ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

E4 - 11486

E.Ivanov, E.Truhlik

HARD PIONS AND AXIAL MESON EXCHANGE CURRENT EFFECTS IN NEGATIVE MUON CAPTURE IN DEUTERIUM

1978

E4 - 11486

E.Ivanov, E.Truhlik*

HARD PIONS AND AXIAL MESON EXCHANGE CURRENT EFFECTS IN NEGATIVE MUON CAPTURE IN DEUTERIUM

Submitted to "Nuclear Physics"

---

* On leave (up to April 30,1978) from Institute of Nuclear Physics, 250 68 Rez n. Prague, Czechoslovakia.

Иванов Е., Труглик Э. E4 - 11486

Метод жестких пионов и эффекты обменных аксиальных токов в реакции захвата отрицательных мюонов в дейтерии

Используя минимальную, кирально- и частично калибровочно-инвариантную лагранжеву модель для системы $A_1 \rho \pi$ мы оценили влияние аксиальных обменных токов на дублетную скорость захвата в реакции $\mu^- + d \to 2n + \nu_\mu$. Показано, что вклад от обычно учитываемого тока типа слабого $\rho - \pi$ распада компенсируется вкладом от подобного процесса, не содержащего A_1 -мезонный полюс, который предсказывается на основании соответствующих принципов инвариантности. Корректный учет импульсной зависимости пропагатора N^* -резонанса в графике, соответствующем процессу N^* -возбуждения с обменом пиона, приводит к подавлению вклада этого тока на $\approx 30\%$.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1978

Ivanov E., Truhlik E. E4 - 11486

Hard Pions and Axial Meson Exchange Current Effects in Negative Muon Capture in Deuterium

The contribution of the axial meson exchange current effects to the doublet transition rate in the reaction $\mu^- + d \to 2n + \nu_\mu$ is calculated by using the minimal, chiral and approximately gauge invariant Lagrangian model for the $A_1 \rho \pi$ system. The contribution from the usually considered $\rho - \pi$ weak decay process current is found to be nearly cancelled by that from the A_1-pole graph which is prescribed by the underlying invariance principles. Correct treatment of the N^*-propagator in the N^*-excitation current of the pion range leads to $\approx 30\%$ suppression of the N^*-effect.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1978

1. INTRODUCTION

Recently, Dautry, Rho and Riska [1] (DDR) have evaluated the axial meson exchange current (MEC) effects in the two-nucleon systems. First, they have shown that the axial MEC (see fig. 1 of ref. [1]) constructed by the standard methods [2] are not in the conflict with the existing data for the reaction

$$p + p \to d + \pi^+ . \tag{1}$$

Further, with the MEC operators so obtained, DRR studied the reaction which is important in the astrophysical scale

$$p + p \to d + e^+ + \nu_e , \tag{2}$$

and also the important reaction of the negative muon capture in deuterium

$$\mu^- + d \to 2n + \nu_\mu . \tag{3}$$

This reaction is one of the best candidates for extracting the neutron-neutron scattering length a_{nn} [1,3-10].

For reaction (3) according to ref. [1], the axial MEC contribution to the doublet transition rate $\Gamma_{1/2}$ increases it by $\approx 6\%$, the most important effect being from the $N^* \equiv N^*(3,3)$ resonance (fig. 1a of ref. [1]). Like as in the reaction

$$^3H \to {}^3He + e^- + \tilde{\nu}_e , \tag{4}$$

the suppression of the N^*-effect may be achieved by

i) inclusion of the N* -piece into the nuclear wave function[11]. While the problem of renormalization of the wave function does not arise in the final neutron-neutron state, it still persists for the deuteron wave function;

ii) taking into account the contribution coming from a graph with the exchange of a heavy meson instead of a pion in fig. 1a[1] and also from some other graphs;

iii) using the correct momentum dependence in the denominator of the N*-propagator in place of the usually accepted static approximation[12,13].

