equations and when restricting ourselves to the lowest hyperspherical harmonics $\mathcal{U}_{K_{min}}$ one is left with a Schrödinger like equation

$$\left(\frac{d^2}{d\rho^2} - \frac{\mathcal{L}(\mathcal{L}+1)}{\rho^2} - \frac{\hbar^2}{2m_N}W(\rho)\right)\chi(\rho) = \frac{\hbar^2}{2m_N}E_{bind}\,\chi(\rho). \qquad (2)$$

Here, the hypermomentum $\mathcal{L} = K_{min} + \frac{3}{2}(A-2)$ includes the contribution from Λ and the hyperpotential contains both V_{NN} and $V_{\Lambda N}$ in the usual way:

$$W(\rho) \equiv W_{NN}(\rho) + W_{\Lambda N}(\rho) = \langle \mathcal{U}_{K_{min}}(\Omega_{3A-4}) | V_{NN} + V_{\Lambda N} | \mathcal{U}_{K_{min}} \rangle. \qquad (3)$$

Calculation of $W_{\Lambda N}(\rho)$ requires generalized Moshinsky brackets for unequal masses $m_\Lambda \neq m_N$ calculated according to ref. [8] and redefinition of Talmi integrals entering [6]. Because B_Λ is a difference of $E_{bind}(^{A-1}Z)$ and $E_{bind}(^{A}_{\Lambda}Z)$, K_{min} approximation should be very good as discussed in [5]. The same applies to the changes of radii and Coulomb energies.

In order to test the Λ-HHM model, we have solved eq. (2) using the same interactions as Rayet did [3], namely B1 interaction for V_{NN} and $V_{\Lambda N} = V_0 e^{-\frac{r^2}{r_0^2}}$ for calculating $^{17}_{\Lambda}O$. In spite of the more involved Hartee-Fock scheme, used in [3], the results are very similar: B (HHM) = 18.70 MeV, B (HF) = 18.73 MeV, δR(HHM) = 4.2%, δR(HF) = 4% (for radii, we however use intrinsic ones). This similarity gives thus HHM credit for description of hypernuclear GS.

On the first inspection, it may seem that hypernuclear radial changes due to Λ particle added are simply related to the nuclear compressibility K, which is a measure of the average ease to move nucleons radially apart. This is corroborated by rough model estimates [9], based on the Fermi gas notion, giving

$$\frac{r_{\Lambda\,core} - r_o}{r_o} = - \frac{B_\Lambda(\infty) + 2\, B_\Lambda(A)}{KA} , \qquad (4)$$

where $B_\Lambda(A)$ is the binding energy of Λ-particle in a system containing (A-1) nucleons. Similar dependence on K may be traced in Rayet´s HF results [3] for $^{17}_{\Lambda}O$, where a Wigner central $V_{\Lambda N}$ was used.

It is worthwhile to scrutinize closer the behaviour of nuclear core when Λ is added. Would a relation similar to (4) hold independently of $V_{\Lambda N}$ it would represent a new independent possibility of a measurement of K connected up to now directly to monopole resonances only. If on the other hand the polarization process is more complicated and depends on the interplay of V_{NN} and $V_{\Lambda N}$, the results may give an additional information on $V_{\Lambda N}$. We have employed a set of effective V_{NN} forces [6], which give reasonable description of ground state properties of all doubly magic nuclei. They are sums of three Gaussians and they purposely differ in the nuclear compressibility yielded, K ranging from 150 - 350 MeV. Coulomb interaction has been included. When studying GS of such hypernuclei as $^{5}_{\Lambda}He$, $^{17}_{\Lambda}O$, $^{41}_{\Lambda}Ca$, the tensor and charge symmetry breaking parts of $V_{\Lambda N}$ potentials do not contribute and in view of constant ratio of the spin-triplet and spin-singlet contributions 3 : 1, spin dependence of $V_{\Lambda N}$

should not play very important role [7]. We shall thus restrict ourselves to central $V_{\Lambda N}$ interactions with Wigner and Majorana exchanges as defined in refs. [3] and [7]:

$$V_{\Lambda N}(r) = (1 + \alpha_x - \alpha_x P_{xm}) \sum_{i=1,2} V_i\, e^{-\frac{r^2}{c_i^2}} \qquad (5)$$

Their parameters are given in tab. 1. Some of the results of an exact solution of eq. (2) are displayed in tab. 2. An estimate of the change of the proton radius of nuclei core is there written only, because for different number of protons and neutral baryons (n's and Λ), HHM needs to approximate $(r_\Lambda - R)^2$ A-body operator. Results on $^{5}_{\Lambda}He$ have not been included in tab. 2, because Rayet´s interaction $V^{a}_{\Lambda N}$ does not describe $^{5}_{\Lambda}He$ well and the comparison with cases b, c would not be fair.

