ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

E4 - 11465

N.I.Pyatov, D.I.Salamov, M.I.Baznat,
A.A.Kuliev, S.I.Gabrakov

SELFCONSISTENT THEORY OF COULOMB MIXING IN NUCLEI

1978

E4 - 11465

N.I.Pyatov, D.I.Salamov,[1] M.I.Baznat,[2]
A.A.Kuliev,[1] S.I.Gabrakov[3]

SELFCONSISTENT THEORY OF COULOMB MIXING IN NUCLEI

Submitted to ЯФ

[1] *Institute of Physics, Akad.Sci.Azerbaijan, Baku.*
[2] *Institute of Applied Physics, Akad.Sci.Moldavia, Kishinev.*
[3] *Faculty of Physics, University of Sofia, Bulgaria.*

Пятов Н.И. и др.

E4 - 11465

Самосогласованная теория кулоновского смешивания по изоспину в ядрах

По заданной форме оболочечного потенциала восстанавливаются в сепарабельной форме остаточные изовекторные силы с помощью принципа изотопической инвариантности ядерных сил. Форма и силовые параметры взаимодействий определяются самосогласованием с потенциалом. Кулоновские силы аппроксимируются эффективным одночастичным потенциалом. Развита теория изобарических состояний в методе случайной фазы. Исследованы эффекты кулоновского смешивания по изоспину в основных и возбужденных 0^+ состояний ядер.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1978

Pyatov N.I. et al.

E4 - 11465

Selfconsistent Theory of Coulomb Mixing in Nuclei

For a given form of the isovector potential the separable residual interactions are constructed by means of the isotopic invariance principle. The strength parameter of the force is found from a selfconsistency condition. The charge dependent force is represented by the Coulomb effective potential. The theory of the isobaric states is derived in the RPA. The Coulomb mixing effects in the ground and isobaric 0+ states of even-mass nuclei are investigated.

The research has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1978

Introduction

At present it is well established that the nuclear force is charge independent with a great accuracy, and that the isobaric symmetry is violated by the electromagnetic force. The isospin mixing of nuclear states is mostly due to the Coulomb effective potential, to be exact, due to the vector part of it which varies in the nuclear volume. For that reason the isotopic spin is a good quantum number in the low-energy states of nearly all nuclei[1].

Nevertheless, the correct description of isospin mixing is important for the isospin forbidden processes[2-4]. It is important to take into account the isospin mixing when evaluating the effective vector coupling constant from the superallowed Fermi transitions[2,3,9,12]. The isospin mixing leads to corrections to the isobaric mass formula[2-5]. The spreading widths of analog states are closely connected with the isospin impurities[4,5].

The isospin impurities in the low-lying nuclear states were calculated in different models. The two-fluid hydrodynamic

model[6] has been introduced to estimate the energy of the isovector monopole state (IMS) with the isospin T_o+1, where T_o is the isospin of the ground state. The admixture of IMS in the ground state which arises due to the Coulomb potential occured to be small (≈ 0.1-0.3%) for stable nuclei with $A > 40$.

The shell model estimates[7-9] are by an order of magnitude larger than those obtained from a hydrodynamic model. The difference of two models may be due to a number of reasons. The shell model calculations were performed in a particle-hole approximation with a restricted number of configurations. The residual interactions were either neglected or taken into account in the Tamm-Dancoff approximation, i.e., the ground state correlations were ignored. The residual interactions used were not consistent with the shell model potential, that is of importance when the latter contains the isovector term. The isovector potential does not commute with $T_\pm$, what leads to an additional isospin mixing. The lack of consistency is always in calculations when one uses the "experimental" one-particle energies or the isotopic invariant residual interactions and the particle-hole basis with broken isotopic symmetry, or the static pairing correlations, etc.

In order to remove those difficulties, it is important to use the isotopic invariant nuclear Hamiltonian (without the electromagnetic force) and to treat the Coulomb mixing effects selfconsistently.

The properties of the isobaric analog states were investigated in the theory of finite Fermi systems[10-13]. The self-consistent equations for those states were derived in ref.[13], but no numerical calculations were performed.

In the present paper we use a more simple method[14] to derive the residual interactions which are consistent with a given form of the shell model potential. A more complete account of the method is given in refs.[15,16], and its application to the case of the broken isotopic symmetry was made in ref.[17].

In the subsequent sections the model Hamiltonian is introduced, and the random phase approximation (RPA) treatment of the isobaric states is given. In so doing we use the effective one-body Coulomb potential instead of the actual two-body force. Then the theory is utilized to calculate the isospin impurities in the ground and isobaric analog states and the matrix elements for the superallowed and isospin forbidden Fermi transitions.

Hamiltonian.

We suppose that the shell model potential is given by

$$U(r) = -U_0 f_0(r) + U_1 f_1(r) t_z + V_c(r), \qquad (1)$$

where $f_0(r)$ and $f_1(r)$ are the radial functions of the isoscalar and isovector potentials, respectively, U_0 and U_1 are the well depth parameters, and $V_c(r)$ is the Coulomb potential

$$V_c(r) = v_c(r)\left(\frac{1}{2} - t_z\right). \qquad (2)$$

We use the following definition for the components of the isospin operator

$$t_\mu = \begin{cases} t_z, & \mu = 0 \\ -\frac{\mu}{\sqrt{2}}\left(t_x + i\mu t_y\right), & \mu = \pm 1, \end{cases} \qquad (3)$$

the eigenvalues of t_z being +1/2 for neutron and -1/2 for proton.

The isotopic symmetry of (1) is broken by both the isovector and Coulomb terms. If we assume that the original nuclear two-body force, which generates the isovector potential, is charge independent, then the broken symmetry should be restored by the residual force h

$$[U(r) - V_c(r) + h, \; t_\mu] = 0 . \qquad (4)$$

In the general case the force h cannot be found uniquely from eq.(4). One needs some assumptions concerning the structure of h . In refs./14-16/ it was suggested to seek the residual force in the separable form by expanding it over the bilinear combinations of commutators of the isovector potential and t_μ. The algebra of these commutators is closed therefore the expansion of h takes a simple form

$$h = \frac{U_1}{2\chi} \sum_{\mu=-1}^{+1} (F_\mu - \chi \delta_{\mu,0})^+ (F_\mu - \chi \delta_{\mu,0}) , \qquad (5)$$

where the operators F_μ are defined by

$$F_\mu = \sum_{i=1}^{A} f_1(r_i) \, t_\mu^{(i)} . \qquad (6)$$

By subatituting (5) into eq.(4) , it is easily verified that the latter is fulfilled with any arbitrary $\chi \neq 0$. The mean value of this parameter is found from the consistency condition

$$\chi = \langle \tilde{0} | F_0 | \tilde{0} \rangle = \frac{1}{2} \left(\langle \tilde{0} | \sum_{i=1}^{N} f_1(r_i) | \tilde{0} \rangle - \langle \tilde{0} | \sum_{i=1}^{Z} f_1(r_i) | \tilde{0} \rangle \right) . \qquad (7)$$

The average here is taken over the noncorrelated Hartree-Fock ground state. It is seen that the quantity χ depends on the neutron-proton difference of the density distribution.

The radial form of the force (5) coincides with that of the isovector potential. In the case of spherical symmetry h contains only the monopole term of the residual isovector force.