In our recent paper[14] we have developed a consistent approach to the axial MEC problem in the nuclear physics based on the hard pion method. This approach allows us to take into account the pion as well as heavy meson exchange graphs on equal footing. The two-body axial MEC operator is reasonably given in the tree-approximation (see fig. 1 in ref.[14]).

Here we study the axial MEC effects, in the framework of accepted model, for reaction (3). The non-Born MEC operators which contribute significantly to the doublet transition rate $\Gamma_{1/2}$ are presented in fig. 1. The Born graph is of a standard form.

Our calculations differ from those by DRR due to

i) much larger contribution from the static N*-excitation current of the pion range (fig. 1a, pion exchanged);

ii) inclusion of other graphs with the non-negligible contribution to the $\Gamma_{1/2}$ (graphs 1a with the A_1 -meson exchange, 1b, 1c and 1e);

iii) correct treatment of the momentum dependence of the N*-propagator in the N*-excitation current of the pion range.

In Sec. 2 we list the axial MEC operators of fig. 1 in terms of form factors. Our numerical results and main conclusions are given and discussed in Sec. 3.

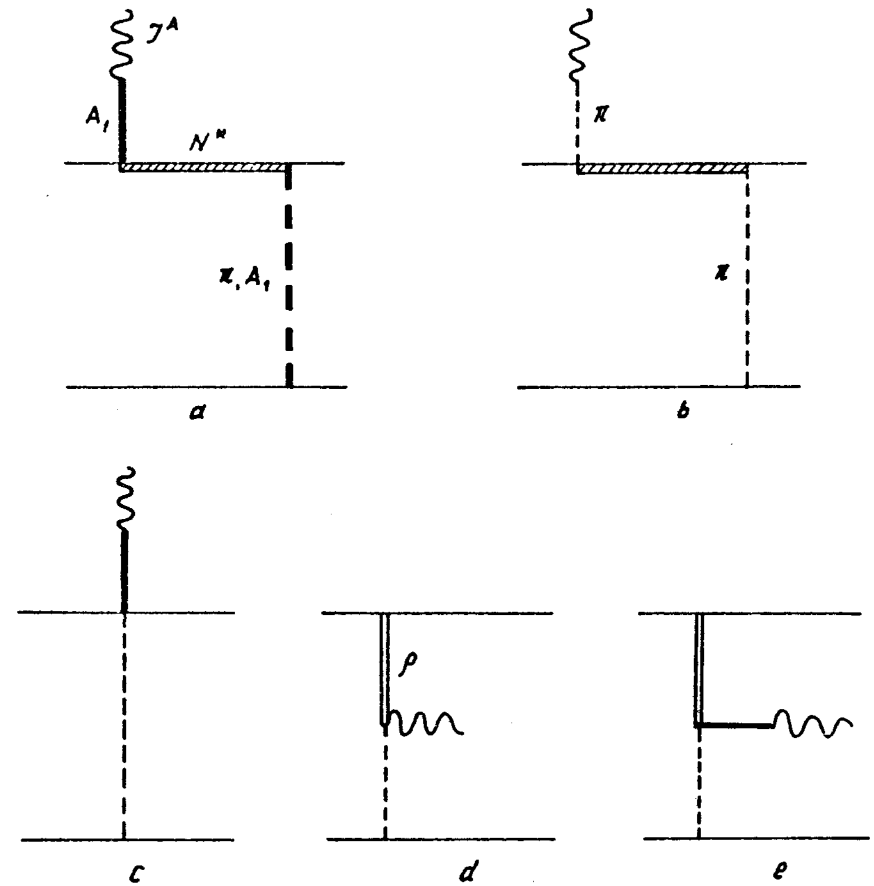

Fig. 1. Feynman graph representation of the axial non-Born MEC operators which contribute to the doublet transition rate $\Gamma_{1/2}$ in reaction (3). a), b) Isobar excitation currents. c) Contact term. d) $\rho-\pi$ weak decay current. e) $A_1\rho\pi$ current. J^A stands for the weak axial-vector current.

