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Метод жестких пионов и обменные слабые аксиальные токи в ядрах

Развит последовательный подход к проблеме аксиальных мезонных обменных токов (МОТ) в ядрах на основе метода жестких пионов. Этот метод представляет собой самосогласованную комбинацию подходов алгебры токов и векторной доминантности и позволяет изучать единым образом обмены как пионами, так и тяжелыми мезонами. Используя минимальный феноменологический лагранжиан для $A_1\rho\pi$-системы мы строим оператор двухнуклонных МОТ в приближении деревьев. Этот оператор автоматически обладает правильными трансформационными свойствами относительно группы $SU_2\times SU_2$ и имеет импульсную зависимость настолько гладкую, насколько это возможно в рамках комбинированного подхода алгебры токов и векторной доминантности. В данной модели мы рассматриваем неборновскую часть амплитуды $N+J^A\to N+\pi$ и демонстрируем, что в мягкопионном пределе она точно воспроизводит предсказания PCAC.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Hard Pions and Axial Meson Exchange Currents in Nuclear Physics

Starting from the hard meson method we develop a consistent approach to the problem of the axial meson exchange currents (MEC). This method incorporates the current algebra and PCAC together with the vector dominance and allows one to study the pion as well as heavy-meson exchanges on equal footing. Using a minimal, chiral and approximately gauge-invariant phenomenological Lagrangian (PL) model for the $A_1\rho\pi$-system we construct the two-nucleon axial MEC operator in the tree-approximation. This operator automatically possesses the correct chiral $SU_2\times SU_2$-transformation properties and has the smoothest momentum dependence which is allowed within the combined current algebra and vector dominance approaches. In the given model, we consider the non-Born part of the amplitude $N+J^A\to N+\pi$ and demonstrate that in the soft pion limit, it exactly coincides with the PCAC prediction.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

The problem of the axial meson exchange current (MEC) effects has been studied (see refs.[1-8] and refs. therein) for about twenty years in the triton β-decay

$$^3H \to {}^3He + e^- + \tilde{\nu}_e . \qquad (1)$$

It is believed that these effects may explain the discrepancy between the experimental value $|M_{GT}|^2 = 2.87\pm0.04$[9,10] of the Gamow-Teller matrix element and the predictions based on the calculations with the single-particle current (impulse approximation) which yield the results smaller by $\approx$ 5-8%.

The necessity of taking into account the axial MEC effects in the reaction

$$\mu^- + {}^3He \to {}^3H + \nu_\mu \qquad (2)$$

was demonstrated in refs.[11,12]. Without them, the extracted value of the weak induced pseudoscalar constant g_P is in direct contradiction with the Goldberger-Treiman relation.

Recently, Dautry, Rho and Riska[13] have evaluated the axial MEC effects in more simple, two-nucleon reactions of principal interest

$$p + p \to d + \pi^+ , \qquad (3)$$

$$p + p \to d + e^{+} + \nu_e , \qquad (4)$$

$$\mu^{-} + d \to n + n + \nu_{\mu} . \qquad (5)$$

They have shown that the calculated MEC effects are non-negligible in all three reactions (3)-(5).

The basic ideas of the modern calculations of the MEC operators are collected in the pioneer work by Chemtob and Rho [3]. There, these operators are constructed in the one-boson exchange approximation by the methods based on the current algebra, PCAC and (to some extent) vector dominance. The meson exchanges, which are usually taken into account, are those due to the pions and rho-mesons. As to the contributions from various nucleon resonances, the $N^* \equiv N^*(3.3)$-isobar contributes predominantly [5]. In the case of reaction (1), the N^* -piece taken together with purely mesonic currents exceeds by 4% [7] the experimental value of M_{GT} .

In the present paper, we construct the two-nucleon axial MEC operators in the one-boson approximation starting with the model which differs from the standard Chemtob-Rho approach. Namely, we consistently exploit the hard pion method [14-16], which incorporates the current algebra and PCAC together with the important concept of the rho-dominance of the isospin current [17]. In our opinion, the hard pion method provides the most suitable framework for the MEC calculations as

i) it fixes the rho-meson couplings. As a result, the important diagrams with the rho-exchange are defined correctly. Note that the current algebra and PCAC fix only the pion couplings but admit uncertainties in treatment of the rho-exchange diagrams. It is not clear, e.g., how many of them should be taken into account.

ii) In the hard pion method, the explicit momentum dependence of various form factors is specified up to ≈ 1 GeV, while the current algebra itself determines only the first terms in the expansion of the proper vertices at low energies (in the soft pion limit). The knowledge of the momentum dependence (other than that coming from propagators) of the MEC operators may turn out to be essential, as the realistic nuclear wave function may be sensitive to it.

The new feature of the MEC calculations within the hard pion method is the appearance of graphs with the A_1 -meson which should be taken into account with necessity for consistency of the current algebra and the vector dominance [14,15]. Let us note that the very existence of the A_1 -meson seems to be now out of question [18]. Possible employment of the A_1-meson in the nuclear physics calculations was already noted in [19].

We use the phenomenological Lagrangian (PL) realization of the hard pion method [15,16]. The PL technique is most convenient for practical calculations. It enables one, applying the standard Feynman rules, to write down all graphs which are required for a given process by the corresponding dynamical principle.

