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G.N.Afanasiev, V.M.Shilov, M.Trajdos

THE ENERGY DISSIPATION IN HEAVY ION REACTIONS WITHIN THE POTENTIAL MODEL.

I.Formalism

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THE ENERGY DISSIPATION IN HEAVY ION REACTIONS WITHIN THE POTENTIAL MODEL.

I.Formalism

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Афанасьев Г.Н., Шилов В.М., Трайдос М. **E4 - 11416**

Диссипация энергии в реакциях с тяжелыми ионами в рамках потенциальной модели. I. Формализм

Предлагается простая потенциальная модель, в которой диссипация энергии в реакциях с тяжелыми ионами связывается с возбуждением уровней гигантского резонанса. Образование составного ядра при этом рассматривается как захват на квазистационарные состояния.

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Afanasiev G.N., Shilov V.M., Trajdos M., **E4 - 11416**

The Energy Dissipation in Heavy Ion Reactions within the Potential Model

A simple potential model is suggested in which the energy dissipation in the heavy ion reactions is treated quantum-mechanically. In the spirit of the early papers on the giant resonance theory we treat it as the mutual oscillations of the proton and neutron nuclear densities. Just due to the excitation of these oscillations the kinetic energy dissipation takes place. The incoming particle can be captured to the quasistationary levels under the Coulomb barrier. The reaction cross-section is defined in a usual way as an infinite sum of the partial penetrabilities. The cross section of the compound-nucleus formation is defined as a part of this sum over those states the kinetic energy of which is below the Coulomb barrier.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. There are only few attemts to treat quantum-mechanically the kinetic energy dissipation in heavy ion reactions. In refs.[1,2] the Hamiltonian of the ion-ion system was composed of 3 parts: the Hamiltonian corresponding to intrinsic motion of the colliding ions, the Hamiltonian of relative motion and the Hamiltonian corresponding to the interaction of these motions. The origin of this last term is due to the deformation of nuclear surfaces resulting from their Coulomb repulsion. As only quadrupole deformation was taken into account, only lowest collective states were excited. It was pointed in refs.[1,2] that taking into account the Coulomb excitation only is insufficient. Nevertheless, the calculations performed in[2] show that Coulomb excitation changes cross sections by an order of 30%. In refs.[3,4] the interaction of intrinsic and relative motions was chosen in the form:

$$H = \Sigma A_{\lambda\mu} V_{\lambda\mu}(r). \qquad (1.1)$$

where $A_{\lambda\mu} = \int dE\, f(E)\, a^{+}_{\lambda\mu}(E)$, and $a^{+}_{\lambda\mu}(E)$ is the creation operator of the state with quantum numbers $(\lambda\mu)$ and energy E ; $f(E)$ is the weight function. As an experiment demonstrates the definite kinetic energy losses, the $f(E)$ was chosen to be a delta function:

$$f(E) = F \cdot \delta(E - h\omega).$$

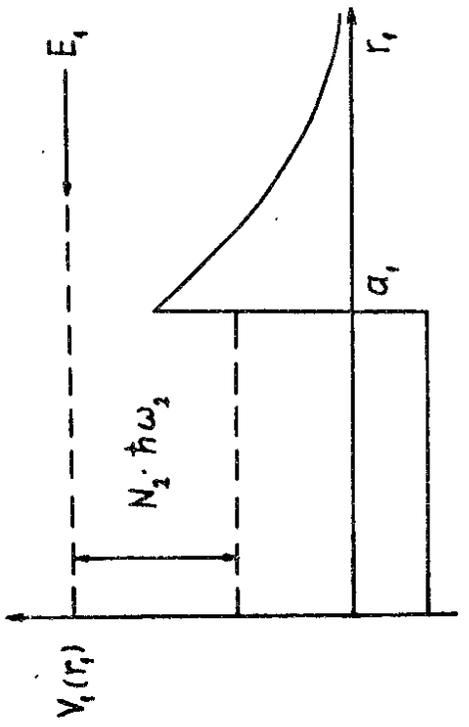
The values $V_{\lambda\mu}(r)$ were defined as the Fourier series components of the following function