2. AXIAL TWO-BODY EXCHANGE CURRENTS

The form of the relevant exchange-current effective operator in the spin-isospin space for reaction (3) was given in eq. (3.15) of ref.[1]. It takes into account the transition to the $^1S_0$ neutron-neutron final state but including the deuteron S- and D-states. We write it here in a symmetric way

$$\vec{M}^2_{\sigma,\tau} = \frac{G}{\sqrt{2}}\{(\vec{\tau}_1\times\vec{\tau}_2)^-[g_I(q^2)(\vec{\sigma}_1\times\vec{\sigma}_2)+g_{II}(q^2)\vec{T}^{(\times)}_{12}(\hat{q})+$$

$$+ig_{III}(q^2)(\vec{\sigma}_1-\vec{\sigma}_2)+ig_{IV}(q^2)\vec{T}^{(-)}_{12}(\hat{q})]+ \tag{5}$$

$$+(\vec{\tau}_1-\vec{\tau}_2)^-[h_I(q^2)(\vec{\sigma}_1-\vec{\sigma}_2)+h_{II}(q^2)\vec{T}^{(-)}_{12}(\hat{q})+ih_{III}(q^2)(\vec{\sigma}_1\times\vec{\sigma}_2)$$

$$+ih_{IV}(q^2)\vec{T}^{(\times)}_{12}(\hat{q})]\},$$

with

$$\vec{T}^{\odot}_{12}(\hat{q}) = (\vec{\sigma}_1\odot\vec{\sigma}_2)\cdot\hat{q}\hat{q} - \frac{1}{3}(\vec{\sigma}_1\odot\vec{\sigma}_2).$$

We now present in terms of the form factors g and h the axial MEC operators which correspond to the graphs 1a with the pion exchange, 1c, 1d and 1e. The relation between our form factors and those by DRR (which are in r.h.s. of eq. (6)) is

$$g_i = g_i\ ,\quad h_i = h_i\ ,\quad i = I, II,$$
$$g_i = -h^{\tau}_{i-II}\ ,\quad h_i = -h^{\sigma}_{i-II}\ ,\quad i = III, IV. \tag{6}$$

2.1. The N*-excitation term with the pion exchange (fig. 1a)

For comparison with ref.[1] in the numerical calculations we also take the quark model prediction for the πNN^*-coupling constant,

$$\frac{f^2_{\pi NN^*}}{4\pi} = 0.23\,, \tag{7}$$

However, from the N*-resonance width[11]

$$\frac{f^2_{\pi NN^*}}{4\pi} = 0.35\,. \tag{8}$$

The form factors are

$$\begin{aligned}
g_I &= -C_1[J^0_0(Y_0)+\frac{1}{\sqrt{8}}J^2_0(Y_2)],\\
g_{II} &= -\frac{3}{2}C_1[J^0_2(Y_2)+\frac{1}{\sqrt{8}}J^2_2(2Y_0-Y_2)],\\
g_{III} &= -\frac{1}{\sqrt{8}}C_1J^2_0(Y_2),\\
g_{IV} &= -\frac{3}{2\sqrt{8}}C_1J^2_2(2Y_0-Y_2),\\
h_I &= C_1[J^0_0(Y_0)-\frac{1}{\sqrt{2}}J^2_0(Y_2)],\\
h_{II} &= 3C_1[-J^0_2(Y_2)+\frac{1}{\sqrt{8}}J^2_2(Y_0+Y_2)],\\
h_{III} &= \frac{1}{\sqrt{2}}C_1J^2_0(Y_2),\\
h_{IV} &= -\frac{3}{\sqrt{8}}C_1J^2_2(Y_0+Y_2).
\end{aligned} \tag{9}$$

In eq. (5)

$$C_1 = \sqrt{\frac{2}{\pi}}\,\frac{2g_A}{27}\,\frac{m_\pi}{M^*-M}\,f^2_{\pi NN^*}\ ,\quad g_A = 1.25$$

and

$$J_K^L(Y_P) = \int_0^\infty r\, j_K(\tfrac{1}{2}qr)\Phi_\kappa(r) Y_P(m_\pi r) u_L(r)\, dr,$$

$$Y_0(x) = \frac{e^{-x}}{x}, \quad Y_2(x) = (1 + \frac{3}{x} + \frac{3}{x^2}) Y_0(x). \tag{10}$$