The isoscalar and isovector terms of the spin-orbital potential may be included into the functions f_0 and f_1 , respectively, which become then momentum dependent. Alternatively, one can derive the residual isovector spin-orbital force.

It is more convenient to use h in terms of the commutators of the shell model Hamiltonian with the components of the total isospin operator $\vec{T}$. If we neglect the neutron-proton mass difference in the kinetic energy, then the operators F_μ can be written as

$$F_{\pm 1} = \pm \frac{1}{U_1} \left[\sum_{i=1}^{A} (H_i^0 - V_c(r_i)), \; T_\pm \right] , \qquad (8)$$

$$F_0 = \frac{1}{U_1} \left[T_+, \left[\sum_{i=1}^{A} (H_i^0 - V_c(r_i)), \; T_- \right] \right] , \qquad (9)$$

where H^0 is the single-particle Hamiltonian. Next, we define the quantity

$$\gamma = U_1 \langle \tilde{0} | F_0 | \tilde{0} \rangle , \qquad (10)$$

which coincides (except for the sign) with the one-particle symmetry energy. Then eq.(5) takes the form

$$h = \frac{1}{2\gamma} \sum_{\mu=\pm 1} \left[\sum_{i=1}^{A} (H_i^0 - V_c(r_i)), \; T_\mu \right]^+ \times \left[\sum_{i=1}^{A} (H_i^0 - V_c(r_i)), \; T_\mu \right]$$

$$+\frac{1}{2\gamma}\left\{\left[T_+,\left[\sum_{i=1}^{A}\left(H_i^o-V_c(r_i)\right),T_-\right]\right]-\gamma\right\}^+$$

$$\times\left\{\left[T_+,\left[\sum_{i=1}^{A}\left(H_i^o-V_c(r_i)\right),T_-\right]\right]-\gamma\right\}. \qquad (11)$$

This form allows one to take into account the Coulomb mixing effects selfconsistently. The spin-orbital potential is assumed to be included in H^o. We should like to emphasize that in the derivation of eq.(5) we have used the only assumption, concerning the separability of the residual force.

Isobaric states.

As the basis one-particle states we choose the eigenfunctions of the Hamiltonian H^o which includes the potential (1) and the spin-orbital potential. This choice allows one to take into account the isovector and Coulomb potentials selfconsistently.

Let us consider the isobaric 0^+ states of an odd-odd nucleus generated from the correlated ground state of an even-even nucleus (parent nucleus) by means of the charge exchange force. The basis set of the particle-hole operators is

$$A_{j_p j_n}=\frac{1}{\sqrt{2j_p+1}}\sum_m a^+_{j_p m}\,a_{j_n m},$$

$$A_{j_n j_p}=A^+_{j_p j_n},\quad |j_p|=|j_n|. \qquad (12)$$

Here, $a^+_{j_p m}$, $a_{j_n m}$ are the proton creation and the neutron annihilation operators, respectively. The angular momenta j_p and j_n are coupled to form the total particle-hole angular momentum J=0. The total isospin of a pair is equal to 1 and its third component $\tau_z=-1$ for the operators $A_{j_p j_n}$ and $\tau_z=+1$ for the operators $A_{j_n j_p}$. Additional quantum numbers are omitted, but it is supposed that the nucleons may occupy the states in different shells. In what follows we use the simplified notation $A_{pn}\equiv A_{j_p j_n}$ and $A_{np}\equiv A_{j_n j_p}=A^+_{pn}$. The Hartree-Fock ground state average for the commutator of these operators is

$$\langle\tilde{0}|\left[A_{np},A_{p'n'}\right]|\tilde{0}\rangle=\delta_{nn'}\delta_{pp'}\left(n_n-n_{p'}\right), \qquad (13)$$

where n_n and n_p are the neutron and proton occupation numbers, respectively.

From the operators (12) we construct a set of Hermitian operators

$$\mathcal{P}_K=\frac{i}{\sqrt{2}}\sum_{pn}\psi^{(K)}_{pn}\left(A_{pn}-A_{np}\right), \qquad (14)$$

$$\mathcal{Q}_K=\frac{1}{\sqrt{2}}\sum_{pn}\varphi^{(K)}_{pn}\left(A_{pn}+A_{np}\right), \qquad (15)$$

where the particle-hole amplitudes ψ and φ are supposed to be real. From the commutation relation

$$\langle\left[\mathcal{Q}_K,\mathcal{P}_{K'}\right]\rangle=i\,\delta_{KK'} \qquad (16)$$

we get the orthonormality equation

$$\sum_{pn}\varphi^{(K)}_{pn}\,\psi^{(K')}_{pn}\left(n_n-n_p\right)=\delta_{KK'} \qquad (17)$$

and the completeness condition

$$\sum_K \varphi^{(K)}_{p'n'} \psi^{(K)}_{pn} = \delta_{pp'} \delta_{nn'} (n_n - n_p). \qquad (18)$$

For the operators $\mathcal{P}_K$ and $\mathcal{L}_K$, one can write the usual RPA equations of motion

$$[H^0 + h, \mathcal{P}_K] = i\omega_K^2 \mathcal{L}_K,$$
$$[H^0 + h, \mathcal{L}_K] = -i\mathcal{P}_K, \qquad (19)$$

where ω_K are the energies of the isobaric states, counted off the correlated ground state of a parent nucleus.

Substituting (14) and (15) into eqs. (19) one gets the following equations for ω_K, $\varphi^{(K)}_{pn}$ and $\psi^{(K)}_{pn}$:

$$(\varepsilon_p - \varepsilon_n)\psi^{(K)}_{pn} - E_{pn}\eta^{(K)}/2\gamma = \omega_K^2 \varphi^{(K)}_{pn},$$

$$(\varepsilon_p - \varepsilon_n)\varphi^{(K)}_{pn} - E_{pn}\xi^{(K)}/2\gamma = \psi^{(K)}_{pn}. \qquad (20)$$

Here, ε_p and ε_n are the one-particle proton and neutron energies, respectively (the eigenvalues of the Hamiltonian H^0) and the quantities $\eta^{(K)}$ and $\xi^{(K)}$ are defined by:

$$\eta^{(K)} = \sum_{pn} E_{pn} \psi^{(K)}_{pn} (n_n - n_p), \qquad (21)$$

$$\xi^{(K)} = \sum_{pn} E_{pn} \varphi^{(K)}_{pn} (n_n - n_p), \qquad (22)$$

$$E_{pn} = (\varepsilon_p - \varepsilon_n)(p\|n) - (p\|v_c\|n), \qquad (23)$$

where $(p\|n)$ are the reduced overlap integrals of the proton and neutron wave functions, and $(p\|v_c\|n)$ the reduced

proton-neutron matrix elements of the Coulomb potential[4].

