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ДУБНА

E4 - 11505

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CALCULATION OF THE LOW ENERGY PION-^{3}He SCATTERING

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Рассеяние $\pi^{\pm}$-мезонов на ^3He в области низких энергий

На основе предложенных авторами приближенных 4-частичных уравнений вычислены длины и фазы рассеяния $\pi^{\pm}$-мезонов на ядре ^3He в области отрицательных энергий. Полученные результаты сравниваются с соответствующими экспериментальными данными по измерению сдвигов и ширин π-мезоатомов.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1978

Belyaev V.B., Wrzecionko J., Sakvarelidze M.I. E4 - 11505

Calculation of the Low Energy Pion- ^3He Scattering

On the basis of approximate four-body equations proposed earlier, the $\pi^{\pm}$-^3He scattering lengths and phase-shifts at negative energies have been calculated. The results of this calculation are compared with the corresponding experimental data on the energy-shifts in π^{-}- ^3He mesoatoms.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1978

The physical problems which arise in the description of the pion-nucleus interaction can naturally be divided into two groups. The first one is related to the description of the collision processes of pions with few nucleon systems (deuterons, ^3He). Here one has the exact dynamic equations which could be solved in principle without any additional model assumptions. The second group of problems concerns the interaction of pions and heavier nuclei. In this case we are forced to use different models, such as optical models, Glauber model, fixed scatterer approximation, and so on. In the last time the pion deuteron interaction is widely discussed in literature /1/. The three-body Faddeev approach to this system is now very well developed. Therefore, on this basis it is possible to investigate quantitatively the role of the inelastic channels related to the absorption and emission of pions.

In this article we shall consider a more complicated problem of the interaction of pions with the 3-nucleon system. As is known, in this case there also exist the exact four body integral equations /2/, which have not yet been applied to the $\pi 3N$ system. It should be mentioned that the exact four-body equations have been solved only for the nuclear systems with very simple two-body forces. Moreover, due to the many dimensional structure of these equations the solution can be obtained after a certain sequence of approximations /3/. Therefore in our opinion it is better to move in the opposite direction, namely, to formulate approximate many-body equations which can be solved practically exactly. An attempt along this line has been undertaken in paper /4/. For the amplitude of elastic scattering of pions on the three nucleon nuclei, the one dimensional integral equation has been obtained *. The essence of the approximation presented in paper /4/ is as follows. Let us write the Hamiltonian of pion-3N system in the form

$$H = h_0 + H_C + V_{\pi N}, \tag{1}$$

where h_0 is the kinetic energy of the relative motion of pions and the center of mass of the 3N system. H_c is the nuclear Hamiltonian. $V_{\pi N} = \sum_{i=1}^{3} v^i_{\pi N}$ is the sum of the elementary pion nucleons potential.

In expression (1) we replace the nuclear Hamiltonian by the approximate one

$$\tilde{H}_C \equiv \epsilon |\chi\rangle\langle\chi|, \tag{2}$$

where ϵ is the binding energy of the nucleus, and $|\chi\rangle$ is the corresponding eigenstate. The problem on the elastic scattering of pions on the three nucleon target can be solved, using the approximation (2), practically exactly. Indeed, for the elastic scattering amplitude $\langle\vec{k}|r|\vec{k}'\rangle$ one can easily obtain the following equation:

$$\langle\vec{k}|r|\vec{k}'\rangle = \langle\vec{k}|r_0|\vec{k}'\rangle + \epsilon\int\frac{d\vec{p}}{(2\pi)^3}\langle\vec{k}|r_0|\vec{p}\rangle G_0(p,E)\, G_0(p,E-\epsilon)\langle\vec{p}|r|\vec{k}'\rangle, \tag{3}$$

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* The similar equation for the wave function was obtained in the problem of nuclear reactions /5/.

where the quantity $\langle\vec{k}|r_0|\vec{k}'\rangle$ is the solution of the equation:

$$\langle\vec{k}|r_0(\vec{r}_{12}\vec{r}_3)|\vec{k}'\rangle = \langle\vec{k}|V_{\pi N}(\vec{r}_{12}\vec{r}_3)|\vec{k}'\rangle - \int\frac{d\vec{q}}{(2\pi)^3}\langle\vec{k}|V_{\pi N}(\vec{r}_{12}\vec{r}_3)|\vec{q}\rangle G_0(q,E)\langle\vec{q}|r_0(\vec{r}_{12}\vec{r}_3)|\vec{k}'\rangle \tag{4}$$

averaged over the ground state wave function χ, $E = \frac{\kappa^2}{2\mu} + \epsilon$ (μ is the pion-nucleus reduced mass) is the total energy of the system, and $\vec{r}_{12}$ and $\vec{r}_3$ are the Jacoby variables of nucleons in nucleus. As is known /6/, the last equation can be exactly integrated for the separable pion-nucleon potential. Its solution $\langle\vec{k}|r_0|\vec{k}'\rangle$ possesses the following properties: i) at the negative total energy E it is real, ii) at the positive energy, it becomes complex, iii) for $\frac{\kappa^2}{2\mu} \gg |\epsilon|$ the function $\langle\vec{k}|r_0|\vec{k}'\rangle$ coincides with the scattering matrix of pions by the three fixed scatterers.

