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WITHIN THE CONTINUUM SHELL-MODEL.

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  Радиационный захват пионов в рамках оболочечной модели
  с континуумом (1) Теория

  В рамках оболочечной модели с континуумом общепринятый форма-
лизм радиационного захвата пионов расширен на случай (π^-, γn) реак-
ции. На основе использования проекционных операторов Фешбаха обсуж-
дается соотношение между прямым и резонансным вкладами реакции.

  Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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      The Radiative Pion Capture within the Comtinuum
      Shell-Model (I) Theory

  In the framework of the continuum shell-model the
standard formalism of the radiative pion capture is
extended to the (π^-, γn) reaction. Using Feshbach's projection
operators the relation between the direct and the reso-
nance reaction mechanisms is discussed.

  The investigation has been performed at the
Laboratory of Theoretical Physics, JINR.

1. INTRODUCTION

During the last years a considerable progress has
been achieved in the description of the nuclear structure
above the nucleon emission threshold. Since the explana-
tion of the giant dipole resonances the shell-model has
succeeded in explaining different excitation modes in this
energy region. Based on the particle- hole $(1p-1h)$ model
the earlier calculations explained the gross structure of
the giant resonances quite well. Recently the description
of the fine structure has been attempted with the help
of more complicated shell-model configurations $(np-nh)$.
This development is hampered by an enormous extension
of the numerical expense. For this reason such calcula-
tions are only feasible for light nuclei and in the region
of magic nuclei, where the effective number of nucleons
taking part in the reaction can be limited.

Another way of improving the shell-model calcula-
tions is the use of an average potential with the correct
asymptotic behaviour. Contrary to the traditional shell-
model (BSM) with bound basis states only, the basis of
the continuum shell-model (CSM) contains also a conti-
nuous spectra generated by an average potential vanishing
for large distances of the nucleon from the centre of
mass. Such a basis is able to describe the emission of
nucleons. In principle, the CSM is capable of describing
bound states, quasi-bound states observed as resonances
in the excitation spectrum, and the region between and
above these resonances. For large excitation energies
the CSM gets near to the distorted wave approximation
(DWA). Therefore, we can use this model to clarify the
relation between the so-called "direct" and "resonance"

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contributions to a nuclear reaction. Besides, the CSM provides the correct relative phases of the contributions of differential multipolarities. This allows the calculation of angular distributions and correlation functions.

The nucleus can be excited in various reactions. Up to now mainly the nuclear photoeffect has been investigated, but some versions of the CSM have also been applied to the description of the high-energy electron[1-4] and proton[2] inelastic scattering and the muon capture[3,5,6]. A review of the CSM is given in refs.[7,17].

In the present paper we investigate the radiative pion capture. The measured spectrum of hard γ-rays[8] suggests an interpretation as a resonance process with a large direct background[9]. On the other hand the same spectrum has been explained quite well within a pure resonance model[10]. The measured $\gamma-n$ angular corralation function[11], however, does not correspond neither to a pure resonance nor to a pure direct process. Here the CSM may help to clarify the relation between the two reaction mechanisms.

The paper is organized as follows. In *Sec. 2* the traditional formalism of the (π^-, γ)-reaction is extended to the case of an unbound nuclear final state. In *Sec. 3* we compare our results with the help of projection operators with the corresponding results obtained in a traditional shell-model calculation and in distorted wave approximation. In a subsequent paper the version of the CSM presented in refs. [18,19] is applied to the numerical calculation of the reactions $^{16}O(\pi^-,\gamma)^{16}N$ and $^{16}O(\pi^-, \gamma n)^{15}N$.