Omitting the details of solving eqs. (20), we give the final dispersion equation for ω_K:

$$\mathcal{F}(\omega_K) \cdot \Phi(\omega_K) = 0, \qquad (24)$$

where

$$\mathcal{F}(\omega_K) = \sum_{pn} [\omega_K (p\|n) - (p\|v_c\|n)] \times \frac{E_{pn}(n_n - n_p)}{\varepsilon_p - \varepsilon_n - \omega_K}, \qquad (25)$$

$$\Phi(\omega_K) \equiv \mathcal{F}(-\omega_K). \qquad (26)$$

It is clear from eqs. (24)-(26) that there exist two branches of isobaric states, corresponding to solutions of eqs:

$$\mathcal{F}(\omega_K) = 0, \qquad (24')$$

$$\Phi(\omega_K) = 0. \qquad (24'')$$

For the particle-hole amplitudes we get

$$\psi^{(K)}_{pn} = \frac{\eta^{(K)}}{2\gamma} \frac{E_{pn}}{\varepsilon_p - \varepsilon_n \mp \omega_K},$$

$$-\varphi^{(K)}_{pn} = \mp \frac{1}{\omega_K} \psi^{(K)}_{pn}, \qquad (27)$$

where the sings $\mp$ corresponds to solutions of eqs. (24') and (24''), respectively. The quantity $\eta^{(K)}$ can be found from the normalization condition

---

[4] Note, that when the isovector potential $U_1 \to 0$ all $E_{pn} \to 0$.

$$1 = \pm \frac{1}{\omega_K} \sum_{pn} |\psi_{pn}^{(K)}|^2 (n_n - n_p) . \qquad (28)$$

The eigenstates of the total Hamiltonian $H^0 + h$ are constructed as one-phonon excitations on the correlated ground state (phonon vacuum $Q_K |0\rangle = 0$) of the parent

$$Q_K^+ |0\rangle = \left(-\frac{i}{\sqrt{2\omega_K}} \mathcal{P}_K + \sqrt{\frac{\omega_K}{2}} \mathcal{L}_K\right)|0\rangle$$

$$= \begin{cases} \frac{1}{\sqrt{\omega_K}} \sum_{pn} \psi_{pn}^{(K)} A_{pn} |0\rangle , & \omega_K \in \mathcal{F}(\omega_K) = 0 , \\ -\frac{1}{\sqrt{\omega_K}} \sum_{pn} \psi_{pn}^{(K)} A_{np} |0\rangle , & \omega_K \in \Phi(\omega_K) = 0 , \end{cases} \qquad (29)$$

$$\langle 0| [Q_K, Q_{K'}^+] |0\rangle = \delta_{KK'} . \qquad (30)$$

Let us discuss the results obtained. From eq. (29) it follows that T_z is strictly conserved in the isobaric states. The states, which correspond to solutions of eq. (24'), have $T_z = T_0 - 1$ and they are located in the nucleus $(N-1, Z+1)$ while those corresponding to solutions of eq. (24'') have $T_z = T_0 + 1$ and they belong to the nucleus $(N+1, Z-1)$. The same conclusions can be drown from eqs. (25) and (26). Solutions to eq. (24') are found between the poles $\varepsilon_p - \varepsilon_n > 0$ $(n_n - n_p > 0)$ which correspond to the neutron hole and proton particle states. Solutions to eq. (24'') are found between the poles $\varepsilon_p - \varepsilon_n < 0$ $(n_n - n_p < 0)$ which correspond to the neutron particle and proton hole states. Both types of particle-hole configurations contribute to the functions $\mathcal{F}(\omega_K)$ and $\Phi(\omega_K)$ due to correlations in the ground state $|0\rangle$. The latter was not taken into account in most of the previous shell model calculations.

The analog state (for nuclei with $N \sim Z$) can be found

only among the solutions to eq. (24'). In the special case

$$(p \| v_c \| n) = \Delta E_c \, (p \| n) , \quad \Delta E_c = const , \qquad (31)$$

eq.(24') contains the solution $\omega_K = \Delta E_c$ which corresponds to the average one-particle Coulomb shift energy for nuclei (N, Z) and $(N-1, Z+1)$. From eqs.(23), (27) and (28), it follows then

$$Q_K^+ |0\rangle_{\omega_K = \Delta E_c} = \frac{1}{\sqrt{T_0}} T_- |0\rangle , \qquad (32)$$

i.e., we have obtained the analog state. According to eq. (30) in this case the isotopic spin is a good quantum number for all the isobaric states. The latter is, of course, a trivial consequence of the isotopic invariance of the total nuclear Hamiltonian.

Fermi beta-transitions.

For future reference we separate the set $\{\omega_K\}$ into the set $\{\omega_{K_\mathcal{F}}\}$ of solutions of eq. (24') and the remainder set $\{\omega_{K_\Phi}\}$ of solutions of eq. (24''). Now we have a complete set of operators $Q_{K_\mathcal{F}}^+$, $Q_{K_\mathcal{F}}$, $Q_{K_\Phi}^+$ and Q_{K_Φ} which can be used to expand over it the components of the total isospin operator

$$T_- = -T_+^+ = \frac{1}{\sqrt{2}} \sum_{pn} (p \| n) A_{pn}$$

$$= \sum_{K_\mathcal{F}} \mathcal{M}_{K_\mathcal{F}} Q_{K_\mathcal{F}}^+ + \sum_{K_\Phi} \mathcal{M}_{K_\Phi} Q_{K_\Phi} . \qquad (33)$$

The expansion amplitudes are none other than the matrix elements of the Fermi decay of the isobaric states to the parent

$$\mathcal{M}_{K_F} = \langle 0 | [Q_{K_F}, T_-] | 0 \rangle = \frac{1}{\sqrt{2\omega_{K_F}}} \sum_{pn} (p\|n) \psi_{pn}^{(K_F)} (n_n - n_p), \quad (34)$$

$$\mathcal{M}_{K\phi} = \langle 0 | [T_-, Q_{K\phi}^+] | 0 \rangle = \frac{1}{\sqrt{2\omega_{K\phi}}} \sum_{pn} (p\|n) \psi_{pn}^{(K\phi)} (n_n - n_p). \quad (35)$$

For the case (31) the only nonvanishing matrix element corresponds to the Fermi decay of the analog state

$$\mathcal{M}_{K_F} (\omega_{K_F} = \Delta E_c) = \sqrt{T_0}. \quad (36)$$

This result can easily be obtaind if one substitutes (27) into eqs.(34) and (35), and compares them with the functions $\mathcal{F}(\omega_K)$ and $\Phi(\omega_K)$.

In the case of arbitrary Coulomb potential, one can use the commutator

$$T_z = [T_-, T_+] \quad (37)$$

to obtaine the sum rule for the Fermi decay of the isobaric states to the parent

$$T_0 \equiv \frac{N-Z}{2} = \frac{1}{2} \sum_{pn} (p\|n)^2 (n_n - n_p) = \sum_{K_F} |\mathcal{M}_{K_F}|^2 - \sum_{K\phi} |\mathcal{M}_{K\phi}|^2. \quad (38)$$

The same sum rule was obtained earlier in ref./13/.

The important consequence of the sum rule (38) is that the Coulomb mixing may result not only in decrease of the matrix element for the superallowed Fermi decay as is usually believed /3,9,12/. In principle the matrix element for the decay of the analog state may exceed the mean value (36).

Isospin structure of the 0^+ states.

Up to now we did not introduced explicitly the isotopic spin of the states considered. The microscopic theory allows one to calculate the energies of the isobaric states and the corresponding Fermi decay matrix elements. Now we show that those quantities allow one to analyze the isotopic structure of the states.