To describe the scattering of pions by ^3He (or triton) nucleus, equations (3) and (4) have to be solved for the given value of the total isospin of the system (in this case T is equal to 1/2 or 3/2). The projection of the potential $V_{\pi N}$ onto the states of the system with the total isospin T has the form

$$V^T_{\pi N}(\vec{r}_{12}\vec{r}_3) = C^T_1 \sum_{i=1}^{3} a^1_i + C^T_3 \sum_{i=1}^{3} a^3_i ,$$
$$C^T_1 = \frac{1}{3}\left[1 + 2(-1)^{T+\frac12}\begin{Bmatrix}1 & 1 & 1\\ \frac12 & T & \frac12\end{Bmatrix}\right], C^T_3 = \frac{2}{3}\left[1-(-1)^{T+\frac12}\begin{Bmatrix}1 & 1 & 1\\ \frac12 & T & \frac12\end{Bmatrix}\right], \tag{5}$$

where $\begin{Bmatrix}1 & 1 & 1\\ \frac12 & T & \frac12\end{Bmatrix}$ is the 6j symbol. In the momentum representation expression (5) can by rewritten in the form

$$\langle \vec{k} | V^T(\vec{r}_{12}\vec{r}_3) | \vec{k}' \rangle = \sum_{i,\alpha} \Lambda^\alpha \eta_i^\alpha(\vec{k}) \bar{\eta}_i^\alpha(\vec{k}'), \quad \alpha = 1,2, \quad i = 1,2,3 \tag{6}$$

where the form factors $\eta_i^\alpha(\vec{k})$ are defined by

$$\eta_i^\alpha(\vec{k}) = \frac{1}{k^2+\beta_\alpha^2} e^{i\vec{k}\vec{z}_i}, \quad \text{where} \quad \vec{z}_1 = \frac{1}{2}\vec{r}_{12} + \frac{1}{3}\vec{r}_3, \quad \vec{z}_2 = -\frac{1}{2}\vec{r}_{12} + \frac{1}{3}\vec{r}_3, \quad \vec{z}_3 = -\frac{2}{3}\vec{r}_3 \tag{7}$$

$$\Lambda^1 = C_1^T \lambda_S^1, \quad \Lambda^2 = C_3^T \lambda_S^3 .$$

The solution of eq. (4) with the potential (6) can be given as follows:

$$\langle \vec{k} | r_0(\vec{r}_{12}\vec{r}_3) | \vec{k}' \rangle = \sum_{\substack{ij\\ \alpha\beta}} \Lambda^\alpha \eta_i^\alpha(\vec{k}) [A^{-1}(E,\vec{r}_{12},\vec{r}_3)]_{ij}^{\alpha\beta} \bar{\eta}_j^\beta(\vec{k}'). \tag{8}$$

The matrix elements of the matrix A are given by the integrals of the form factors $\eta_i^\alpha(\vec{k})$ and free Green functions as usual in the fixed scattering model with the separable elementary pion-nucleon potentials. The inverse matrix $A^{-1}(E,\vec{r}_{12},\vec{r}_3)$ at negative energies E as a function of $\cos\theta = \frac{\vec{r}_{12}\cdot\vec{r}_3}{|\vec{r}_{12}|\cdot|\vec{r}_3|}$ and the lengths of the vectors $|\vec{r}_{12}|,|\vec{r}_3|$ have been investigated. The matrix elements of this matrix have been expanded over the set of Legendre polynomials $P_\ell(\cos\theta)$. Only the first term ($\ell=0$) gives the contribution (with a few per cent accuracy). Moreover, in the wide interval $0{,}25\,\text{fm} \le |\vec{r}_{12}|,|\vec{r}_3| \le 3\,\text{fm}$ the matrix A^{-1} is the smooth function of the parameters r_{12}, r_3. Therefore, in the calculation of the average value of expression (8) over the wave function of the ground state, the matrix A^{-1} has been taken out from the integrals at some middle points $\bar{r}_{12}, \bar{r}_3$. The remaining integrals which contain fast oscillating factors η_i^α have been calculated analytically. With the function thus obtained $\langle \vec{k} | r_0 | \vec{k}' \rangle$ equation (3) has been solved for the S wave component at the negative energy of the system. The scattering lengths and the phase shifts in the states with total isospin T=1/2 and T=3/2, have been computed.