The function $\Phi_\kappa(r)$ describes the two neutrons in the final $^1S_0$ state. For the neutrons without interaction $\Phi_\kappa(r) = j_0(\kappa r)$; $\kappa = \frac{1}{2}|\vec{n}_1 - \vec{n}_2|$. The momentum transfer $q = \nu$, where ν is the neutrino momentum. Our form factors, eq. (9), differ from those by DRR. In order to shed light on our calculations we present in the Appendix the basic formulae and steps in obtaining eq. (9). We note that in our model /14/, the considered N*-excitation current, constructed for the triton β-decay, is the same as that already used in refs. /15, 16/.

2.2. The contact term (graph 1c)

Only the form factors g are present

$$g_I = 2C_2[J_0^0(Y_0) + \frac{1}{\sqrt{8}} J_0^2(Y_2)],$$

$$g_{II} = 3C_2[J_2^0(Y_0) + \frac{1}{\sqrt{8}} J_2^2(2Y_0 - Y_2)],$$

$$g_{III} = \frac{1}{\sqrt{2}} C_2 J_0^2(Y_2), \tag{11}$$

$$g_{IV} = \frac{3}{\sqrt{8}} C_2 J_2^2(2Y_0 - Y_2),$$

$$C_2 = -\frac{g_A(1+\kappa_V)}{24\sqrt{2\pi}} \frac{m_\pi}{M} (\frac{m_\pi}{f_\pi})^2,$$

with $\kappa_V = 3.7$, $f_\pi = 92$ MeV.

2.3. The $\rho - \pi$ weak decay current contribution (graph 1d)

All what matters is

$$g_I = C_3[J_0^0(W_1) + \frac{1}{\sqrt{8}} J_0^2(W_2)], \tag{12}$$

$$g_{III} = \frac{C_3}{\sqrt{8}} J_0^2(W_2), \quad C_3 = -\sqrt{\frac{\pi}{2}} \frac{(1+\kappa_V) g_A}{3M} \frac{g_\rho^2}{4\pi}.$$

The ρNN-coupling constant $g_\rho \approx 5.9$ is taken from the KSFR relation

$$2 f_\pi^2 g_\rho^2 = m_\rho^2,$$

and $m_\rho = 770$ MeV. The functions J_K^L are defined in eq. (10), but now

$$W_i = \int_0^1 f(r,t) g_i(r,t)\, dt,$$

$$g_1(r,t) = a - \frac{2}{r}, \quad g_2(r,t) = a + \frac{1}{r}, \tag{13}$$

$$f(r,t) = e^{-ar} j_0(qrt),$$

$$a \equiv a(q,t) = [t(1-t)q^2 + t(m_\rho^2 - m_\pi^2) + m_\pi^2]^{\frac{1}{2}}.$$

In obtaining eqs. (12) and (13) we have transformed the functional form of the amplitude of the graph 1d

$$J(\vec{r}) = \int \frac{e^{i\vec{p}\cdot\vec{r}}\, d\vec{p}}{[(\vec{p}-\vec{q})^2 + m_\rho^2][\vec{p}^2 + m_\pi^2]} \tag{14}$$

into the convenient integral representation

$$J(\vec{r}) = \pi^2 \int_0^1 \frac{dt}{a} e^{-ar+it\vec{q}\cdot\vec{r}} . \qquad (15)$$