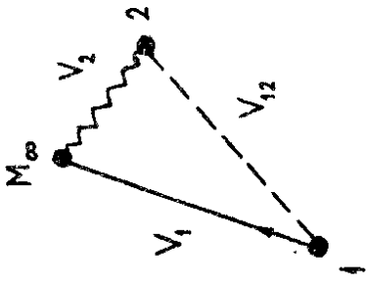
$$V(r) = V_0 \frac{\delta(r-R_0)}{r^2} \delta(\cos\theta - \cos\theta_\ell) = \Sigma V_{\lambda\mu}(r).$$

At last θ_ℓ were defined as the deflection angles of the Coulomb classical trajectories. The interaction (1.1) was treated in the first order of perturbation theory to calculate both reaction and compound nucleus cross sections. So, this model uses the receipts of the classical mechanics in order to fix the parameters of the model. Further calculations are performed using the rules of the quantum mechanics. The fitting parameters are ω, R_0, V_0. The drawbacks of this model are evident: i) the energy losses are put into it by "hand" (i.e., there is no mechanism of the energy losses); ii) it uses the receipts of the classical mechanics, iii) there is no mechanism of the compound nucleus formation.

2. We suggest the following simple quantum-mechanical model which permits us to treat the energy dissipation in heavy ion reactions. The incident ion (see the *figure*) is scattered inelastically on the nuclear target and excites the intrinsic degrees of motion. We adopt the simplest possibility and treat these intrinsic degrees as the giant resonance oscillation modes of the target nucleus. For simplicity we treat these modes in the spirit of the early papers on this subject (cf. [5]), i.e., we consider these modes as the mutual oscillations of the proton and neutron components of the nuclear density.

The following simplifications are also made: a) we neglect the recoil effects experienced by the proton component of the nuclear density; this is equivalent to using of the infinite mass of this component; b) the proton and neutron components being treated as structureless objects (the same is true for the incident ion) are interacting by the oscillator law; c) the interaction of the incident ion with the proton component (i.e., with the fixed force center located at the origin) is composed of the attractive square well at small distances and of the Coulomb repulsion at the greater ones; d) the interaction of the incident ion with the neutron component of the density





Schematic presentation of the treated model.

is short range and is treated in the first order perturbation theory using the Fermi's Golden rules. Point a) is needed to escape the solution of the full 3-body problem and permits us to be limited to the consideration of the scattering of particle 1 on particle 2 in the field[6]. Point b) is needed to exclude the fission channel of the neutron and proton density components. At last, points c) and d) are aimed at presenting results in an explicit form using the special functions of the mathematical physics.

3. The total Hamiltonian may be written as

$$H = H_1 + H_2 + V_{12}(r_{12}) .$$

Here H_2 is the oscillator Hamiltonian of particle 2 in the field of the force center:

$$H_2 = -\frac{h^2}{2m_2}\Delta_2 + V_2(r_2), \qquad V_2 = \frac{k_2 r_2^2}{2},$$

H_1 is the Hamiltonian of the incident ion 1 in the field of the force center:

$$H_1 = -\frac{h^2}{2m_1}\Delta_1 + V_1(r_1)$$

$$V_1(r_1) = \begin{cases} -V_{01}, & r_1 < a_1 \\ \dfrac{Z_1 Z_2 e^2}{r_1}, & r_1 > a_1 \end{cases} .$$

At last, $V_{12}(r_{12})$ is the interaction of the incident ion 1 with oscillator 2. We consider two variants of the V_{12} : a) delta function $V_{12} = V_{012}\cdot\delta(\vec{r}_1 - \vec{r}_2)$ and b) square well: $V_{12} = -V_{012}$ for $r_{12} < a_{12}$, $V_{012} = 0$ for $r_{12} > a_{12}$.