2. THE PROBABILITY OF THE RADIATIVE PION CAPTURE FOLLOWED BY NEUTRON EMISSION

The probability of the radiative pion capture with a bound nuclear final state

$$\pi^- + A(N,Z) \rightarrow B(N+1, Z-1) + \gamma \qquad (2.1)$$

can be written (see, e.g., ref.[12])

$$dw_{i \rightarrow f} = c(1 + \frac{m_\pi}{M})^2 \frac{E_\gamma}{E_\pi} |<\Phi_f |H_{int}|\Phi_i>|^2 \times \qquad (2.2)$$

$$\times \delta(E_i - E_f) d\Omega_\gamma dE_\gamma,$$

with the pion rest mass m_π, the pion energy E_π, the photon energy E_γ, the nucleon mass M, the emission direction Ω_γ of the photon and the wave functions $\Phi_{i,f}$ of the initial and the final nuclear state, respectively. The δ-function ensures the conservation of the total energy $E_{i,f}$. The effective interaction operator in impulse approximation

$$H_{int}(n_\pi, \ell_\pi, \sigma_\pi, \lambda, \vec{k}_\gamma) = \sum_j e^{-i\vec{k}_\gamma \vec{r}_j} f_j \phi_{n_\pi \ell_\pi}^{\sigma_\pi}(\vec{r}_j) \qquad (2.3)$$

is a sum over all nucleons j of the nucleus and contains the wave function $\phi_{n_\pi \ell_\pi}^{\sigma_\pi}(\vec{r})$ of the pion in the atomic orbit characterized by the principal quantum number n_π, the angular momentum ℓ_π and its projection σ_π. The reaction amplitude f_j is taken from the elementary process

$$\gamma + n \rightarrow p + \pi^- . \qquad (2.4)$$

It can be approximated by the expression

$$f_j = i r_j^- \{ A\vec{\sigma}_j \cdot \vec{\epsilon}_\lambda + B\vec{\sigma}_j \cdot \vec{\epsilon}_\lambda \vec{k}_\gamma \cdot \vec{k}_\pi +$$

$$\qquad (2.5)$$

$$+ C\vec{\sigma}_j \cdot \vec{k}_\gamma \vec{\epsilon}_\lambda \cdot \vec{k}_\pi + iD\vec{\epsilon}_\lambda \cdot (\vec{k}_\pi \times \vec{k}_\gamma) \} ,$$

with the nucleon spin operator $\vec{\sigma}_j$, the photon polarization vector $\vec{\epsilon}_\lambda$ and the pion and photon momentum vectors $\vec{p}_{\pi,\gamma} = \hbar \vec{k}_{\pi,\gamma}$. The operator r_j^- transforms a proton into a neutron. The parameters A,B,C and D can be fitted by the experiment corresponding to reaction (2.4). Using eq. (2.2) we neglected the recoil of the final nuc-

leus caused by the photon emission and the differences between the centre of mass systems of the reactions (2.1) and (2.4) [12].

The formalism of the description of the radiative pion capture has already been presented by several authors [9,12,13] in a rather detailed form. We only generalize it to the case of an unbound final state, where the emitted photon can be measured in coincidence with the emitted nucleon.

Because the annihilation energy of the pions is sufficiently large, final states above the nucleon emission threshold can be excited. In the region of light nuclei the following reaction dominates:

$$\pi^- + A(N,Z) \to T(N, Z-1) + n + \gamma , \qquad (2.6)$$
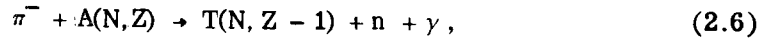
with an emitted neutron and a residual nucleus T. In this case the reaction probability reads

$$dw_{i \to f} = \alpha(1 + \frac{m_\pi}{M})^2 \frac{E_\gamma}{E_\pi} |<\Psi_{E,f}^{(-)} |H_{int} | \Phi_i >|^2 \times$$

$$\times \delta(E_i - E_f) d\Omega_\gamma d\Omega_f dE dE_\gamma , \qquad (2.7)$$