2.4. The $A_1\rho\pi$ contribution (graph 1e)

In the same notation as for the graph 1d, we have

$$g_I = C_4[J_0^0(W_3) + \frac{1}{\sqrt{8}} J_0^2(W_4)],$$

$$g_{II} = -\frac{9}{4\sqrt{2}} C_4 J_0^2(W_5), \qquad g_{III} = \frac{1}{\sqrt{8}} C_4 J_0^2(W_4), \qquad (16)$$

$$g_{IV} = g_{II}, \qquad C_4 = \frac{C_3}{m_\rho^2}.$$

The form of the W_i is given in eq. (13), where now

$$g_3(r,t) = a^2(a - \frac{4}{r}),$$

$$g_4(r,t) = a^2(a - \frac{1}{r}) - \frac{6}{r^2}(a + \frac{1}{r}), \qquad (17)$$

$$g_5(r,t) = \frac{a^2}{3}(a - \frac{7}{r}) + \frac{2}{r^2}(a + \frac{1}{r}).$$

In the limit $q=0$ ("deuteron β-decay")

$$W_1(x) = D[Y_0(x) - z^3 Y_0(y)],$$

$$W_2(x) = D[Y_2(x) - z^3 Y_2(y)],$$

$$W_3(x) = m_\rho^2 D [z^{-2} Y_0(x) - z^3 Y_0(y)],$$

$$W_4(x) = m_\rho^2 D [z^{-2} Y_2(x) - z^3 Y_2(y)],$$

$$D = \frac{2m_\pi^3}{m_\rho^2 - m_\pi^2}, \quad z = \frac{m_\rho}{m_\pi}, \quad x = m_\pi r, \quad y = zx. \qquad (18)$$

3. NUMERICAL RESULTS AND DISCUSSION

In calculating the contribution to the doublet transition rate $\Gamma_{1/2}$ from the processes illustrated in fig. 1, we have used the following part of the single-particle weak interaction Hamiltonian [17]

$$\Delta H_\mu = \frac{G}{\sqrt{2}} \cos\theta \chi_\nu^+ (1 - \vec{\sigma}_\ell \cdot \hat{\nu}) \sum_i e^{-i\vec{\nu}\cdot\vec{r}_i} [G_A(\vec{\sigma}_\ell \cdot \vec{\sigma}_i) + G_P(\vec{\sigma}_i \cdot \hat{\nu})] \tau_i^- \chi_\mu, \qquad (19)$$

where

$$G_A = -g_A - \frac{\nu}{2M}(g_V - 2Mg_W),$$

$$G_P = \frac{\nu}{2M}[g_A + m_\mu g_P - g_V + 2Mg_W], \qquad (20)$$

$$g_V - 2Mg_W = 4.7, \quad g_A = 1.25, \quad g_P = -\frac{2Mg_A}{m_\pi^2 + q^2}.$$

The q-dependence of the form factors g_A, g_V and g_W plays no role here. The deuteron wave function was taken in the standard form,

$$\Psi_d(\vec{r}) = \frac{1}{\sqrt{4\pi}\, r}[u_0(r) + \frac{1}{\sqrt{8}} S_{12}(\hat{r}) u_2(r)] \chi_{1M} . \qquad (21)$$

The functions u_0, u_2 and also the neutron-neutron wave function in the $^1S_0$ state were generated by exploiting the RSC potentials /18/ in the $^3S_1 - ^3D_1$ and $^1S_0$ channels, respectively. The numerical results together with those by DRR are given in the table. The evaluation of other processes, presented in fig. 1 of ref. /14/ but not here, shows that their effect is negligible.

In our scheme with the pseudovector πN-coupling the negative-energy Born term does not contribute. The positive-energy Born term can be considered to be incorporated into the nuclear wave function. However, it is not clear à priori if this procedure is equivalent to that used in the standard calculations with the pseudoscalar πN-coupling. In order to clarify the situation we passed to the standard scheme by the equivalence transformation of the nucleon field. As we have shown in ref. /14/, a new pion pole graph appears (which does not contribute) and the minimal $A_1 \pi N$ coupling in the graph 1c is renormalized (see eq. (21) in ref. /14/). The piece $\sim -2g_A^2$ is nothing but "the PCAC constraint term". Numerically, this term gives a contribution $\Delta\Gamma_{1/2} = -2\ \mathrm{sec}^{-1}$ to the $\Gamma_{1/2}$. Now also the negative-energy Born term contributes to $\Gamma_{1/2}$ by 6.7 sec^{-1}. We see that these two terms do not cancel each other. This means that the incorporation of the positive-energy Born term into the nuclear wave function is a model dependent procedure. From a more general point of view, the definition of the Born term via the pseudoscalar πN-coupling should be preferred. The numerical results are presented (table) for both types of coupling.