The dynamical principle which governs the structure of the hard pion PL is the chiral gauge invariance (the invariance under the coordinate dependent $SU_2 \times SU_2$ -transformations). It is assumed to be broken only due to the non-zero mass m_ρ and m_A of the rho- and A_1 -mesons. This form of breakdown ensures the universality of the minimal rho-meson couplings and besides, leads to the field-current identities which manifestly realize the vector dominance idea. The hard pion PL possesses also the usual constant-parameter chiral invariance broken by the pion mass. Hence, such a PL reproduces in the soft pion limit all standard PCAC results.

The main feature of our study should be seen in the consistent combination of the chiral approach with the vector dominance concept. It makes possible an unambigous counting of the pion and heavy-meson exchange graphs. The two-nucleon axial MEC operator is defined as a transition amplitude (a set of all possible graphs) constructed in the tree-approximation from the hard pion PL. Due to the basic properties of the PL, such an operator automatically possesses the correct chiral $SU_2 \times SU_2$ -transformation properties.

The standard ideology accepted in the elementary particle physics is that the hard pion method works up to the energy scale $\approx$ 1 GeV ($\approx m_\rho, m_A$) and in this range the corresponding PL's provide a reasonable approximation at the tree-level to the hadron amplitudes. This is the reason why in constructing the MEC operator we also restrict ourselves to the tree-approximation. In other words, it makes no sense to calculate loops in the approach we deal with.

We note that the one-loop approximation constructed from the soft pion PL is nevertheless used in the calculations dealing with the elementary particle processes[20]. Like the hard pion method it also allows one to pass towards the higher energies in comparison with the soft pion tree-approximation which works at the threshold.

A direct important consequence of our consistent approach for reaction (5) is[21] that the contribution to the doublet transition rate from the usually accepted rho-meson weak current (see fig. 1b in[13]) turns out to be nearly cancelled by the analogous graph containing the A_1-meson pole in addition. This shows the real importance of taking into account the A_1 -meson contributions.

The paper is organized as follows. In Sec. 2 we present the hard pion PL model used here. For the $A_1 \rho\pi$ -sector, it coincides with the minimal model by Ogievetsky and Zupnik[16]. Having in mind the results of the previous investigations[1-8,13] we include together with the A_1, ρ, π, N also the N^*-isobar (which is not required by the model itself, however). In Sec. 3, to illustrate the power of the hard pion PL technique, we consider the non-Born part of the weak pion production amplitude $N + J^A \to N + \pi$

and show that its soft pion limit exactly coincides with the PCAC prediction. We explain why analogous calculation in the Chemtob-Rho method[3] leads to the deviations from the PCAC results.

2. FORMALISM

There exist several different PL formulations of the hard pion method[15,16]. We prefer the PL model by Ogievetsky and Zupnik[16] because it provides the smoothest momentum dependence of the proper vertices. The corresponding PL contains no more than two derivatives in each term. In this sense, it is the minimal hard pion PL.

2.1. The Lagrangian for the $A_1 \rho\pi$ System

The effective PL for the $A_1\rho\pi$ system in the Ogievetsky-Zupnik model is completely determined by four phenomenological parameters m_ρ , m_A and the rho- and A_1 -meson coupling constants g_ρ and g_{A_1}, respectively. The pion decay constant f_π is related to them by the first Weinberg sum rule[22]

$$f_\pi^2 = \left(\frac{m_\rho}{g_\rho}\right)^2 \left[1 - \left(\frac{m_\rho g_{A_1}}{m_A g_\rho}\right)^2\right], \tag{6}$$

which is, in fact, the consistency condition between the current algebra and the vector dominance. To simplify the subsequent formulae, it is convenient to use the phenomenological KSFR relation

$$2 f_\pi^2 g_\rho^2 = m_\rho^2 , \tag{7}$$

which holds within 15%. We exploit in the calculations also the second Weinberg sum rule[22]

$$g_{A_1}^2 = g_\rho^2 , \tag{8}$$

which together with eqs. (6), (7) implies the remarkable Weinberg relation[22]

$$m_A^2 = 2m_\rho^2 \ . \qquad (9)$$

Eq. (9) seems now to be confirmed experimentally [18].

Inserting eqs. (6) and (7) into the general form of the Ogievetsky-Zupnik PL [16], we extract from it the elementary vertices relevant to the contsruction of the MEC operators

$$\mathcal{L}_{\rho\pi\pi} = g_\rho \vec{\rho}_\mu \cdot \vec{\pi} \times \partial_\mu \vec{\pi},$$

$$\mathcal{L}_{A_1 A_1 \rho} = g_\rho (\vec{\rho}_\mu \times \vec{a}_\nu - \vec{\rho}_\nu \times \vec{a}_\mu) \cdot \partial_\mu \vec{a}_\nu \ , \qquad (10)$$

$$\mathcal{L}_{A_1\rho\pi} = -\frac{g_{A_1}}{g_\rho f_\pi} \vec{\rho}_{\mu\nu} \cdot \vec{a}_\mu \times \partial_\nu \vec{\pi} - \frac{1}{4}\frac{g_{A_1}}{g_\rho f_\pi}\left(2 - \frac{g_\rho^2}{g_{A_1}^2}\right) \vec{\rho}_{\mu\nu} \cdot \vec{\pi} \times \vec{a}_{\mu\nu},$$

where $\vec{\rho}_\mu$, $\vec{a}_\mu$ are the rho- and A_1-meson fields, respectively, and

$$\vec{\rho}_{\mu\nu} = \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu \ , \qquad \vec{a}_{\mu\nu} = \partial_\mu \vec{a}_\nu - \partial_\nu \vec{a}_\mu \ .$$