As in atomic physics [7-10] (the difference is that there is no potential screening in the treated case) the exact wave function may be developed into the eigenfunctions of particle 2 in the field of the force center:

$$\psi = \phi(\vec{r}_1)u_0(\vec{r}_2) + \sum F_\mu(\vec{r}_1)u_\mu(\vec{r}_2) . \qquad (3.1)$$

Here $u_\mu(\vec{r}_2)$ are the oscillator functions:

$$u_\mu \equiv u_{n_2\ell_2 m_2}(\vec{r}_2) = R^{osc}_{n_2\ell_2}(r_2)Y_{\ell_2 m_2}(\theta_2\phi_2)$$

$\phi(\vec{r}_1)$ is the wave function of incident particle 1 in the field of the origin, which has the following asymptotic behaviour

$$\phi(\vec{r}_1) \to \exp[ik_1 z_1 + i\alpha_1 \ln 2k_1(r_1 - z_1)] +$$

$$+ \frac{f_c(\theta_1)+f_N(\theta_1)}{r_1}\exp[i(k_1 r_1 - \alpha_1 \ln 2k_1 r_1)] \qquad (3.2)$$

$$\left(k_1 = \frac{2m_1 E_1}{h^2}, \quad \alpha_1 = \frac{Z_1 Z_2 e^2}{2m_1 v_1^2}\right).$$

In order to make the main text to be more readable, we collected all the details concerning expression (3.2) and the next ones in ***Appendix***. The functions $F_\mu(\vec{r}_1)$ satisfy the following system of the differential equations

$$\left[\Delta_1 + k_{1\nu}^2 - \frac{2m_1 V_1(r_1)}{h^2}\right]F_\nu(\vec{r}_1) = \frac{2m_1}{h^2}w_0(\vec{r}_1) +$$

$$+ \frac{2m_1}{h^2}\sum_{\mu\neq 0} w_{\nu\mu}(\vec{r}_1)F_\mu(\vec{r}_1) \qquad (3.3)$$

$$\left(k_{1\nu}^2 = \frac{2m_1}{h^2}(E_1 - E_\nu^{osc} + E_0^{osc}), \quad \nu = n_2\ell_2 m_2 ,\right.$$

$$E_\nu^{osc} = h\omega_2(2n_2 + \ell_2 + 3/2).$$

Neglecting (point c)) the sum in the r.h.s. of (3.3) and using the Green functions formalism, one easily finds F_μ:

$$F_\mu(\vec{r}_1) = \frac{2m_1}{h^2}\sum Y_{\ell m}(\vec{n}_1)\int G_\ell^\mu(r_1 r_1')w_{\ell m}^\mu(r_1')r_1'^2\,dr_1' . \qquad (3.4)$$

At last, finding the asymptotic value of (3.4) one easily gets the scattering amplitude and cross section for the excitation of the oscillator level with quantum numbers $n_2\ \ell_2\ m_2$

$$f_{n_2\ell_2 m_2}(\theta_1\phi_1) = \frac{2m_1}{h^2}\sqrt{\frac{2\ell_2+1}{4\pi}}\sum_{\ell\ell_1} Y_{\ell,-m_2}(\theta_1\phi_1) i^{\ell_1-\ell} \times$$

$$\times e^{i(\eta_\ell+\eta_{\ell_1})}\sqrt{2\ell+1}\,(2\ell_1+1)\begin{pmatrix}\ell_2 & \ell & \ell_1\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}\ell_2 & \ell & \ell_1\\ -m_2 & m_2 & 0\end{pmatrix} \times$$

$$\times C(\ell_1\ell\ ; n_2\ell_2); \qquad (3.5)$$

$$\sigma_{n_2\ell_2} = \frac{k_{1\nu}}{k_1}\sum_{m_2}\int|f_{n_2\ell_2 m_2}(\theta_1\phi_1)|^2 d\Omega_1 =$$

$$= \left(\frac{2m_1}{h^2}\right)^2 \frac{2\ell_2+1}{4\pi}\,\frac{k_{1\nu}}{k_1}\sum_{\ell\ell_1}(2\ell+1)(2\ell_1+1) \times$$

$$\times \begin{pmatrix}\ell_2 & \ell & \ell_1\\ 0 & 0 & 0\end{pmatrix}^2 |C(\ell_1\ell\ ; n_2\ell_2)|^2 .$$