Recently, Jaus has reported in the triton β-decay calculations /13/ the large damping effect ($\approx 50\%$) due to the momentum dependence of the N^*-propagator taken into account. It is seen from the table, that in our case, with the exact two-nucleon wave functions, this effect is about 30%, i.e., somewhat smaller. In order to perform reliably this type of calculation, we passed to the momentum representation.

If we add to this damping effect also the contribution from two other N^*-excitation processes considered (see the table), we obtain the reduction of $\approx 50\%$ of the N^*-piece with the pion exchange. According to ref. /11/ more suppression could be achieved by the generation of the N^*N^*-configuration in the deuteron.

From the table we also see the near cancellation of the effects from the graphs 1d and 1e. As was already discussed in our earlier paper /14/ within the hard pion method, the process presented in graph 1e is prescribed by the gauge chiral invariance principle. As a consequence, the effective $\rho\pi J^A$ vertex (for small momentum transfer) is /14/

$$g_{\rho\pi J^A} = f_\pi g_\rho \left(1 - \frac{p^2}{m_\rho^2}\right), \tag{22}$$

instead of

$$g_{\pi\rho J^A} = 2 f_\pi g_\rho{}^{/2/} . \tag{23}$$

In eq. (22), p is the virtual pion four-momentum. Only for $p^2 = -m_\rho^2$ the value of the $g_{\rho\pi J^A}$ in eq. (22) is also equal to $2f_\pi g_\rho$. It is the p-dependent part of the $g_{\rho\pi J^A}$ which enters into the graph 1e. The near cancellation just mentioned above is a consequence of the calculation of the nuclear matrix elements. This can be seen more clearly in the case of zero momentum transfer, $q = 0$ ("deuteron β-decay"). Then the functions W_i are given in eq. (18). For the graph 1e, the functions W_3 and W_4 determine the contribution to $\Gamma_{1/2}$. Comparing with the functions W_1 and W_2, the functions $Y_L(x)$ are now multiplied by $z^{-2} = (m_\pi / m_\rho)^2 \approx 1/30$ and the negative terms $-z^3 Y_L(y)$ prevail. We suppose that this effect takes place also in the case of the triton β-decay and may bring the theoretical predictions of ref. /19/ into a better agreement with the experiment.

We note that the whole amount of the contribution from the graph 1d is due to the deuteron D-state. Analogous cancellation of the pion and rho-meson propagators for the S-state part of the matrix element in the triton β-decay was noted in refs.[2,20,21].

We have also evaluated other two N^*-excitation processes, represented in the graphs 1a with the A_1-meson exchange and in the graph 1b. The contribution from them is comparable with that from the graph 1d or 1e. In our model, there exists also a graph of the type 1a with the rho-meson exchange. Because of the uncertainty in the sign of the $f_{\pi NN^*}$ and f_π, only the absolute value of the effect can be estimated. Fortunately, it turns out to be $\sim 1\ \mathrm{sec}^{-1}$, and therefore, it can be neglected.

If we take the value for the $f_{\pi NN^*}$ from the N^*-resonance width (eq. (8)), then for the total MEC effects, we finally have

$$\delta\Gamma_{1/2} \simeq 29\ \mathrm{sec}^{-1} \tag{24}$$

instead of 23.2 sec^{-1}. However, it is the $\delta\Gamma_{1/2} \simeq 37\ \mathrm{sec}^{-1}$ which should be compared with the $\delta\Gamma_{1/2} = 24\ \mathrm{sec}^{-1}$ obtained by DRR.

Let us stress that the main difference with the calculations of ref.[1] is due to

i) different estimation of the N^*-excitation graph;
ii) taking into account the contact graph 1c;
iii) employing the momentum dependent $\rho\pi J^A$ vertex.