2.2. Nucleon Part of the Lagrangian

Although the nucleon sector has not been considered in ref. [16] the relevant PL can be simply constructed by using the general prescriptions of refs. [23]. The PL which is invariant under the gauge isospin and non-linear gauge chiral transformations is given by

$$\mathcal{L}_N^{str} = -\bar{N}\gamma_\mu D_\mu N - M\bar{N}N - i\frac{g_A}{f_\pi}\bar{N}\gamma_\mu\gamma_5\vec{\tau}N \cdot \nabla_\mu\vec{\pi} + \frac{1}{4} g_\rho \frac{\kappa_V}{2M} \bar{N}\sigma_{\mu\nu}\vec{\tau}N \cdot \vec{\tilde{\rho}}_{\mu\nu} \ , \qquad (11)$$

where the covariant derivatives $D_\mu N$, $\nabla_\mu \vec{\pi}$ and the covariant curl $\vec{\tilde{\rho}}_{\mu\nu}$ * are determined following the procedure of refs. [23]. In eq. (11) $g_A = 1.25$ (we have used here the Goldberger-Treiman relation) and $\kappa_V = 3.7$. The definition of the metrics and Dirac matrices is the same as in ref. [1].

After removing the unphysical $A_1\pi$-mixing, the piece of the Lagrangian (11) which we need, reads as follows:

$$\mathcal{L}_{NA_1\rho\pi} = -\bar{N}\gamma_\mu\partial_\mu N - M\bar{N}N - i\frac{g_A}{2f_\pi}\bar{N}\gamma_\mu\gamma_5\vec{\tau}N(\partial_\mu\vec{\pi} - g_\rho\vec{\rho}_\mu \times \vec{\pi}) - i\frac{g_\rho}{2}\bar{N}\gamma_\mu\vec{\tau}N\cdot\vec{\rho}_\mu - i\frac{g_{A_1}}{2f_\pi}\bar{N}\gamma_\mu\vec{\tau}N\cdot\vec{\pi}\times\vec{a}_\mu - \qquad (12)$$
$$- i g_A g_{A_1}\bar{N}\gamma_\mu\gamma_5\vec{\tau}N\cdot\vec{a}_\mu + \frac{1}{4}g_\rho\frac{\kappa_V}{2M}\bar{N}\sigma_{\mu\nu}\vec{\tau}N\cdot\vec{\tilde{\rho}}'_{\mu\nu} \ ,$$

where

$$\vec{\tilde{\rho}}'_{\mu\nu} = \vec{\rho}_{\mu\nu} + \frac{g_{A_1}}{f_\pi g_\rho}(\vec{a}_\mu \times \partial_\nu\vec{\pi} - \vec{a}_\nu \times \partial_\mu \vec{\pi}) + \frac{g_{A_1}}{f_\pi g_\rho}\vec{\pi}\times\vec{a}_{\mu\nu} \ .$$

Likewise, the part of the PL containing the N*-resonance is

$$\mathcal{L}_{N^*NA_1\rho\pi} = 2\frac{f_{\pi NN^*}}{m_\pi}\bar{N}^*_\nu \vec{T}N \cdot \nabla_\nu\vec{\pi} - g_\rho \frac{G_1}{M}\bar{N}^*_\mu \gamma_5\gamma_\nu \vec{T}N\cdot\vec{\tilde{\rho}}_{\mu\nu} +$$

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* In contrast with the linearly transforming curls $\vec{R}_{\mu\nu}$, $\vec{S}_{\mu\nu}$ from ref. [16] $\vec{\tilde{\rho}}_{\mu\nu}$ transforms like the covariant derivative $\nabla_\mu\vec{\pi}$, i.e., nonlinearly. It is constructed out of the pion field and the curls $\vec{R}_{\mu\nu}$, $\vec{S}_{\mu\nu}$ by general rules of Sec. 4 of the first of refs. [23].

$$+g_\rho \frac{G_2}{2M^2}\bar{N}^*_\mu \gamma_5 (\vec{\partial}_\nu - \overleftarrow{\partial}_\nu)\vec{T}N\cdot\vec{\rho}_{\mu\nu} + \text{h.c.} \qquad (13)$$

Here $f_{\pi NN^*}$ is the πNN^*-coupling constant, m_π is the pion mass, and $\vec{T}$ is an operator which transforms the nucleon isospin wave function into the isobar one [24]. Further, according to ref. [25]

$$G_1 \approx 1.3\,(G_1)_{SU_6}\,,\quad G_2 = -\frac{M}{M^*}(G_1)_{SU_6}\,,\quad (G_1)_{SU_6} \approx 2,$$

where M* is the N*-isobar mass. For our purpose, only the linear terms in the covariant derivative $\nabla_\mu \vec{\pi}$ and the curl $\vec{\rho}_{\mu\nu}$ are of interest

$$\nabla_\mu \vec{\pi} = \frac{1}{2}\partial_\mu \vec{\pi} + f_\pi g_{A_1}\vec{a}_\mu + O(\pi^2),\quad \vec{\rho}_{\mu\nu} = \rho'_{\mu\nu} + O(\pi). \qquad (14)$$