The expansion (33) can be used to calculate the mean square of the total isospin in the ground state

$$\langle 0 | T^2 | 0 \rangle = \langle 0 | T_z^2 | 0 \rangle + \langle 0 | T_z | 0 \rangle + 2 \langle 0 | T_- T_-^+ | 0 \rangle = T_0 (T_0 + 1) + 2 \sum_{K\phi} |\mathcal{M}_{K\phi}|^2. \quad (39)$$

The isospin mixing in the case considered is caused by the vector part of the Coulomb potential. Therefore, the ground state of the parent nucleus contains only the admixture of the isospin $T_0 + 1$. Let us expand the ground state over the pure isospin states $|T, T_z\rangle$:

$$|0\rangle = a |T_0, T_0\rangle + b |T_0 + 1, T_0\rangle, \quad a^2 + b^2 = 1. \quad (40)$$

For the mean square of the total isospin we have then

$$\langle 0|T^2|0\rangle = T_0(T_0+1) + 2b^2(T_0+1). \qquad (41)$$

Comparing eqs. (41) and (39), one can see that the admixture of the isospin T_0+1 in the ground state of the parent is determined through the sum of Fermi decays of all the isobaric states in the nucleus $(N+1, Z-1)$:

$$b^2 = (T_0+1)^{-1} \sum_{K\phi} |M_{K\phi}|^2. \qquad (42)$$

This is the new definition (and it is, in fact, model independent) of the isospin impurity, because usually one considers the Coulomb mixing of states in the same nucleus[1]. The latter is always model dependent. However, the two definitions are not different in principle, because the isobaric states $Q^+_{K\phi}|0\rangle$ belong to the same isospin multiplet as the isovector monopole state in the parent. Indeed, using eq.(33) we construct in the nucleus $(N+1, Z-1)$ the pure isospin state of the multiplet mentioned

$$|T_0+1, T_0+1\rangle = -|\langle 0|T_- T_+|0\rangle|^{-1/2} T_+|0\rangle$$

$$= \frac{1}{b\sqrt{T_0+1}} \sum_{K\phi} M_{K\phi} Q^+_{K\phi}|0\rangle. \qquad (43)$$

This state is not the eigenstate for our Hamiltonian H^0+h. Nevertheless, we may estimate its energy with respect to the ground state of the parent

$$E_{T_0+1, T_0+1} = \langle T_0+1, T_0+1|H^0+h|T_0+1, T_0+1\rangle$$

$$= \frac{1}{b^2(T_0+1)} \sum_{K\phi} |M_{K\phi}|^2 \omega_{K\phi}. \qquad (44)$$

Now we may estimate the energy of the isovector monopole state in the parent

$$E_{T_0+1, T_0} = E_{T_0+1, T_0+1} + \Delta E_c' - \Delta m_{np}, \qquad (45)$$

where $\Delta E_c'$ is the Coulomb shift energy and Δm_{np} the proton-neutron mass difference. The remainder term of the isospin multiplet $T = T_0+1$ with $T_z = T_0-1$ (the double analog state) lies in the nucleus $(N-1, Z+1)$ and the admixtures of this state may be found in all the isobaric states $Q^+_{K\phi}|0\rangle$ (see text below).

From eq. (43) it follows, that the amplitude of the isospin T_0+1 in any state $Q^+_{K\phi}|0\rangle$ is equal to

$$B_{K\phi} = \frac{M_{K\phi}}{b\sqrt{T_0+1}}, \qquad (46)$$

i.e., it is defined by the matrix elelement of the Fermi transition to the parent. The sum rule for those amplitudes reads

$$\sum_{K\phi} |B_{K\phi}|^2 = 1. \qquad (47)$$

This result is easily understood, because from the ground state of the parent one can generate by means of operators $T_\pm$ the only collective state (43) with the isospin T_0+1. This state is distributed among the isobaric states $Q^+_{K\phi}|0\rangle$.

Using the T_--operator one can construct the collective analog state[4] in the nucleus $(N-1, Z+1)$:

$$|A\rangle = |\langle 0|T_+ T_-|0\rangle|^{-1/2} T_-|0\rangle$$

$$= \frac{a\sqrt{T_0}}{\sqrt{T_0+b^2(T_0+1)}} |T_0, T_0-1\rangle + \frac{b\sqrt{2T_0+1}}{\sqrt{T_0+b^2(T_0+1)}} |T_0+1, T_0-1\rangle, \qquad (48)$$

which is not the eigenstate for the Hamiltonian $H^o + h$ (except for the case (31)). This state is distributed among the isobaric states $Q^+_{K_\mathcal{F}}|0\rangle$. The corresponding admixtures $\delta_{K_\mathcal{F}}$ are defined through the Fermi decay matrix elements

$$\mathcal{M}_{K_\mathcal{F}} = \langle 0|[Q_{K_\mathcal{F}}, T_-]|0\rangle = \delta_{K_\mathcal{F}} \langle A|T_-|0\rangle$$
$$= \delta_{K_\mathcal{F}} \sqrt{T_o + b^2(T_o+1)}\,, \qquad (49)$$

from where it follows:

$$\delta_{K_\mathcal{F}} = \mathcal{M}_{K_\mathcal{F}} / \sqrt{T_o + b^2(T_o+1)}\,. \qquad (50)$$

The amplitudes $\delta_{K_\mathcal{F}}$ are subject to the sum rule (see eqs. (38) and (42)):

$$\sum_{K_\mathcal{F}} |\delta_{K_\mathcal{F}}|^2 = 1. \qquad (51)$$

Now we can estimate the impurities of the isospins T_o and T_o+1 in any isobaric state $Q^+_{K_\mathcal{F}}|0\rangle$

$$\alpha_{K_\mathcal{F}}(T_o) = \langle 0|Q_{K_\mathcal{F}}|T_o, T_o-1\rangle$$
$$= \mathcal{M}_{K_\mathcal{F}} \sqrt{(1-b^2)T_o}\,/[T_o + b^2(T_o+1)]\,, \qquad (52)$$

$$\beta_{K_\mathcal{F}}(T_o+1) = \langle 0|Q_{K_\mathcal{F}}|T_o+1, T_o-1\rangle$$
$$= \mathcal{M}_{K_\mathcal{F}}\, b\sqrt{2T_o+1}\,/[T_o + b^2(T_o+1)]\,. \qquad (53)$$

It is seen that the largest impurities are expected for the state with the largest value of $\mathcal{M}_{K_\mathcal{F}}$, i.e., for the analog state. This fact emphasizes the importance of the impurity of the isospin T_o+1 when considering the superallowed Fermi

transition, because this impurity may compensate the decrease of the Fermi matrix element due to the admixture of the isospin T_o-1. The admixture of the isospin T_o+1 is most noticeable in the analog state of odd-odd nuclei with $T_z = 0$ $(T_o = 1)$:

$$\beta_{K_\mathcal{F}}(T=2) \approx b\sqrt{3}\,, \qquad (54)$$

i.e., the mean value of this impurity exceeds the corresponding impurity in the ground state of the parent.