The theoretical prediction for $\Gamma_{1/2}$ seems now to be well established [1,3,4,7]. Without the MEC effects

$$\Gamma_{1/2} \approx 380\ \mathrm{sec}^{-1}, \quad g_P \approx -7\, g_A .$$

On the other hand, the experimental situation is critical. The old experiment in the liquid matter[9] yields

$$\Gamma_{1/2} = 365 \pm 96\ \mathrm{sec}^{-1},$$

Table

The contributions $\Delta\Gamma_{1/2}$ (in sec^{-1}) to the doublet transition rate for reaction (3) from various meson exchange processes of fig. 1. A - results obtained with $G_P = 0$ in eq. (19). B - G_P from eq. (20). C - results from ref.[1] (a) pseudovector πN-coupling, (b) pseudoscalar πN-coupling.

$\Gamma_{1/2}$ \ graph		1a pion exchange	negative-energy Born term	1c	1d	1e	1a A_1-meson exchange	1b	total MEC effect
A	(a)	25.0	0	7.8	4.4	−3.4	−2.6	−2.6	28.6
	(b)		7.8	5.6[e]					34.2
B	(a)	21.6	0	6.8	3.8	−2.8	−2.2	−2.2	25.0
	(b)		6.7	4.8[e]					29.7
	(a)	15.1[d]	0	6.8					18.5
	(b)		6.7	4.8[e]					23.2
C	(b)	13	6		5				24

d) the momentum dependence of the N^*-propagator taken into account
e) the PCAC constraint term included.

whereas from the recent CERN experiment [10] in the gaseous p-d mixture

$$\Gamma_{1/2} = 445 \pm 60 \text{ sec}^{-1}.$$

However, according to ref. [22] the conditions of the last experiment correspond rather to the measuring of the statistical average Γ_{tot} of the doublet and quadruplet $\Gamma_{3/2}$ transition rates. The theoretical prediction for Γ_{tot} is

$$\Gamma_{tot} \approx 135 \text{ sec}^{-1}.$$

As was noted in ref. [1], even a good experimental knowledge of $\Gamma_{1/2}$ would be useless for the direct evidence of the axial MEC in reaction (3), because of uncertainty in the g_P which also affects the value of the $\Gamma_{1/2}$. Recently, the sensitivity of the ratio $\Gamma_{3/2}/\Gamma_{1/2}$ to the value of the g_P has been pointed out in ref. [23]. It is clear that in the ultimate analysis, the influence of the MEC effects should be taken into account. They should be also considered when extracting the neutron-neutron scattering length from the spectra measured in reaction (3).

We thank Dr. R.A.Eramzhyan and Dr. M.Gmitro for discussion. The help in programming in SCHOONSHIP by O.V.Tarasov is acknowledged.

APPENDIX

Here we expose the main formulae needed to obtain eq. (9). Starting from the hard pion Lagrangian model [14] and applying the standard Feynman rules, we get for the N^*-excitation current of the pion range the following formula

$$\vec{H}_{N^*} = \frac{G}{\sqrt{2}} \frac{C}{2} \delta(\vec{r}_1 - \vec{r}_1{}')\delta(\vec{r}_2 - \vec{r}_2{}')(e^{-i\vec{\nu}\cdot\vec{r}_1} + e^{-i\vec{\nu}\cdot\vec{r}_2})\vec{H}_{N^*}, \quad (A.1)$$

where

$$\vec{H}_{N^*} = \{(\vec{\tau}_1 - \vec{\tau}_2)^{-}[(\vec{\sigma}_1 - \vec{\sigma}_2)Y_0(x) + 3\vec{T}_{12}^{(-)}(\hat{r})Y_2(x)]$$

$$+ (\vec{\tau}_1 \times \vec{\tau}_2)^{-}[-(\vec{\sigma}_1 \times \vec{\sigma}_2)Y_0(x) + \frac{3}{2}\vec{T}_{12}^{(\times)}(\hat{r})Y_2(x)]\}, \quad (A.2)$$