2.3. Weak Currents

The vector and the axial-vector currents are [16]

$$\vec{J}^V_\mu = \frac{m_\rho^2}{g_\rho}\rho_\mu\,,$$

$$\vec{J}^A_\mu = g_{A_1}\frac{m_\rho^2}{g_\rho^2}\vec{a}_\mu - f_\pi \partial_\mu \vec{\pi} + f_\pi g_\rho \vec{\rho}_\mu \times \vec{\pi} + O(\pi^2), \qquad (15)$$

where we have used the relation (6) and have explicitly written down in $\vec{J}^A_\mu$ only terms up to the second order in the fields. The vector current obeys the exact field-current identity and is dominated by the rho-meson. At the same time, the axial-vector current consists of two parts. The 1^+-part is dominated by the field $\vec{a}_\mu$, the 0^--piece is mediated by the pion pole in accordance with the general prescription of the current algebra. Note also that by the equivalence redefinition of the $\vec{a}_\mu$-field, $\vec{a}_\mu \to \vec{a}'_\mu = \vec{a}_\mu + (g_\rho f_\pi / g_{A_1} m_\rho^2)\vec{\rho}_\mu \times \vec{\pi} + O(\pi^2)$ we might reduce $\vec{J}^A_\mu$ to the form

$$\vec{J}^A_\mu = g_{A_1}\frac{m_\rho^2}{g_\rho^2}\vec{a}'_\mu - f_\pi \partial_\mu \vec{\pi}\,,$$

in which the dominance of the 1^+-term by the axial-vector field becomes evident (of course, such a redefinition would entail an appropriate rearrangement of the elementary vertices in our PL).

The constant-parameter chiral invariance of the present PL is also broken, as usual, by adding the term $m_\pi^2 f_\pi^2 [1-(\vec{\pi}/f_\pi)^2]^{1/2}$ which supplies the pion field with the mass. This breakdown ensures that the axial current satisfies the PCAC

$$\partial_\mu \vec{J}^A_\mu = m_\pi^2 f_\pi \vec{\pi}.$$

Of course, the vector current remains conserved.

2.4. Two-Body Axial MEC Operator

We now present in the given model, the two-body axial MEC operator (fig. 1). It is the set of all admissible graphs in the tree-approximation.

The sign of some of the coupling constants in eqs. (10)-(13) is unknown. No difficulty arises, if a graph is composed of the vertices eqs. (10), (12) (only the squares of the g_{A_1}, g_ρ, f_π appear). It is not so in the case of the graphs 1a and 1b with $\mathfrak{N} = N^*$ and $B = \rho$, Fortunately, the contribution from these graphs is negligible [21].

As the PCAC is satisfied, the divergence of the set of graphs in fig. 1 should be zero in the limit of zero pion mass. We have verified that the sum of graphs 1a-1b (for each intermediate baryon and for each boson exchanged), 1c-1f and 1g-1j are divergenceless separately.

The graphs 1a+1b with $\mathfrak{N} = N, B = \pi$ (Born terms), 1a with $\mathfrak{N} = N^*, B = \pi$ (N* -excitation current) and 1d ($\rho - \pi$ weak decay current) are of the same form as the