However, the total impurity of the isospin T_o+1 in all isobaric states $Q^+_{K_\mathcal{F}}|0\rangle$ is relatively small $(b^2 \ll 1)$:

$$\sum_{K_\mathcal{F}} |\beta_{K_\mathcal{F}}(T_o+1)|^2 = \frac{b^2(2T_o+1)}{T_o + b^2(T_o+1)} \ll 1 \qquad (55)$$

and it decreases with increasing T_o.

The admixture of the isospin T_o-1 is estimated from the following equation:

$$\gamma^2_{K_\mathcal{F}}(T_o-1) = 1 - \alpha^2_{K_\mathcal{F}}(T_o) - \beta^2_{K_\mathcal{F}}(T_o+1)\,. \qquad (56)$$

Here we have neglected the small admixtures of the isospin T_o+2 in the isobaric states $Q^+_{K_\mathcal{F}}|0\rangle$.

Calculations and discussion of the results.

The Woods-Saxon potential in the form /18/ has been used in our calculations. For the isovector potential, one supposes

$$f_1(r) = f_o(r)\,, \quad U_1 = 2\eta\,\frac{N-Z}{A}\,U_o\,. \qquad (57)$$

The Coulomb potential was taken in the form which corresponds to the uniform distribution of the charge in the sphere of the radius $R_c = r_c A^{1/3}$:

$$V_c(r) = \begin{cases} \frac{(Z-1)e^2}{R_c}\left[\frac{3}{2} - \frac{1}{2}\left(\frac{r}{R_c}\right)^2\right], & r \le R_c \\ \frac{(Z-1)e^2}{r}, & r > R_c \end{cases} \quad . \qquad (58)$$

In ref./18/ the following mean values of the parameters were found from the calculation of the separtion energies $U_0 =$ = 53.3 MeV, η = 0.63 and $r_c = r_0$ = 1.24 fm. In nuclei with small neutron excess the separation energies depend weakly on η . Therefore, in our calculations we have determined η from the observed energies of splitting of the analog and antianalog states in ^{64}Ga and ^{66}Ga. A reasonable agreement of calculations with experiment was obtained for $\eta \approx 0.7$. The Coulomb radius r_c , which is very important for all the characteristics of the isobaric states, was not fixed strictly. Below we give the results of calculation as functions of r_c . Note, at last, that we have used a little bit smaller values for the diffusiness parameter $\alpha = 1.4\ fm^{-1}$ and for the spin-orbital constant $\lambda^2 = 0.23\ fm^2$ than in ref. /18/.

The one-particle basis was calculated according to the method given in ref./19/. The computing code is given in ref. /20/. The basis includes all the bound and quasistationary states, i.e., the continuum was approximated by the quasistationary states. The left-hand side of the sum rule (38), which includes all the overlap integrals $(p\|n)$ with $\Delta n = 0, 1$ and 2 was fulfilled with an accuracy up to (0.1-0.3)%.

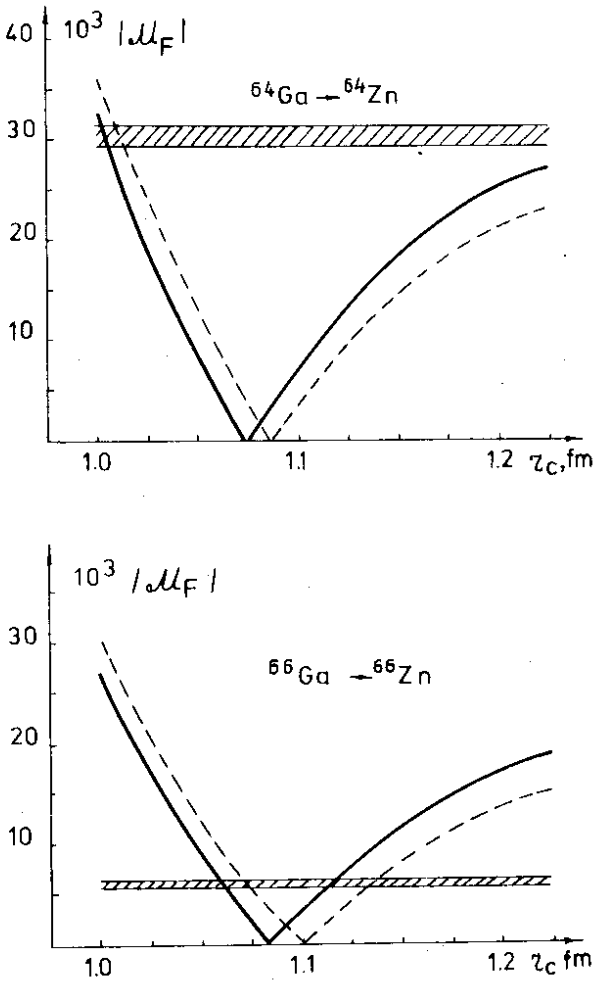

Fig.1. The dependence of the isospin forbidden Fermi decay matrix element on r_c . Calculations are performed with $\eta = 0.7$ and the diffuseness parameter $\alpha = 1.4\ fm^{-1}$ (solid curves) and $1.5\ fm^{-1}$ (dotted curves). The dashed areas represent the experimental values of $|M_F|$ /3/.

Due to the $\Delta\ell = 1$ and 2 transitions the isobaric spectra considered include not only the analog and antianalog states but also the high-lying states which form the isovector monopole resonance. Below we discuss the results of calculations.

a) Isospin forbidden $0^+ \to 0^+$ transitions.

We consider as a key problem the description of the isospin forbidden Fermi transitions which are strongly dependent on both the Coulomb mixing and the nucleon correlations (11). The calculations were performed for the Fermi decay of the antianalog states (ground states) of ^{64}Ga and ^{66}Ga to the ground states of the parent nuclei ^{64}Zn and ^{66}Zn, respectively. In both cases the residual interactions (11) mix strongly the $(n^{-1}p)$ configurations $[(2p_{3/2})^{-1}(2p_{3/2})]_{0^+}$ and $[(1f_{5/2})^{-1}(1f_{5/2})]_{0^+}$ which dominate in the structure of the ground states of ^{64}Ga and ^{66}Ga. The contributions of these configurations to the matrix element (34) are of different signs, so that the latter depends strongly on the Coulomb radius r_c (fig.1). The small variation of r_c results in the strong change of $|M_F|$. The latter may serve as an explanation of the observed large difference of $\log ft$ in the nuclei considered /3/. The variation of both η and the diffuseness parameter α of the potential results in the shift of the curves shown in fig.1, but it does not affect the strong dependence of $|M_F|$ on r_c. We should like to emphasize, however, that the strong dependence of $|M_F|$ on r_c is not caused mostly by the Coulomb potential itself but it is due to the nucleon correlations. For that reason we believe that the selfconsistency is of great importance in such calculations.