with the excitation energy E of the intermediate nucleus (T+n) and the emission direction Ω_f of the neutron. The wave function $\Psi_{E,f}^{(-)}$ describes the residual nucleus, the emitted neutron and their relative motion. The channel quantum number f denotes simultaneously the spin projection σ_f of the emitted neutron, the spin I_f of the residual nucleus and its projection K_f and the direction Ω_f of the emitted neutron. For large separations of the two fragments the wave function $\Psi_{E,f}^{(-)}$ describes a plane wave with the quantum numbers f and incoming spherical waves in all open channels. The energy E is given by the kinetic energy of the relative motion with the momentum $\vec{p}_f = \hbar \vec{k}_f$ and the excitation energy ϵ_T of the residual nucleus

$$E = \frac{\hbar^2 k_f^2}{2\mu_f} + \epsilon_T , \qquad (2.8)$$

with the reduced mass μ_f of the system (T+n). The final state wave function is normalized to a delta-function

$$<\Psi_{E,f}^{(-)} | \Psi_{E',f'}^{(-)} > = \delta_{ff'} \delta(\Omega_f - \Omega_{f'}) \delta(E - E') . \qquad (2.9)$$

For our further calculation we use the channel representation with a channel ket-vector

$$|c> = \sum_{K_c m_c} <I_c K_c j_c m_c | J_c M_c > |T; I_c K_c > |\ell_c j_c m_c > , \qquad (2.10)$$

where the Clebsch-Gordan coefficients [15] couple the state $|T; I_c K_c >$ of the residual nucleus (I_c: total spin, K_c its projection) and the spin-angular part of the relative motion

$$<\Omega |\ell_c j_c m_c > = i^{\ell_c} \sum_{\lambda_c \sigma_c} <\ell_c \lambda_c \tfrac{1}{2}\sigma_c | j_c m_c > Y_{\ell_c \lambda_c}(\Omega) \chi_{\frac{1}{2}\sigma_c} . \qquad (2.11)$$

In this representation the wave function $\Psi_{E,f}^{(-)}$ with the asymptotic behaviour described above can be written

$$<rc | \Psi_{E,f}^{(-)} > = \frac{1}{\hbar} \sqrt{\frac{2\mu_f}{\pi k_f}} \sum_{\substack{\ell_f \lambda_f j_f m_f \\ J_f M_f}} <\ell_f \lambda_f \tfrac{1}{2}\sigma_f | j_f m_f > \times$$

$$\times <I_f K_f j_f m_f | J_f M_f > Y^*_{\ell_f \lambda_f}(\Omega_f) u_c^{f(-)}(r) . \qquad (2.12)$$

The function $u_c^{f(-)}(r)$ describes the relative motion in the channel c as a function of the distance r of the two fragments. It has the asymptotic behaviour

$$u_c^{f(-)}(r) \equiv <rc|u^{f(-)}> \xrightarrow{r \to \infty} \delta_{fc} \sin(k_c r - \ell_c \pi/2) +$$

$$+ \eta_{cf} I_c(k_c r) , \qquad (2.13)$$

with the amplitudes η_{cf} of the incoming spherical waves in the channel c

$$I_c(k_c r) \xrightarrow{r \to \infty} \exp[-i(k_c r - \ell_c \pi/2)]. \qquad (2.14)$$

The effective operator (2.3) is decomposed into spherical tensor operators H^M_{LJ} in a standard way /13/

$$H_{int}(n_\pi, \ell_\pi, \sigma_\pi, \lambda, \vec{k}_\gamma) = \sqrt{2\pi} \sum_{L\Lambda JM} (-i)^{L+1} \sqrt{2L+1} \times$$

$$\times \langle L\Lambda \ell_\pi \sigma_\pi | JM \rangle D^{(L)}_{\Lambda,\lambda}(\Omega_\gamma) H^M_{LJ}(n_\pi, \ell_\pi, \lambda, k_\gamma), \qquad (2.15)$$

where $D^{(L)}_{\Lambda,\lambda}(\Omega)$ is the rotation matrix /15/ $D^{(L)}_{\Lambda,\lambda}(\alpha, \beta, \gamma)$ with $(\beta, \alpha) = \Omega$ and $\gamma = 0$. The dependence on the photon spirality λ can be separated in the following way:

$$H^M_{LJ} = \lambda B(J; L, \ell_\pi) + iA(J; L, \ell_\pi), \qquad (2.16)$$

with the operators A and B defined in ref. /13/. For our further considerations we mention that the tensors A and B have opposite parity π, namely

$$\pi_A = (-1)^{\ell_\pi + L + 1},$$

$$\qquad (2.17)$$

$$\pi_B = (-1)^{\ell_\pi + L} = -\pi_A.$$

Inserting the expansions (2.12) and (2.15) into eq. (2.7) we get

$$dw_{i \to f} = 2c(1 + \frac{m_\pi}{M})^2 \frac{E_\gamma}{E_\pi} \frac{2\mu_f}{\hbar^2 k_f} \times$$

$$\times | \sum_{L\Lambda JM} (-i)^{L+1} \langle L\Lambda \ell_\pi \sigma_\pi | JM \rangle D^{(L)}_{\Lambda,\lambda}(\Omega_\gamma) \times$$

$$\times \sum_{\substack{\ell_f \lambda_f j_f m_f \\ J_f M_f}} \langle \ell_f \lambda_f \tfrac{1}{2}\sigma_f | j_f m_f \rangle \langle I_f K_f j_f m_f | J_f M_f \rangle \times$$

$$\times \langle J_i M_i JM | J_f M_f \rangle \hat{L} \hat{j}_f^{-1} Y_{\ell_f \lambda_f}(\Omega_f) U^{LJ}_{if} |^2 \times$$

$$\times \delta(E_i - E_f) d\Omega_\gamma d\Omega_f dE dE_\gamma, \qquad (2.18)$$

with $\hat{J} = \sqrt{2J+1}$ and the reduced matrix elements

$$U^{LJ}_{if}(n_\pi, \ell_\pi, \lambda, k_\gamma) \equiv (-1)^{2J} \langle u^{f(-)} || H_{LJ} || \Phi_i \rangle =$$

$$= \hat{J}_f^{-1} \sum_{M_i MM_f} \langle J_i M_i JM | J_f M_f \rangle \langle u^{f(-)} | H^M_{LJ} | \Phi_i \rangle. \qquad (2.19)$$

Combining the properties (2.16) and (2.17) we see that the reduced matrix elements with $\lambda = \pm 1$ differ only by a phase

$$U^{LJ}_{if}(n_\pi, \ell_\pi, \lambda = +1, k_\gamma) =$$

$$= \pi_i \pi_T (-1)^{\ell_f + \ell_\pi + L + 1} U^{LJ}_{if}(n_\pi, \ell_\pi, \lambda = -1, k_\gamma) \qquad (2.20)$$

with the parities π_i and π_T of the initial and the residual nucleus, respectively.

Let us consider the capture from a single atomic orbit n_π, ℓ_π. We calculate now the photon emission probability per energy unit $d\Lambda(n_\pi, \ell_\pi, T)/dE$ corresponding to the state T of the residual nucleus. By integrating eq. (2.18) over the directions Ω_γ and Ω_f, summing (averaging) over the projection quantum numbers K_f, σ_f, $\lambda, (\sigma_\pi, M_i)$ we get with the help of the phase relation (2.20)

$$\frac{d\Lambda(n_\pi, \ell_\pi, T)}{dE} = \frac{1}{\hat\ell_\pi^2 \hat J_i^2} \sum_{\sigma_\pi M_i} \sum_{K_f \sigma_f \lambda} \int_{\Omega_f} \int_{\Omega_\gamma} \frac{dw_{i \to f}}{dE} =$$

<div align="right">(2.21)</div>

$$= 16\pi c \left(1 + \frac{m_\pi}{M}\right)^2 \frac{E_\gamma}{E_\pi} \frac{2\mu_f}{\hbar^2 k_f} \frac{1}{\hat\ell_\pi^2 \hat J_i^2} \sum_{j_f J_f L J} |U_{if}^{LJ}(n_\pi, \ell_\pi, \lambda = -1, k_\gamma)|^2 .$$