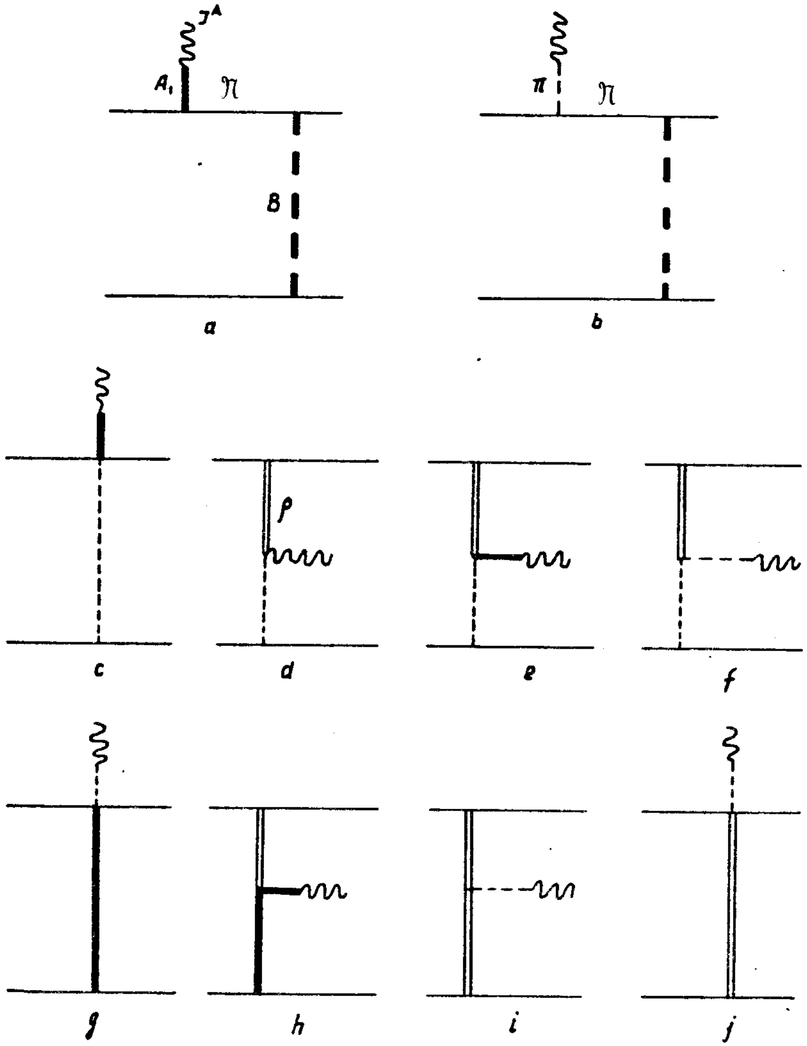

Fig. 1. The two-nucleon axial MEC operator in the tree-approximation. The wavy line represents the axial-vector current. In the graphs a and b, $\mathfrak{N}$ stands either for the nucleon or N^*-isobar, $B \equiv \pi, A_1, \rho$. Otherwise heavy double and light dashed lines are for the A_1-meson, rho-meson and pion, respectively.

graphs usually calculated in the standard approach with the pseudoscalar (p.s.) πN-coupling. Because we are working with the pseudovector (p.v.) πN-coupling, our Born terms differ from the standard ones. The problem of eliminating the piece, which is already included in the nuclear wave function, will be discussed in the next paper where also the numerical results for reaction (5) will be published.

3. WEAK PION PRODUCTION IN THE HARD MESON METHOD

In the present section we consider the non-Born part of the amplitude for the pion production from the nucleon by the axial-vector current

$$N(p_1) + J^{Aj}_{\lambda}(k) \to N(p_2) + \pi^n(q) \qquad (16)$$

in the hard meson method. In eq. (16), $J^{Aj}_{\lambda}(k)$ is the axial-vector current with the isospin index j and $\pi^n(q)$ stands for the pion in the isospin state n. This amplitude enters en bloc into the axial MEC operator (graphs a-f in fig. 1) and, hence, has a direct relevance to the main subject of our study. We show here that the consistent application of the hard pion PL method enables one to avoid discrepancies with the pure PCAC approach, which are present in Alder's [26] and Chemtob-Rho's [3] analysis of this amplitude. We restrict ourselves to the two important limits, as $k \to 0$ or $q \to 0$.

In the limit $k = 0$, we parametrize the non-Born term $\bar{M}^A_\lambda$ of the amplitude M^A_λ of the process (16) as follows:

$$\bar{M}^A_\lambda(q^2, k=0) = i\bar{u}(p_2)\{a^{(+)}_{nj}\gamma(q^2)q_\lambda + a^{(-)}_{nj}[i\beta'(q^2)\times \qquad (17)$$

$$\times \sigma_{\lambda\nu} q_\nu - 2i\alpha'(q^2) M\gamma_\lambda]\}u(p_1),$$

where $a^{(\pm)} = \frac{1}{4}[\tau_n, \tau_j]_\pm$ are the isospin projection operators, and $\alpha'(q^2), \beta'(q^2), \gamma(q^2)$ are the form factors which survive in the limit $k = 0$ [$\alpha'(q^2) \equiv \alpha'(q^2, k=0)$etc.]. We choose the covariants $\sigma_{\lambda\nu} q_\nu$ and γ_λ instead of $(p_1 + p_2)_\lambda$ and γ_λ because sandwiched between the nucleon spinors they remain independent also in the limit $q = 0$. Therefore, the form factors α', β', γ remain also independent in this limit, and the analysis turns out to be more simple than in refs. [3,26]. Our form factors α', β' are related to α, β from [3] as follows:

$$\beta'(q^2) = \beta(q^2), \quad \alpha'(q^2) = \alpha(q^2) + \beta(q^2). \tag{18}$$

Due to the PCAC, the form factors α', β', γ are related to the non-Born part of the πN-scattering amplitude taken at zero momentum of one of the pions involved (see, e.g., eq. (2.15) in ref. [3]). These relations hold also in our model.

More detailed information is provided by the current algebra and PCAC for the form factor $\alpha'(q^2)$ when $q = 0$ [27]

$$\alpha'(0) = \frac{1}{2Mf_\pi}(1 - g_A^2), \tag{19}$$

where $g_A \equiv g_A(0)$ is the axial β-decay constant.

It was observed in refs. [3,26], that the amplitude (17) evaluated within the pure rho-exchange approximation shows some deviation from eq. (19). We show that the discrepancy disappears if the A_1-meson contribution is correctly taken into account.