In these calculations the one term separable potentials, which reproduced the pion-nucleon S phase shifts (at energies $0 \le E \le 80$ MeV) and the scattering lengths in the states with $t_{\pi N} = 1/2$ and 3/2 have been used*.

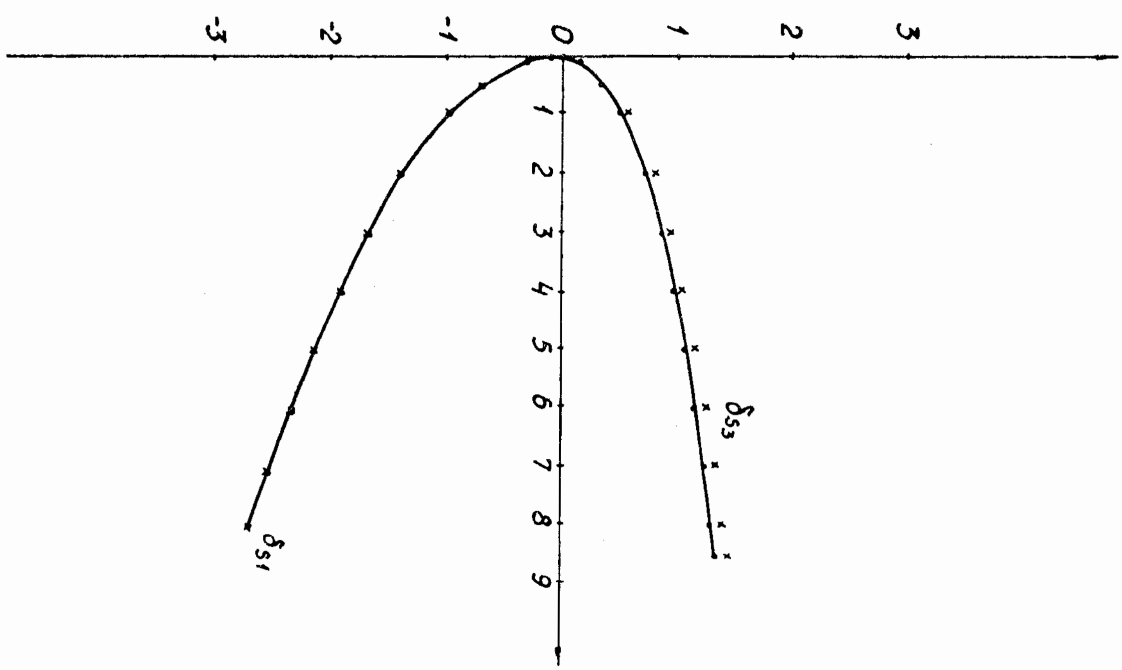
The ground state wave function $\chi(\vec{r}_{12},\vec{r}_3)$ of the three-nucleon system has been chosen in the form proposed by Irving[8]. The parameters of this function provide the experimental value of the mean square root radius of ^{3}He nucleus. The spatial part of it is fully symmetric with respect to the permutation of three particles. The results of our calculations are presented in the Table and Figure. As one can see from the table for the $\pi^- {}^3$He scattering, the strong cancellation of the pion-nuclear amplitude takes place though the amplitudes with the given total isospin are essentially different from zero. Here arises the situation opposite to the pion-deuteron scattering. Indeed, in the πd scattering even the impulse approximation leads to the strong cancellation of $t_{\pi N}=1/2$ and $t_{\pi N}=3/2$ amplitudes, and for the real part of the π^-d scattering length, one has $a^{\pi^- d}_{imp} = -0.009 m_\pi^{-1}$. The inclusion of the multiple scattering effects in the framework of the Faddeev

*In this paper the S-wave pion-nucleon potential for the isospin $t_{\pi N}$=1/2 is the same as in[7]. For the S -wave potential in $t_{\pi N}$ = 3/2 state, we omit the exponent factor and change slightly its parameters. Due to this change we are able to perform all integrations in matrix A analytically, but the range of the reproduction of the phase-shift has been narrowed to [0, 80 MeV].