In the experiment, the capture processes from different atomic orbits, and resulting in different states T of the residual nucleus can hardly be distinguished. Up to now the only measurable quantity is the averaged ratio

$$\frac{dR}{dE} = \sum_{n_\pi \ell_\pi T} \frac{dR(n_\pi, \ell_\pi, T)}{dE} = \sum_{n_\pi \ell_\pi T} \frac{\omega_{n_\pi \ell_\pi}}{\Lambda_{n_\pi \ell_\pi}^{tot}} \frac{d\Lambda(n_\pi, \ell_\pi, T)}{dE} ,$$

<div align="right">(2.22)</div>

with the total capture possibility $\Lambda_{n_\pi \ell_\pi}^{tot}$ from the orbit n_π, ℓ_π and the relative capture probability $\omega_{\ell_\pi n_\pi}$ normalized as $\sum_{n_\pi \ell_\pi} \omega_{n_\pi \ell_\pi} = 1$.

The γ-n correlation function $d\Lambda(n_\pi, \ell_\pi, T)/(d\Omega_\gamma d\Omega_f dE)$ is derived from the general expression (2.18) by summing (averaging) over the projections K_f, σ_f, λ, (σ_π, M_i)

$$\frac{d\Lambda(n_\pi, \ell_\pi, T)}{d\Omega_\gamma d\Omega_f dE} = \frac{1}{\hat\ell_\pi^2 \hat J_i^2} \sum_{\sigma_\pi M_i} \sum_{K_f \sigma_f \lambda} \frac{dw_{i \to f}}{d\Omega_\gamma d\Omega_f dE} =$$

$$= c\left(1 + \frac{m_\pi}{M}\right)^2 \frac{E_\gamma}{E_\pi} \frac{\mu_f}{\hbar^2 k_f \pi} (-1)^{\ell_\pi - J_i + I_f + \frac{1}{2}} \times$$

$$\times \frac{1}{\hat\ell_\pi^2 \hat J_i^2} \sum_{\substack{j_f j_{f'} J_f J_{f'} \\ LL'JJ'\kappa}} i^{L'-L}\, \hat j_f \hat j_{f'} \hat J_f \hat J_{f'} \hat L \hat L' \hat J \hat J' \times$$

$$\times [1 + (-1)^{\ell_f + \ell_{f'} + \kappa}] \Delta(\ell_f, \ell_{f'}, \kappa) \times$$

$$\times \langle L1L'-1|\kappa 0\rangle \langle j_f -\tfrac{1}{2}\, j_{f'}\, \tfrac{1}{2}|\kappa 0\rangle \times$$

$$\times \begin{Bmatrix} J & J' & \kappa \\ L' & L & \ell_\pi \end{Bmatrix} \begin{Bmatrix} J_f & J_{f'} & \kappa \\ j_{f'} & j_f & I_f \end{Bmatrix} \begin{Bmatrix} J & J' & \kappa \\ J_{f'} & J_f & J_i \end{Bmatrix} \times$$

$$\times U_{if}^{LJ}(n_\pi, \ell_\pi, \lambda = -1, k_\gamma)\, U_{if'}^{L'J'*}(n_\pi, \ell_\pi, \lambda = -1, k_\gamma)\, P_\kappa(\cos\theta)$$

<div align="right">(2.23)</div>

with the Legendre polynomial $P_\kappa(\cos\theta)$, the 6-j symbols in curly brackets, and with the triangular relation

$$\Delta(\ell_1, \ell_2, \ell_3) = \begin{cases} 1 & \text{if } |\ell_1 - \ell_2| \le \ell_3 \le \ell_1 + \ell_2 \\ 0 & \text{otherwise.} \end{cases}$$

According to the theorem of Treiman and Young [16] the correlation function depends only on the angle θ between the directions Ω_γ and Ω_f.