The graphs which contribute to the amplitude $\bar{M}_\lambda^A(q^2, k=0)$ are given in fig. 2. Before evaluating them, let us note that the decomposition of the pion production amplitude into the Born and non-Born parts depends on the form of the πN-coupling (i.e., the non-Born terms are different for the p.s. and p.v. πN-couplings). In order to make the comparison with the results of refs. [3,26] unambiguous, we

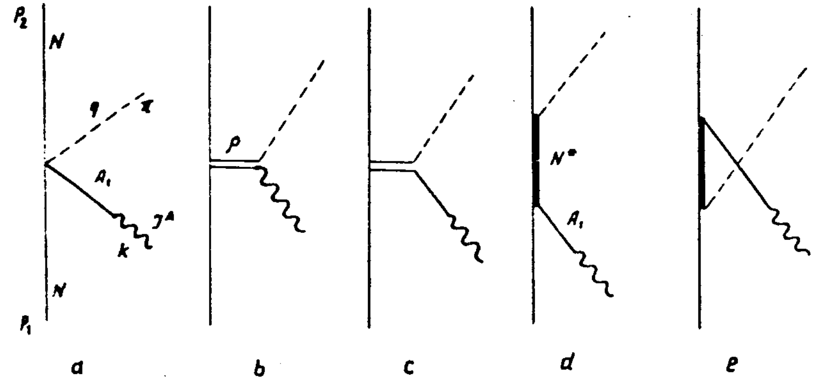

Fig. 2. The contribution from different processes admitted by the PL eq. (12) to the non-Born part $\bar{M}_\lambda^A$ of the pion weak production amplitude.

pass in our PL (12) to the p.s. πN-coupling, which was used there. It can be made by the standard equivalence redefinition of the nucleon field

$$N \to N' = e^{i\frac{g_A}{2f_\pi}\gamma_5(\vec{\tau}\cdot\vec{\pi})} N. \tag{20}$$

This replacement leads also to the renormalization of the coupling constants in a number of the contact vertices and, generally, produces some new vertices. This is the reason why the non-Born terms are different for the p.s. and p.v. πN-couplings. The effect of the canonical transformation (20) on the amplitude (17) is

i) the renormalization of the minimal $A_1 \pi N$ coupling constant in the graph 2a

$$g_{\pi N A}^{min} = -\frac{1}{2f_\pi} g_{A_1} \to -\frac{1}{2f_\pi} g_{A_1}(1 - 2g_A^2), \tag{21}$$

ii) the appearance of a new graph (fig.3) due to the non-derivative πN-coupling $M(g_A^2/2f_\pi^2)\bar{N}N\vec{\pi}^2$ which comes from the nucleon mass term (the derivative

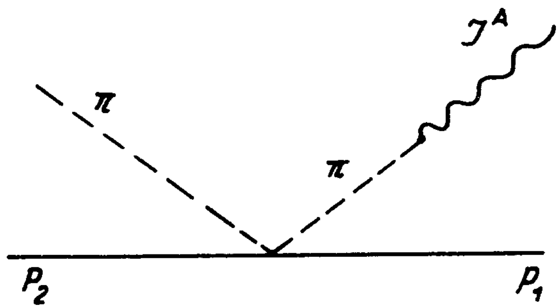

Fig. 3. The graph which appears after passing in the PL eq. (12) to the pseudoscalar πN -coupling.

coupling $\sim \bar{N}\gamma_\mu \vec{\tau} N \cdot \vec{\pi} \times \partial_\mu \vec{\pi}$ which arises from the nucleon kinetic energy term makes no contribution in either limit we deal with).

Using the PL (12) with these modifications taken into account, we evaluate the contributions of the graphs in figs. 2 and 3 to the form factors α', β', γ. They are listed in table 1.

Summing up the contributions and neglecting for simplicity the terms $\sim (q/M^*)^2$, we have

$$\gamma(q^2) = \frac{16}{9}\,\frac{f^2_{\pi NN^*} f_\pi}{M^2}\,\frac{1}{M^*-M} + 2M\,\frac{g^2_A}{f_\pi}\,\frac{1}{q^2+m^2_\pi},$$

$$\beta'(q^2) = -\frac{\kappa_V}{2Mf_\pi}\,\frac{m^2_\rho}{m^2_\rho+q^2} - \frac{4}{9}\,\frac{f^2_{\pi NN^*} f_\pi}{M^2}\,\frac{1}{M^*-M}, \qquad (22)$$

$$\alpha'(q^2) = \frac{1}{2Mf_\pi}\left[1-g^2_A - \frac{q^2}{q^2+m^2_\rho}\right].$$

It is seen from eqs. (22) that the low energy theorem (19) is exactly satisfied in our model. The origin of the controversy with the current algebra predictions in [3,26] lies in the overpassing the

Table 1

The contributions of the graphs in figs. 2 and 3 to the form factors α', β' and γ in the limit $k \to 0$.

graphs \ form factors	$\gamma(q^2)$	$\beta'(q^2)$	$\alpha'(q^2)$
2a	–	$-\frac{1}{4}\,\frac{\kappa_V}{Mf_\pi}$	$\frac{1}{2Mf_\pi}\left(\frac{1}{2}-g^2_A\right)$
2b	–	$-\frac{1}{4}\,\frac{\kappa_V}{Mf_\pi}\,\frac{m^2_\rho}{m^2_\rho+q^2}$	$\frac{1}{4Mf_\pi}\,\frac{m^2_\rho}{m^2_\rho+q^2}$
2c	–	$\frac{1}{4}\,\frac{\kappa_V}{Mf_\pi}\,\frac{q^2}{m^2_\rho+q^2}$	$-\frac{1}{4Mf_\pi}\,\frac{q^2}{m^2_\rho+q^2}$
2d+2e	$\frac{16}{9}\,\frac{f^2_{\pi NN^*} f_\pi}{M^2}\,\frac{1}{M^*-M}\left(1+\frac{q^2}{4M^{*2}}\right)$	$-\frac{4}{9}\,\frac{f^2_{\pi NN^*} f_\pi}{M^2}\,\frac{1}{M^*-M}\left(1-\frac{q^2}{2M^{*2}}\right)$	$\frac{1}{9}\,\frac{f^2_{\pi NN^*} f_\pi}{M^2}\,\frac{(2M+M^*)}{M^*-M}\,\frac{q^2}{MM^*}$
3	$2M\frac{g^2_A}{f_\pi}\,\frac{1}{q^2+m^2_\pi}$	–	–