There are many hypernuclear features nicely seen in tab. 2. B_Λ does not depend strongly on N-N potential for $V_{\Lambda N}$ fixed. In the wide range of K, B_Λ changes by 1 MeV. As expected, the stronger is the Majorana component α_x, the faster is the saturation in B_Λ. Only Wigner Λ-N interaction V^{a}_{N} [3] compresses nuclear core in approximate accordance with Feshbach relation (4). This may give a rough estimate on the binding of Λ in infinite nuclear matter, yielding $B_\Lambda(\infty) \sim 30$ MeV (using $^{17}_{\Lambda}O$ results) or $B_\Lambda(\infty) \sim 45$ MeV (using $^{41}_{\Lambda}Ca$ results). When however, a Majorana exchange is switched on ($V^{b}_{\Lambda N}$ or $V^{c}_{\Lambda N}$), nuclear core in presence of Λ expands in contrast to eq. (4). Nevertheless, the expansion is still approximately inversely proportional to the compressibility K. This expansion may be understood when realizing that such $V_{\Lambda N}$ potentials are prevailingly repulsive in relative p-waves and prevailingly attractive in s-waves, acting thus differently on nucleons from different shells. Consequently, the process of radial excitation of the nuclear core by Λ-particle is basicly different from monopole excitations [5], where all nucleons are uniformly excited by harmonic like force. Those radial changes may be further illustrated by the Coulomb energies E_c (tab. 3). E_c can be calculated exactly in ΛHHM (being expectation value of an intrinsic two-body operator). The smaller the value of K the larger the changes in Coulomb energies due to Λ-particle presence. Moreover, δE_c is much larger, when Majorana component is included. Would the radial changes or Coulomb energy changes be known, tables 2 and 3 may suggest a possibility to extract an independent estimate on K and α_x. More explicitly, the value and sign of both δE_c and δr_c^p or $(r_\Lambda - r)$ indicate presence and approximate value of the Majorana mixture (α_x). The rate of change of δE_c or δr_c^p with change of A depends on K as seen in the last two columns of table 3. Those two examples are, however, only qualitative and deserve further study. Especially, the range of repulsive core of $V_{\Lambda N}$ may simulate Majorana exchange to some extent, influencing the compression of the nuclear core. The above analysis demonstrates usefulness of further studying hypernuclear collective degrees of freedom by means of models suited just to a pertinent variable chosen (here hyperradius vs HHM).

Table 1.

Notation	α_x	V_1	μ_1	V_2	μ_2	Ref.
a	0	-36.3	1.044	0		[3]
b	-1	-418.0	1.2	586.6	1.	[7]
c	-0.9	-418.0	1.2	586.6	1.	[7]

Parameters of the ΛN potential used.

Table 2.

$K_\infty(V_{NN})$	$V_{\Lambda N}$	$^{17}_{\Lambda}O$ B_Λ	$^{17}_{\Lambda}O$ $r_\Lambda - r_0$	$^{17}_{\Lambda}O$ δr_c^p	$^{41}_{\Lambda}Ca$ B_Λ	$^{41}_{\Lambda}Ca$ $r_\Lambda - r_0$	$^{41}_{\Lambda}Ca$ δr_c^p
150	a	15.28	-0.088	-2.3	26.13	-0.075	-1.6
	b	11.09	0.024	2.2	13.70	0.019	1.3
	c	14.23	0.009	1.6	20.03	0.002	0.8
250	a	15.15	-0.074	-1.7	25.87	-0.060	-1.1
	b	11.02	0.017	1.9	13.66	0.008	0.9
	c	14.21	0.004	1.4	20.04	-0.004	0.6
350	a	15.78	-0.059	-1.1	26.75	-0.048	-0.7
	b	10.38	0.013	1.8	12.83	0.003	0.8
	c	13.77	0.003	1.4	19.55	-0.007	0.5

HHM Λ-binding energies B_Λ [MeV], difference of $^{A}_{\Lambda}Z$ and ^{A-1}Z mass rms radii $r_\Lambda - r_0$ [fm] and relative radial core distortion of proton distributions due to Λ added $\delta r_c^p = (r_c^p - r_0^p)/r_0^p$ [%] for various N-N potentials [6], denoted by their K_∞ value.

Table 3.

K_{-} (V_{NN})	$V_{\Lambda N}$	δE_c ${}^{17}_{\Lambda}O$	δE_c ${}^{41}_{\Lambda}Ca$	S_r	S_c
150	a	0.4	1.0	1.4	0.3
	b	-4.1	-1.8	1.7	2.3
	c	-3.5	-1.3	2	2.7
250	a	-0.1	0.5	1.5	-0.2
	b	-3.9	-1.4	2.1	2.8
	c	-3.3	-1.1	2.3	3
350	a	-0.7	0.2	1.6	-3.5
	b	-3.7	-1.3	2.3	2.8
	c	-3.3	-1.1	2.8	3

Coulomb energy differencies $\delta E_c = [E_c({}^{A}_{\Lambda}Z) - E_c({}^{A-1}Z)] / E_c({}^{A-1}Z)$ in % and rate of radial and Coulomb changes: $S_r = \delta r_c^p(O) / \delta r_c^p(Ca)$, $S_c = \delta E_c(O) / \delta E_c(Ca)$.

References:

[1] A. Gal et al., Ann. Phys. (N.Y.) 72 (1972) 445.

[2] A. Bouyssy, Nucl. Phys. A290 (1977) 324.

[3] M. Rayet, Ann. Phys. (N.Y.) 102 (1976) 226.

[4] A. I. Baz and M. V. Zhukov, Sov. J. Nucl. Phys. 11 (1970) 435.

[5] M. Sotona and J. Žofka, Phys. Lett. 57B (1975) 27.

[6] M. Sotona and J. Žofka, Czech. J. Phys. B28 (1978) 593.

[7] B. A. Khrylin and A. B. Kochetov, Phys. Lett. 42B (1972) 38.

[8] J. Dobeš, J. Phys. A 10 (1977) 2053.

[9] H. Feshbach in Proc. Meet. on K-Physics (ed. H. Palevsky, BNL, December 1976).

Received by Publishing Department
on June 21 1978.