In analogy to the averaged quantity (2.22), we define the averaged correlation function

$$\frac{dR(T)}{dE\, d\Omega_\gamma\, d\Omega_f} = \sum_{n_\pi \ell_\pi} \frac{d\Lambda(n_\pi, \ell_\pi, T)}{dE\, d\Omega_\gamma\, d\Omega_f} \frac{\omega_{n_\pi \ell_\pi}}{\Lambda_{n_\pi \ell_\pi}^{tot}} .$$

<div align="right">(2.24)</div>

All the quantities from (2.21) to (2.24) are sensitive to the nuclear initial and final states via the reduced matrix-elements (2.19). The capture probability (2.21) and (2.22) depends only on the absolute values of the matrix-elements while the correlation function depends also on their relative phases.

3. THE NUCLEAR FINAL STATE
IN THE CONTINUUM SHELL-MODEL

In this section we outline the main ideas of the CSM which will be used for the description of the nuclear final state in the transition matrix element (2.7). Especially we investigate how does the continuum modify the parameters of the single resonances, and, on the other hand, how does the channel-coupling influence the direct reaction mechanism.

In the framework of the CSM a wave function is given in a basis defined by a shell-model Hamiltonian H^0 with an average potential $V^0(r)$ vanishing at large separations r. Such a basis contains both bound and unbound single-particle states, and is capable of describing bound and decaying states[17]. Because the behaviour of bound and unbound states is rather different from the physical and computational point of view, we split the basis into two subspaces:

(i) the Q-space where all A nucleons of the nucleus are in bound or quasi-bound states, and

(ii) the P-space containing all residual basis states. In our numerical calculations we shall take into account only unbound states with one nucleon in the continuum, but the formal considerations in this section can easily be extended to the case of more than one particle in the continuum. The inclusion of the quasi-bound states (single-particle resonances) in the Q-subspace has some advantages. Doing so the Q-space corresponds to the basis of the traditional shell-model, and the P-space gives a contribution which is a smooth function of the excitation energy, i.e., it contains no resonances. In this case the resonance parameters can be determined by diagonalizing an effective Hamiltonian within the Q-subspace.

In analogy to Feshbach[14] we define projection operators projecting onto the corresponding subspaces

$$Q = \sum_R |\Phi_R><\Phi_R| \quad \text{and} \quad P = 1 - Q , \quad (3.1)$$

with the complete set of basis states $|\Phi_R>$ of the Q-subspace.

A wave function $\Psi_{E,f}$ which is eigenfunction of the Hamiltonian

$$H = H^0 + V^{res} , \quad (3.2)$$

with a residual nucleon-nucleon interaction V^{res} can be decomposed in the following way[18]

$$\Psi^{(\pm)}_{E,f} = \xi^{(\pm)}_{E,f} + \sum_\rho \frac{<\tilde{\Phi}^{(\pm)}_\rho|H|\xi^{(\pm)}_{E,f}>}{E - E_\rho \pm i/2\,\Gamma_\rho} \tilde{\Omega}^{(\pm)}_\rho . \quad (3.3)$$

The first component $\xi_{E,f}$ is determined by the P-space only, and satisfies the equation

$$[E - PHP]\xi^{(\pm)}_{E,f} = 0 , \quad (3.4)$$

with the P-P-component of the Hamiltonian (3.2). The second term describes a number of resonances. Their positions and widths are given by the complex eigenvalues of the operator

$$H^{eff(\pm)}_{QQ} = QHQ + QHG^{(\pm)}_p HQ \quad (3.5)$$

according to

$$<\tilde{\Phi}^{(\pm)}_{\rho'}{}^*|H^{eff(\pm)}_{QQ}|\tilde{\Phi}^{(\pm)}_\rho> = (\tilde{E}_\rho \mp \frac{i}{2}\tilde{\Gamma}_\rho)\delta_{\rho\rho'} . \quad (3.6)$$

The Green operator

$$G^{(\pm)}_p(E) = P(E^\pm - PHP)^{-1}P, \quad (E^\pm = E \pm i\epsilon) \quad (3.7)$$

generates outgoing (+) or incoming (−) waves in the P-subspace. The function

$$\tilde{\Omega}^{(\pm)}_\rho = \tilde{\Phi}^{(\pm)}_\rho + G^{(\pm)}_p(E)H\tilde{\Phi}^{(\pm)}_\rho \quad (3.8)$$

differs from the resonance wave function $\tilde{\Phi}^{(\pm)}_\rho$ by an, in general, small component in the P-space, and is called the modified resonance wave function.