graphs with the A_1-meson pole. Just these graphs ensure the compatibility of the current algebra and the rho-dominance for the given process. Note that the contact $A_1 \pi N$-coupling (graph 2a) is prescribed by the gauge chiral invariance principle and is present in any PL realization of the hard pion method*.

The second reason why the calculations of ref. /3/ and partly of ref. /26/ deviate from the current algebra predictions is the incorrect treatment of the off-the-mass-shell behaviour of the $\rho\pi J^A$-vertex in the rho-meson weak decay current. It is usually assumed that this vertex is well approximated by its soft pion value

$$\langle \pi^n(0) | J_\lambda^{Aj}(k) | \rho_\mu^\ell \rangle = g_{\rho\pi J^A} \left(\delta_{\lambda\mu} + \frac{k_\lambda k_\mu}{m_\rho^2}\right)\epsilon^{nj\ell}, \qquad (23)$$

where

$$g_{\rho\pi J^A} = \frac{g_{\pi NN}(0)}{M g_A} \frac{m_\rho^2}{g_\rho} \quad /3/ .$$

The Goldberger-Treiman relation $f_\pi g_{\pi NN} = M g_A$ together with the KSFR relation (7) yield

$$g_{\rho\pi J^A} = 2 f_\pi g_\rho . \qquad (24)$$

Then this value of $g_{\rho\pi J^A}$ is substituted /3/ into the weak decay $\rho-\pi$ graph at $k \sim 0$ (small momentum transfer).

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* We might remove the $A_1 \pi N$-coupling by the equivalence transformation of the $\vec{\rho}_\mu'$-field, $\vec{\rho}_\mu \to \vec{\rho}_\mu' = \vec{\rho}_\mu + (g_{A_1}/g_\rho f_\pi)(1-2g_A^2)\vec{\pi}\times\vec{a}_\mu$. As a result, we would be left only with the ρ'-exchange graphs. In contrast with $\vec{\rho}_\mu$, the $\vec{\rho}_\mu'$ is not transversal off-the-mass-shell and the terms $q_\mu q_\nu / m_\rho^2$ in the ρ'-propagator would contribute instead of the contact graph.

However, such an approximation is justified only if the off-the-mass-shell form factor $g_{\rho\pi J^A}(q=0, k^2)$ is assumed to be nearly constant in a rather wide range $0 < -k^2 < m_\rho^2$. Within the hard pion method, it is possible to estimate explicitly the correction terms which arise when extrapolation from $k^2 = -m_\rho^2$ to $k = 0$ is made. The off-the-mass-shell $\rho\pi J^A$-vertex is now composed of the contact and A_1-pole parts which enter into the graphs 2b and 2c, respectively,

$$\langle \pi^n(q) | J_\lambda^{Aj}(k) | \rho_\mu^\ell \rangle = \epsilon^{nj\ell} \left[\left(\delta_{\lambda\mu} + \frac{(q+k)_\lambda (q+k)_\mu}{m_\rho^2}\right) \times \right. \qquad (25)$$

$$\left. \times f_\pi g_\rho \frac{m_A^2 - 2q^2}{m_A^2 + k^2} + O(qk) \right].$$

When the rho-meson is on-the-mass-shell, the soft pion limit $(q=0, k^2 = -m_\rho^2)$ of eq. (25) coincides with eq. (24). We obtain the same value of $g_{\rho\pi J^A}$ also if $k=0$ and $q^2 = -m_\rho^2$. However, in the combined limit $q=0$ and $k=0$ (the rho-meson is off-the-mass-shell), the vertex (25) has the value

$$g_{\rho\pi J^A}(q=0, k=0) = f_\pi g_\rho , \qquad (26)$$

i.e., it is twice smaller in comparison with (24). This shows the importance of the off-the-mass-shell corrections. In our scheme, just the value of $g_{\rho\pi J^A} = f_\pi g_\rho$ enters into the graph 2b. Consequently, the contribution from this graph is twice smaller as compared to the standard calculations /3/. Note that the hard pion momentum dependence of the $\rho\pi J^A$-vertex has been taken into account by Adler /26/ who, however, did not consider the contact graph 2a, and therefore, did not obtain the complete agreement with the PCAC results.

The momentum dependence of the $\rho\pi J^A$-vertex turns out to be important in the practical calculations. Indeed, we have shown[21] for reaction (5) that the contribution to the doublet transition rate from the graph 1d is nearly cancelled by that due to the A_1-pole graph 1e. This type of cancellation may also bring the calculations of ref.[7] for reaction (1) into better agreement with experiment.