The earlier calculations /21,22/ suffer from the lack of

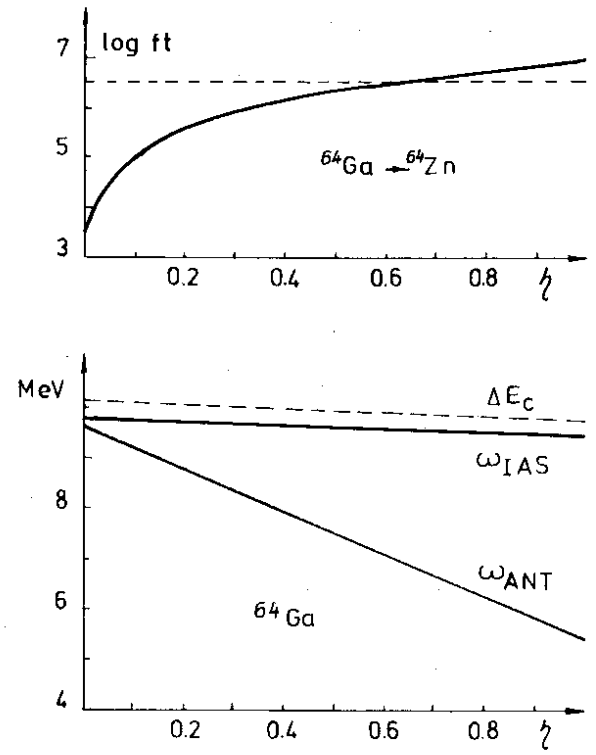

Fig.2. The $\log ft$ of the isospin forbidden transition $^{64}Ga \to ^{64}Zn$ versus the isovector parameter η (above; the dotted line corresponds to experimental value of logft /3/). The lower part shows the dependence on η of ΔE_c, ω_{IAS} and ω_{ANT} in the nucleus ^{64}Ga (see text). Calculations are performed with $r_c = 1.2\,fm$ and $\alpha = 1.4\,fm^{-1}$.

selfconsistency in the configuration mixing. Moreover, in ref. /21/ the static pairing correlations with broken isotopic symmetry were taken into account. The latter serves as an additional source of the isospin mixing and distort the true role of the Coulomb mixing.

Fig.2 shows the dependence on η of the $\log ft$ value for the isospin forbidden Fermi transition $^{64}Ga \rightarrow {}^{64}Zn$. The lower part of fig. 2 shows also the energy of the analog (ω_{IAS}) and antianalog (ω_{ANT}) states as well as the average one-particle Coulomb shift energy, defined as

$$\Delta E_C = \langle\tilde{0}|[T_+,[V_c, T_-]]|\tilde{0}\rangle / \langle\tilde{0}|[T_-,T_+]|\tilde{0}\rangle = \frac{\sum_{pn} (p\|v_c\|n)(p\|n)(n_n - n_p)}{\sum_{pn} (p\|n)^2 (n_n - n_p)} \quad (59)$$

As it was expected in a self-consistent approach there is a very weak dependence of ω_{IAS} on the isovector parameter η associated with the nucleon correlations. The location of the analog state with respect to the ground state of the parent is defined mainly by the Coulomb potential.
The splitting of the analog and antianalog states, on the contrary, is proportional to the isovector parameter. When $\eta \to 0$ those states are strongly mixed by the Coulomb potential (we mean the isospin mixing), what leads to the sharp decrease of the $\log ft$ value for the Fermi decay of the antianalog state shown in fig.2. The analogous results were obtained for the decay $^{66}Ga \rightarrow {}^{66}Zn$.

The energy of the analog state ω_{IAS} in different medium weight nuclei calculated with the parameters $\eta \approx 0.7$

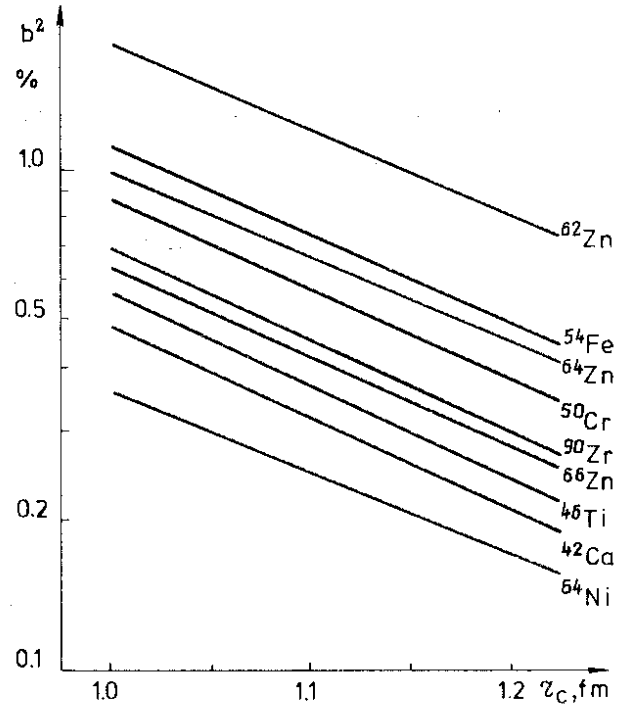

Fig.3. The impurity of the isospin $T_0 + 1$ in the ground state of a number of even-even nuclei versus r_c. Calculations are performed with $\eta = 0.7$ and $\alpha = 1.4\ fm^{-1}$.

and $r_c \approx 1.2\ fm$ is usually by about 0.5 MeV smaller than the experimental Coulomb shift energy. This difference may to some extent be associated with our choice of the model isovector potential. Some contributions are expected to come from the tensor part of the Coulomb potential as well as from the spin-orbital correction, etc. (see, estimates in refs. /2,4,5/).

b) Isospin impurities and superallowed Fermi transitions.

We have calculated the impurity of the isospin T_0+1 in the ground state of spherical even-even nuclei with $A \geq 42$. Fig. 3 shows the calculated β^2 values (eq.(42)) for a number of nuclei as functions of the Coulomb radius r_c. The value of β^2 decreases slightly with increasing η, i.e., with increasing correlations the isospin impurity in the ground state decreases. The behaviour of β^2 as a function of T_0 and of charge number Z corresponds to the predictions of the hydrodynamic model[6]. However, the calculated impurities are much larger than those estimated by Bohr and Mottelson. The reason for such a difference is that the calculated energies E_{T_0+1, T_0} (eq.(45)) of the isovector monopole state are of an order of 26-30 MeV, i.e., noticeably smaller than those predicted by the hydrodynamical model ($\approx 170 A^{-1/3}$ MeV). Our estimate of E_{T_0+1, T_0} agrees well with that in the theory of final Fermi systems [23] as well as with the estimate obtained by means of the sum rule method[5]. Due to the self-consistency of our model the mean value of E_{T_0+1, T_0} depends rather weakly on η and r_c (when selfconsistency is broken the energy of the isovector monopole state increases with increasing the strength of the isovector force).

On the other hand the calculated values of β^2 are much smaller than the shell model estimates[7,8], in which the nucleon correlations were ignored. This result agrees well with that obtained in ref.[5]. Thus, the selfconsistency and the nucleon correlations are important when estimating the isospin impurities.

The results shown in fig.3 are obtaind when only the quasistationary states with the width $\Gamma \leq 0.5$ MeV and the energy $\varepsilon \gg \Gamma$ were included in calculations. The value of β^2 may increase when the quasistationary states with the larger width are included, but the physical meaning of those states is doubtfull. Therefore, a more reliable estimate of β^2 requires a more correct inclusion of the continuum into calculations.