The sign $(\pm)$ of the wave functions $\Psi_{E,f}$ and $\xi_{E,f}$ denotes their asymptotic behaviour in the usual way: a plane wave in the channel f and outgoing $(+)$ or incoming $(-)$ spherical waves in all open channels. In the functions $\tilde{\Phi}_\rho$ and $\tilde{\Omega}_\rho$ the plane wave is absent.

Let us now consider the transition from a ground state Φ_i to an excited state $\Psi_{E,f}$ caused by an interaction H_{int} which can be described in first order Born approximation. The corresponding transition probability $dw_{i \to f}$ is determined by the transition matrix element $<\Psi_{E,f}^{(-)}|H_{int}|\Phi_i>$. Using representation (3.3) we get

(i) a term describing the direct transition to the P-space, and

(ii) some resonance terms at the energies E_ρ

$$<\Psi_{E,f}^{(-)}|H_{int}|\Phi_i> = <\xi_{E,f}^{(-)}|H_{int}|\Phi_i> +$$

$$+ \frac{1}{\sqrt{2\pi}} \sum_\rho \frac{\tilde{\gamma}_{f\rho} \tilde{W}_{\rho i}}{E - \tilde{E}_\rho + i/2\, \tilde{\Gamma}_\rho}. \qquad (3.9)$$

Because we have excluded the quasi-bound states from the P-space the first term only weakly depends on the energy E. All the resonance behaviour is described by the second term. The amplitude of the resonances is determined by two factors: the partial width amplitude

$$\tilde{\gamma}_{f\rho} = \sqrt{2\pi} <\xi_{E,\rho}^{(-)}|H|\tilde{\Phi}_\rho^{(+)}> \qquad (3.10)$$

describing the decay of the state ρ into the channel f caused by the nuclear interaction H, and the matrix element of the external interaction H_{int}

$$\tilde{W}_{\rho i} = <\tilde{\Omega}_\rho^{(-)}|H_{int}|\Phi_i> \qquad (3.11)$$

determining the excitation of the state ρ.

If we denote the first term of eq. (3.9) as the "direct" contribution and the second one as the "resonance" contri-

bution, we see that the total transition probability is not equal to the arithmetical sum of both the partial probabilities, but contains an interference term. A certain separation of both the reaction mechanisms is got only for some energy regions: around a narrow resonance and at large excitation energies. Here, one of the mechanisms dominates over the other. Let us discuss these cases in detail.

In the neighbourhood of an isolated, narrow resonance the matrix element (3.9) can be approximated by one resonance term. In this case the transition probability "via the resonance ρ" can be defined

$$\Lambda_\rho = \int dE \oint_f dw_{i \to f}^{(\rho)} = \frac{1}{2\pi} \int dE \frac{\oint_f |\tilde{\gamma}_{f\rho}|^2}{(E-\tilde{E}_\rho)^2 + (\tilde{\Gamma}_\rho/2)^2} |\tilde{W}_{\rho i}|^2, \qquad (3.12)$$

where we have summed over all open final channels and integrated over all emission direction Ω_f (denoted as $\oint_f$). For narrow, isolated resonances the integration over the energy E can formally be extended to $\pm\infty$. Because the energy dependence of the quantities $\tilde{E}_\rho$, $\tilde{\Gamma}_\rho$ and $\tilde{\Omega}_\rho$ is determined by the P-subspace only, we can replace them in eq. (3.12) by the fixed values $E_\rho = \tilde{E}_\rho(E_\rho)$, $\Gamma_\rho = \tilde{\Gamma}_\rho(E_\rho)$ and $\Omega_\rho = \tilde{\Omega}_\rho(E_\rho)$. As for the partial widths, we use the fact that their sum $\sum_f |\tilde{\gamma}_{f\rho}|^2$ is equal to the total width $\tilde{\Gamma}_\rho$, what strictly holds also only for isolated resonances. Within these approximations we get [17,18]

$$\Lambda_\rho = |W_{\rho i}|^2 = |<\Omega_\rho^{(-)}|H_{int}|\Phi_i>|^2. \qquad (3.13)$$