In the limit $q=0, k\neq 0$, we parametrize the amplitude $\bar{M}^A_\lambda$ by analogy with eq. (17)

$$\bar{M}^A_\lambda(q=0,k^2)=i\bar{u}(p_2)\{a^{(+)}_{nj}\rho(k^2)k_\lambda+a^{(-)}_{nj}[i\lambda(k^2)\sigma_{\lambda\nu}k_\nu- \tag{27}$$

$$-2i\alpha'(k^2)M\gamma_\lambda]\}u(p_1),$$

where now $\alpha'(k^2)\equiv\alpha'(q=0,k^2)$, etc. It follows from eqs. (17) and (27) that α' is the only form factor which survives in the combined limit $q=0$, $k=0$.

Generally, the form factors ρ, λ and α' are restricted by the current algebra and PCAC as follows[26]

$$\rho(k^2)=-\frac{1}{f_\pi}g_A g_P(k^2),$$

$$\lambda(k^2)=\frac{1}{f_\pi}F_2^V(k^2), \tag{28}$$

$$\alpha'(k^2)=\frac{1}{2f_\pi M}[F_1^V(k^2)-g_A g_A(k^2)],$$

where g_P, g_A are the standard weak induced pseudoscalar and axial-vector form factors, $F_1^V(k^2)=1+O(k^2)$ and $F_2^V(k^2)=\frac{\kappa_V}{2M}+O(k^2)$ are the vector current form factors.

In the framework of our definition of the non-Born part $\bar{M}^A_\lambda$ of the amplitude M^A_λ (figs. 2 and 3), $\rho(k^2)$ is given by the graph in fig. 3 and contains only the pion-pole piece. Notice, however, that our $g_P(k^2)$ consists of the pion-pole as well as A_1-meson pole parts (the last one coming from the term $\sim k_\nu k_\lambda$ in the A_1-meson propagator which mediates the weak axial-vector current in the NNJ^A vertex). The seeming inconsistency with the first of eqs. (28) is removed by putting our Born terms into the form used in refs.[3,26] (they remained different even after passing to the p.s. πN-coupling). Then we obtain for $\rho(k^2)$ the missing A_1-meson pole part as demanded by the first of eqs. (28).

The remaining part of the amplitude (27) is given (see table 2) by the graphs 2a-2c (N^*-isobar makes no contribution in the limit $q\to 0$). Summing

Table 2

The contributions of the graphs 2a-2c to the form factors λ and α' in the limit $q\to 0$.

form factors / graphs	$\lambda(k^2)$	$\alpha'(k^2)$
2a	$\frac{\kappa_V}{2Mf_\pi}\frac{m_\rho^2}{m_A^2+k^2}$	$\frac{1}{2Mf_\pi}\frac{m_\rho^2}{m_A^2+k^2}(1-2g_A^2)$
2b	$\frac{\kappa_V}{4Mf_\pi}\frac{m_\rho^2}{m_\rho^2+k^2}$	$\frac{1}{4Mf_\pi}\frac{m_\rho^2}{m_\rho^2+k^2}$
2c	$-\frac{\kappa_V}{4Mf_\pi}\frac{m_\rho^2 k^2}{(m_\rho^2+k^2)(m_A^2+k^2)}$	$-\frac{1}{4Mf_\pi}\frac{m_\rho^2 k^2}{(m_\rho^2+k^2)(m_A^2+k^2)}$

up the contributions and using the first Weinberg sum rule (6), we finally have

$$\lambda(k^2) = \frac{\kappa_V}{2Mf_\pi} \frac{m_\rho^2}{m_\rho^2 + k^2},$$

$$\alpha'(k^2) = \frac{1}{2Mf_\pi}\left(\frac{m_\rho^2}{m_\rho^2 + k^2} - g_A^2 \frac{m_A^2}{m_A^2 + k^2}\right). \qquad (29)$$

Keeping in mind that in the present model

$$F_1^V(k^2) = m_\rho^2/(m_\rho^2 + k^2), \quad F_2^V(k^2) = (\kappa_V/2M) m_\rho^2/(m_\rho^2 + k^2)$$

(rho-dominance), and besides $g_A(k^2) = g_A m_A^2/(m_A^2 + k^2)$, we observe complete agreement between our calculations and the general PCAC predictions (26). Again, the crucial role in attaining this agreement is played by the contact graph 2a. Note, that in the limit $k \to 0$, we again recover the Adler-Weisberger theorem (19).

Let us emphasize that the inclusion of other particles into the present scheme (such as the nucleon isobars with $J = 1/2$) cannot affect eqs. (29) as far as it is made in a gauge chiral invariant way. Note also, that the minimal model by Ogievetsky and Zupnik corresponds to the choice $\delta = -1$ for the anomalous magnetic momentum δ of the A_1-meson in the general hard pion vertices /14/. Any other choice leads to a stronger momentum dependence of the form factors. However, in the limit $q \to 0$, the weak pion production amplitude does not depend on the δ, and the relations (29) hold in any realization of the hard pion method.

Throughout this section we used the KSFR relation (7) alongside with the first Weinberg sum rule (6). However, the correct PCAC behaviour of the amplitude is governed by the first Weinberg sum rule alone. In particular, the derivation of eq. (29) is completely independent of employing the KSFR relation.

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