$$C = \frac{4}{27} \frac{g_A m_\pi}{M^* - M} \frac{f^2_{\pi N N^*}}{4\pi}.$$

The form of the operator $\vec{T}_{12}^{\odot}(\hat{r})$ is given in eq.(5). The transition operator from the nuclear ${}^3S_1 - {}^3D_1$ state to the neutron-neutron ${}^1S_0$ state in the spin-isospin space is as follows

$$\vec{M}^2_{\sigma,\tau}(N^*) = \frac{G}{\sqrt{2}} C \int e^{-\frac{i}{2}\vec{\nu}\cdot\vec{r}} \sqrt{2}\,\Phi_\kappa(r)\vec{H}_{N^*}\frac{1}{\sqrt{4\pi}\,r}[u_0(r) + \frac{1}{\sqrt{8}} S_{12}(\hat{r})u_2(r)]\,dr. \quad (A.3)$$

Performing some vector algebra calculations and using the formulae

$$\int e^{-\frac{i}{2}\vec{\nu}\cdot\vec{r}} \Phi_\kappa(r) Y_p(x) \frac{u_L(r)}{r} d\vec{r} = 4\pi J_0^L(Y_p),$$

$$\int e^{-\frac{i}{2}\vec{\nu}\cdot\vec{r}} \Phi_\kappa(r) Y_p(x) \vec{T}_{12}^{\odot}(\hat{r}) \frac{u_L(r)}{r} d\vec{r} = -4\pi J_2^L(Y_p)\vec{T}_{12}^{\odot}(\hat{\nu}), \quad (A.4)$$

we obtain the operator $\vec{M}^2_{\sigma,\tau}(N^*)$ in the form (5) with the g,h in eq. (9).

REFERENCES

1. Dautry F., Rho M., Riska D.O. Nucl.Phys., 1976, A264, p.507.
2. Chemtob M., Rho M. Nucl.Phys., 1971, A163,p.1.
3. Ho-Kim Q., Lavine J.P., Picket H.S. Phys.Rev., 1976, C13, p.1966.

4. Dogotar G.E., Salganic Yu.A., Eramzhyan R.A. Yad.Fiz., 1975, 22, p.472.
5. Cremmer E. Nucl.Phys., 1967, B2, p.409.
6. Pascual P., Tarrach R., Vidal F. Nuovo Cim., 1972, 12A, p.241.
7. Sotona M., Truhlik E. Nucl.Phys., 1974, A229, p.471.
8. Wang I.T. Phys.Rev., 1965, 139, B1539.
9. Wang I.T. et al. Phys.Rev., 1965, 139, B1229.
10. Bertin A. et al. Phys.Rev., 1973, D8, p.3774.
11. Green A .M. Rep. Progr.Phys., 1976, 39, p.1109.
12. Ichimura M., Hyuga H., Brown G.E. Nucl.Phys., 1972, A196, p.17.
13. Jaus W. Nucl.Phys., 1976, A271, p.495.
14. Ivanov E., Truhlik E. JINR, E4-11477, Dubna, 1978.
15. Cheng W.K. Ph.D.Thesis. University of Pennsylvania, 1966, unpublished.
16. Kim Y.E. et al. In: Few Particle Problems in Nuclear Interaction, ed. I.Slaus et al. North-Holland, Amsterdam, 1972, p.433.
17. Blin-Stoyle R.J. Fundamental Interactions and Nucleus, North-Holland, Amsterdam, 1973.
18. Reid R.V., Jr. Ann.of Phys., N.Y., 1968, 50, p.411.
19. Onta K., Wakamatsu M. Preprint UT-Komaba 75-8.
20. Riska D.O., Brown G.E. Phys.Lett., 1970, 32B, p.662.
21. Fischbach E. et al. Phys.Lett., 1972, 38B, p.8.
22. Bystritsky V.M. Int. Symp. on Meson Chemistry and Mesomol. Processes in Matter, Dubna, 1977. JINR, D1,2,14-10408, Dubna, 1977.
23. Dogotar G.E., Eramzhyan R.A. JINR, P2-9469, Dubna, 1976.

Received by Publishing Department
on April 17 1978.