In the framework of the traditional shell-model the corresponding expression reads

$$\Lambda_\rho^{BSM} = |<\Phi_\rho^{BSM}|H_{int}|\Phi_i>|^2, \qquad (3.14)$$

where Φ_ρ^{BSM} is an eigenfunction of the Hamiltonian $Q H Q$. Both expressions differ from each other in the description of the resonance state ρ. Unlike the shell-model wave function Φ_ρ^{BSM}, the function Φ_ρ is eigenfunction of the antihermitian Hamiltonian H_{QQ}^{eff} (3.5) taking

into account the coupling with the single-particle continuum. Furthermore the function Ω_ρ in expression (3.13) contains in comparison to (3.14) the additional term $\langle \Phi_\rho^{(-)} | H G_P^{(+)}(E) H_{int} | \Phi_i \rangle$ describing the excitation of the resonance ρ via the continuum. This term is proportional to the direct matrix elements

$$W_{fi}^{dir} = \langle \xi_{E,f}^{(-)} | H_{int} | \Phi_i \rangle \tag{3.15}$$

(first term in eq. (3.9)) and to the matrix elements of the nuclear Hamiltonian H. Consequently, the larger the direct contribution to a reaction the stronger the resonances are modified by the continuum.

Eq. (3.13) has been derived for narrow isolated resonances. In the limit $\Gamma_\rho \to 0$, i.e., for bound final states the conditions for the validity of eq. (3.13) are exactly fulfilled. But even in this case the single-particle continuum influences the transition matrix element. The final state described by the modified wave function Ω_ρ contains twice the influence of the continuum, via the additional term $G_P(E) H \Phi_\rho$ in eq. (3.8) and via the effective Hamiltonian $H_{QQ}^{eff}(3.5)$.

Above the threshold of nucleon emission the quantity (3.13) characterizes only approximately the transition probability. Narrow, isolated resonances are well described, but with increasing density and width of the resonances the parameter (3.13) loses by and by its physical significance. In the region of broad overlapping resonances only the transition probability per energy unit as a function of the energy gives an adequate description of the reaction.

At excitation energies far above the resonance energies E_ρ the "direct" term in eq. (3.9) dominates over the resonance terms. This "direct" matrix element (3.15) differs from that in distorted wave approximation only by the following particulars:

(i) The wave function $\xi_{E,f}$ describing the final state contains a channel-channel coupling within the P-subspace. Therefore the emission of those particles is possible which are not directly affected by the interaction H_{int}.

(ii) The matrix element (3.15) contains no single-particle resonances which has been excluded from the P-space.

4. CONCLUSIONS

In the foregoing the direct and resonance mechanisms have been explained as boundary cases of a general reaction mechanism. This so-called "unified" mechanism allows an adequate description of a reaction in a wide energy range. But this "universality" has to be payed by a formalism much more complicated than the traditional approaches.

As is shown above, for a narrow resonance and at large excitation energies the unified description gives only some modifications of the traditional approaches. In the intermediate region of broad overlapping resonances, however, the unified description seems to be the only adequate way.

The necessity of the unified description is determined also by other aspects. There are reactions which are realized by practically only one reaction mechanism: In a reaction with a very small transferred momentum ($q < 0.5$ fm^{-1}), for instance, the direct contribution can be neglected in the whole energy range. On the other hand, not all physical quantities are in the same degree sensitive to a small contribution resulting from another reaction mechanism. In the radiative pion capture the γ-n angular correlation function is much more sensitive than the γ-spectrum, because the first quantity is a coherent sum and the latter, an incoherent sum of the contributions of different multipolarities.

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