The isospin impurities are much larger in the isobaric states. Of special interest is the isotopic structure of the analog states which may contain an admixture of the isospin T_0+1. The calculated Coulomb mixing amplitudes α_{IAS} and β_{IAS} in analog states of a number of nuclei with $N=Z$ are shown in fig.4. It is seen that the isospin purity of the analog state as well as of the ground state of the parent depends on the Coulomb radius r_c. With decreasing r_c the depending on r part of the Coulomb potential is more localized in the nuclear volume that results in the increase of the isospin impurities. The impurities of the isospin T_0+1 in $N=Z$ nuclei are of the same order as those of the isospin T_0-1 (the latter are not shown in fig. 4 but may be estimated from eq. (56)). The reason for that is that the analog state is constructed on the correlated ground state of the parent which contains already a noticeable admixtures of the isospin T_0+1. With increasing T_0 and the mass number, the admixtures of the isospin T_0+1 decreases slightly while the admixtures of the isospin T_0-1 increase. To illustrate this we give below the isospin mixing amplitudes in analog states of some nuclei, calculated with $\eta = 0.7$ and $r_c = 1.2$ fm :

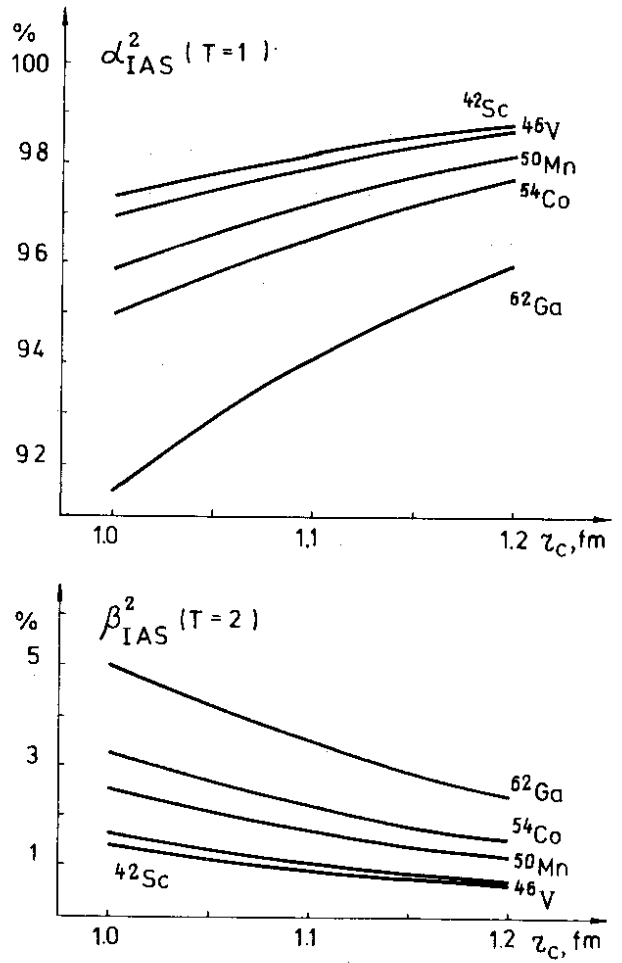

Fig.4. The Coulomb mixing amplitudes for the isospins $T=1$ (above) and $T=2$ (below) in the analog states of some nuclei with $N=Z$ as functions of r_c. Calculations are performed with $\eta = 0.7$ and $\alpha = 1.4\,fm^{-1}$.

$$\alpha^2_{IAS}(T=4) = 98.9\%\,;\quad \beta^2_{IAS}(T=5) = 0.4\%\,;$$
$$\gamma^2_{IAS}(T=3) = 0.7\% \quad in \quad {}^{64}_{29}Cu\,;$$
$$\alpha^2_{IAS}(T=5) = 98.4\%\,;\quad \beta^2_{IAS}(T=6) = 0.6\%\,;$$
$$\gamma^2_{IAS}(T=4) = 1\% \quad in \quad {}^{90}_{41}Nb\,;$$
$$\alpha^2_{IAS}(T=22) = 88.3\%\,;\quad \beta^2_{IAS}(T=23) = 0.7\%\,;$$
$$\gamma^2_{IAS}(T=21) = 11\% \quad in \quad {}^{208}_{83}Bi\,. \qquad (60)$$

The values of α^2_{IAS} listed agree qualitatively with the estimates obtained in ref.[5], but differ noticeably from the results given in ref.[24]. In the latter paper the calculations were performed in the model with broken isotopic symmetry, what may be the reason for such a difference.

As to the β^2_{IAS} values we did not know any other calculations to compare with.

We have mentioned above that the admixture of the isospin T_o+1 in the analog state may affect the value of the correction to the superallowed Fermi transition matrix element. Usually this matrix element is presented in the form[2,5,6,9,12]

$$\mathcal{M}_F^2 = T_o\,(1-\delta_c)\,, \qquad (61)$$

where δ_c is the correction associated with the isospin mixing. In terms of the isospin mixing amplitudes the correction is

$$\delta_c = 1 - \left(\sqrt{1-\beta^2}\,\alpha_{IAS} + b\,\beta_{IAS}\sqrt{\frac{2T_o+1}{T_o}}\right)^2. \qquad (62)$$

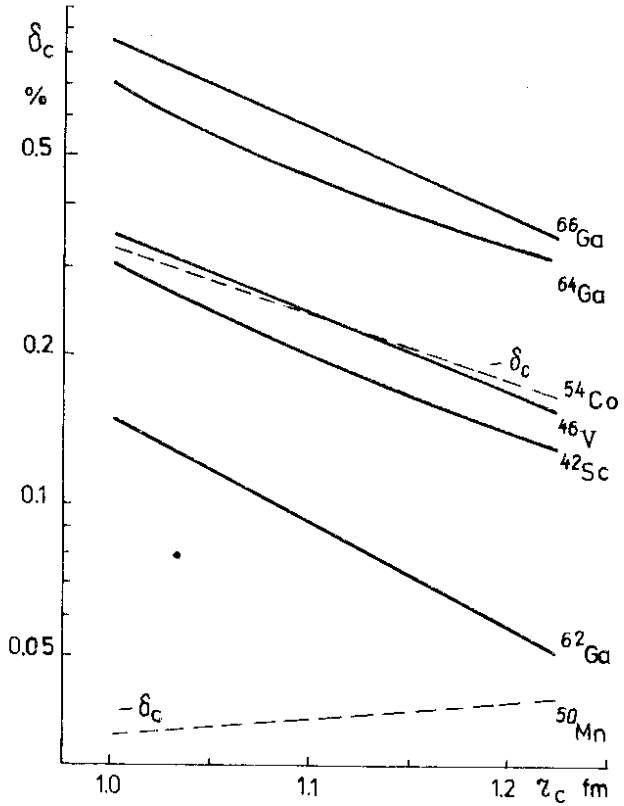

Fig.5. The dependence on r_c of the Coulomb mixing correction to the superallowed Fermi decay of the analog states. The negative sign corrections are shown with dotted curves. Calculations are performed with $\eta = 0.7$ and $\alpha = 1.4\,fm^{-1}$.

When the impurity of the isospin T_0+1 in the analog state is neglected ($\beta_{IAS} = 0$), the correction $\delta_c > 0$ as is usually believed. The admixture of the isospin T_0+1 may result in the negative sign of δ_c.