Table

	$\lambda_3 = -13.77\ fm^{-3}$ $\beta_3 = 3.045\ fm^{-1}$				$\lambda_3 = -8.883\ fm^{-3}$ $\beta_3 = 2.745\ fm^{-1}$			
$\bar r_{12}$ fm, $\bar r_3$ fm	$a^1_{\pi He^3}$ (fm)	$a^3_{\pi He}$ (fm)	$a_{\pi^- He}$ (fm)	$a_{ch\ exch}$ (fm)	$a^1_{\pi He^3}$ (fm)	$a^3_{\pi He^3}$ (fm)	$a_{\pi^- He^3}$ (fm)	$a_{ch\ exch}$ (fm)
$\bar r_{12} = 1$, $\bar r_3 = 0.8$	0,1053	-0,2101	0,00016	-0,1487	0.1135	-0.2096	0,0058	-0,1523
$\bar r_{12} = 2$, $\bar r_3 = 0.8$	0,1043	-0,2173	-0,00289	-0,1516				

Experiment: $a^3_{\pi He^3} = -0.226\ fm$ [13] : $a_{\pi^- He^3} = [0{,}050 \pm 0{,}005 + i(0{,}034 \pm 0{,}012)]\ m_\pi^{-1}$ [11]



Figure

equations[9] increases these values to $0.05\,m_\pi^{-1}$. On the contrary in the case of $\pi^- \, ^3He$ scattering the inclusion of the multiple scattering effects leads to considerable cancellation (see the table) in comparison with the value $a_{imp}^{\pi^- \, ^3He} \approx 0.1$ fm calculated in the impulse approximation.

At present there are no direct measurements of the $\pi \, ^3He$ (triton) scattering length. The value of these quantities are usually extracted from the data on the shifts and widths of the corresponding π-mesonic atoms[10]. The recent measurements[11] of the level shifts and widths in $\pi^- \, ^3He$ atom bring to the following value $a_{\pi^- \, ^3He} = 0.050 \pm 0.005 + i(0.034 \pm 0.012) m_\pi^{-1}$. This value should be compared with the corresponding number presented in the table. The point is that this experimental value includes both the elastic multiple scattering of pions and the inelastic processes, connected, for example, with the absorption and emission of pions in the intermediate states.

Let us assume the Bruckner mechanism for the formation of the level shifts. Then for the measurable shift, we get the following expression[12]

$$\Delta E_{exp.} = \Delta E_{el.} + 1.05\,\Gamma_{exp.} \qquad (9)$$

Comparing the last expression with the experimental data[11] we see that the "elastic" part of the $\pi^- \, ^3He$ scattering lengths comprise about 5 per cent of the experimental value. Within the experimental errors it agrees with our calculations for the elastic part of the scattering length. As one can see from the table, this value is extremely sensitive both to the parameters of the πN interaction and to the approximations used in the calculation $\langle \vec{k} | r_0 | \vec{k}' \rangle$. At the same time the $\pi \, ^3He$ (triton) amplitudes corresponding to the states with the given total isospin (1/2, 3/2) are not sensitive to these effects.

To compare the calculated $\pi^+ \, ^3He$ scattering length with experiment we make use of the results of paper[13]. In this article the level shifts and widths of $\pi^- \, ^3H$ atom are extracted on the basis of the interpolation formula on A and Z of the existing data of other light nuclei. With the help of the mentioned Bruckner procedure the authors extract the elastic part of the $\pi^+ \, ^3He$ scattering lengths which equals -0.226 fm. This value is also in good agreement with our calculation. From the aforesaid we conclude that at very low energy the $\pi^- \, ^3He$ amplitude is formed mainly due to the inelastic processes while $\pi^+ \, ^3He$ is mainly elastic. This qualitatively agrees with the pure "triton-like" (T=1/2) picture of the intermediate state. As one can see from the Figure, the phase shifts in the states T = 1/2 and T = 3/2 qualitatively has the same behaviour as the corresponding pion-nucleon ones. However, in the absolute value the $S_{1/2}$ phase shift $\pi \, ^3He$ is less than $S_{1/2}$ of πN whereas the $S_{3/2}$ phase shift of $\pi \, ^3He$ is larger than the corresponding pion-nucleon one.

As has been mentioned above a rather simple wave function of ^3He nucleus has been used in the calculation. Its parameter has been fixed to reproduce the mean square root of ^3He. We have performed the supplementary calculations of the pion ^3He scattering lengths and phase shifts by changing the value of the mean square root of the ^3He-nucleus by a factor of 1.5. It happened that scattering lengths have not been changed significantly. This fact can demonstrate the reliability of the approximation (2) in the considered energy range.

For completeness we present the value of the charge exchange amplitude at zero energy (see the table).

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Received by Publishing Department
on April 19 1978.