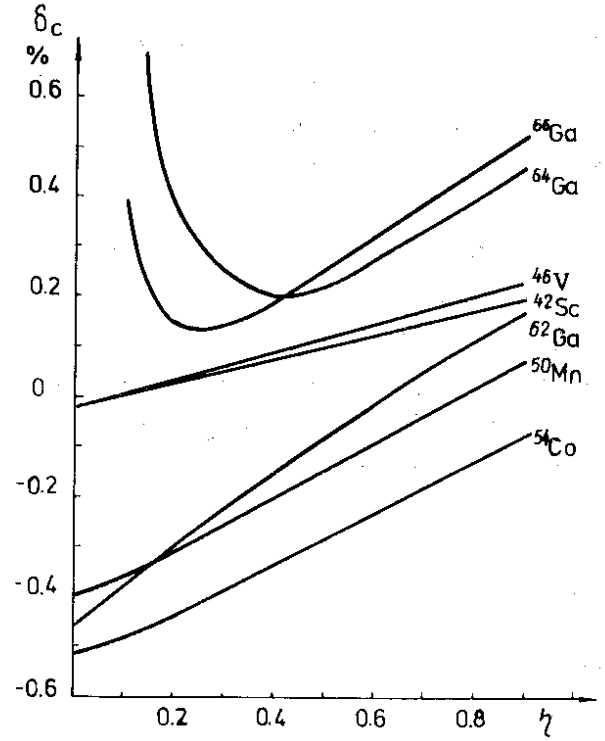

Fig.6. The Coulomb mixing correction δ_c versus η. Calculations are performed with $r_c = 1.2\,fm$ and $\alpha = 1.4\,fm^{-1}$.

Numerical calculations have shown that δ_c depends essentially on both the Coulomb radius r_c (fig.5) and the isovector parameter η (fig.6). For two cases (^{50}Mn and ^{54}Co) the negative corrections were obtained. The sign of δ_c may change with changing η, as is shown in fig. 6. In ^{64}Ga and ^{66}Ga nuclei the fast change of δ_c when $\eta \lesssim 0.2$ is related to the strong mixing of the analog and antianalog states (see also fig.2). The other nuclei shown in figs. 5,6 have no antianalog states.

The calculated values of $|\delta_c^{\sim}|$ are in qualitative agreement with the estimates[9,12]. The new quality consists in the possibility of the negative correction $\delta_c^{\sim}$.

We should like to mention that the value (and even the sign) of $\delta_c^{\sim}$ may change when the quasistationary states with large width $\Gamma \gtrsim 1\,MeV$ and the energy $\mathcal{E} \sim \Gamma$ are included into calculations. Therefore, a more reliable estimate of $\delta_c^{\sim}$ may be given when the continuum is included explicitly. For that reason in the present paper we emphasize only the fact that the correction $\delta_c^{\sim}$ may be negative and did not perform the analysis of the effective $\mathcal{F}t$ values for the superallowed Fermi transitions.

Conclusions.

The theory of the isobaric states derived in the present paper is based on the assumption that the residual interactions which restore the broken isotopic symmetry of a given nuclear potential may be sought in a separable form. Then the model considered does not involve any additional parameter. In our consideration of the isospin mixing the two-body Coulomb force was replaced by the effective Coulomb potential. The tensor part of the Coulomb force and the spin-orbital correction were not taken into account.

The RPA treatment of the isobaric states allows one to take into account the ground state correlations and to reveal the new qualitative effects in the isotopic structure of the 0^+ states. In particular, it is shown that the analog states in nuclei with $N \approx Z$ contain a noticeable admixture of the isospin $T_0 + 1$, what may affect both the Coulomb mixing

correction in the superallowed Fermi transition matrix element and the spreading width of the analog state.

We have given here the new definition of the isospin impurities in the ground state of (N, Z) nucleus in terms of the matrix elements of the Fermi decay of the isobaric states in the nucleus $(N+1, Z-1)$. This definition as well as the analysis of the isospin structure of the isobaric states may be considered, to a large extent, as model independent, because, it involves only the Fermi decay matrix elements.

Application of the theory to the isospin forbidden Fermi decay have shown a reasonable agreement of calculations with experiment.

In conclusion we would like to thank B.L.Birbrair and F.A.Gareev for many helpful discussions, and R.M.Jamaleev and V.V.Palchik for help in numerical calculations.

References:

1. A.Bohr, B.R.Mottelson, Nuclear Structure v.1 (W.A.Benjamin, Inc., New York, 1969).
2. R.J.Blin-Stoyle, Fundamental Interactions and the Nucleus (N.-H., Amsterdam, 1973).
3. S.Raman, T.A.Walkiewicz, H.Behrens, Atomic Data and Nucl. Data Tables 16, 451 (1975).
4. N.Auerbach, J.Hüfner, A.K.Kermann, C.M.Shakin, Rev.Mod.Phys., 44, 48 (1972).
5. A.M.Lane, A.Z.Mekjian, Adv.Nucl.Phys. 7, 97 (1973).
6. A.Bohr, J.Damgaard, B.R.Mottelson. In: Nuclear Structure (N.-H., Amsterdam, 1967), p.1; NORDITA Publications, No.244.

7. L.A.Sliv, Yu.I.Kharitonov, Phys.Lett. 16, 176 (1965).
8. S.B.Khadkikar, C.S.Warke, Nucl.Phys. A130 577 (1969).
9. I.S.Towner, J.C.Hardy, Nucl.Phys. A205, 33 (1973).
10. D.F.Zaretsky, M.G.Urin, JETPh, 53, 324 (1967).
11. Yu.V.Gaponov, Yu.S.Lutostanski, Journ.Nucl.Phys.(Sov.) 16, 484 (1972).
12. S.A.Fayans, Phys.Lett. 37B, 155 (1971).
13. B.L.Birbrair, V.A.Sadovnikova, Journ. Nucl.Phys. (Sov.) 20, 654 (1974).
14. N.I.Pyatov, JINR Report P4-8380, Dubna, 1974.
15. N.I.Pyatov, Proc Xth Winter School of LNPhI on Nuclear Physics and Elementary Particles, part 1, p. 232, Leningrad 1975.
16. N.I.Pyatov, D.I.Salamov, Nucleonica, 22, 127 (1977).
17. A.A.Kuliev, D.I.Salamov, Int.Topical Conf. on Effective Interactions and Operators in Nuclei, v.1, p.4, Arizona, 1975.
18. V.A.Chepurnov, Journ. Nucl.Phys. (Sov.) 6, 955 (1967).
19. J.Bang, F.A.Gareev, I.V.Puzynin, R.M.Jamaleev, Nucl.Phys. A261, 59 (1976).
20. M.X.Gizzatkulov, I.V.Puzynin, R.M.Jamaleev, JINR Report P11-10029, Dubna, 1976.
21. A.Ikeda, Prog.Theor.Phys., 38, 832 (1967).
22. C.T.Yap, C.S.Tee, Nucl.Phys. A194, 573 (1972).
23. J.Damgaard, V.V.Gortchakov, G.M.Vagradov, A.Molinari, Nucl.Phys. A121 , 625 (1968).
24. G.Ja.Tertychny, E.L.Yadrovski, Bull. of Acad. of Sci, USSR, ser.fyz., 41, 57 (1977).

Received by Publishing Department on
April 10, 1978.