The reaction cross section is defined in the usual way:

$$\sigma_R = \sum_{n_2\ell_2}\sigma_{n_2\ell_2} . \qquad (3.6)$$

4. Now about the compound nucleus cross section. Due to the excitation of the oscillator levels the kinetic energy is decreasing and may be below the Coulomb barrier. In this case the capture to the quasistationary levels inside this barrier is possible. Nevertheless, taking into account only the single-particle quasistationary levels does not give good results: the compound nucleus cross section after passing the maximal value decreases too rapidly as a function of the incident ion energy [11]. One may easily understand this if takes into consideration the fact that compound nucleus is essentially many-particle formation. So, it is not enough to treat only single-particle quasistationary levels. In refs. [12,13] we tried to simulate the many-particle nature of the compound nucleus with the use of the appropriate boundary condition. Here we adopt a different approach. In view of lack of the information about the nature of many-particle quasistationary states we postulate that each time as kinetic energy of the incident ion (due to the excitation of the oscillator levels) drops below the Coulomb barrier, the formation of the compound nucleus takes place with the probability equal to 1. After this logical jump we note that mathematically this is equivalent to summing up in (3.6) over the oscillator states within the energy interval:

$$E_1 - \frac{Z_1 Z_2 e^2}{a_1} \le E_{n_2\ell_2} \le E_1 \qquad (4.1)$$

(E_1 is the energy of the incident ion, $E_{n_2\ell_2}=(2n_2+\ell_2+3/2)\hbar\omega_2$). The compound nucleus cross section at first grows as a function of energy and after passing the maximum decreases. The first is due to the fact that cross section being equal to zero, if the kinetic energy is less than the energy of the first excited state, grows with increasing number of the excited states. On the other hand, for E_1 sufficiently large there is relatively small number of the excited states in the interval (4.1).

The r.h.s. of inequality (4.1) means that the kinetic energy loss must be less than the energy of the incident ion. Otherwise, the formation of the bound state takes place (force center + particle 1 + particle 2). But this is possible only for discrete values of the incident ion energy. We want to point out the relation of the given model to others (see, for example, review paper [14]) in which the capture to the potential pocket is due to the using of the classical equations of motion with a friction force. In the present quantum-mechanical model this capture is due to the excitation of the intrinsic degrees of motion.

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APPENDIX

For the sake of completeness we give here the normalized wave functions used in the text. The radial oscillator function

$$R^{osc}_{n_2 \ell_2} = \left[\frac{2\cdot\Gamma(n_2+\ell_2+3/2)}{n_2!\, a_{osc}^3}\right]^{1/2} \cdot \rho_2^{\ell_2} \cdot \exp(-\rho_2^2/2) L^{\ell_2+\frac{1}{2}}_{n_2}(\rho_2^2)$$

$$\rho_2 = \frac{r_2}{a_{osc}}, \quad a^2_{osc} = \frac{h}{\sqrt{k_2 m_2}}.$$

Function $\phi(\vec{r}_1)$ used in (3.2) is equal to

$$\phi(\vec{r}_1) = \frac{1}{r_1} \Sigma \phi_\ell(r_1) P_\ell(\cos\theta_1)(2\ell+1) i^\ell \cdot e^{i\eta_\ell}, \qquad (A.1)$$

where for the treated potential well $V_1(r_1)$

$$\phi_\ell(r_1) = \begin{cases} -\dfrac{j_\ell(k_{01}r_1)}{k_1 j_\ell(k_{01}a_1)\dot{f}^{(1)}_\ell(k_1a_1) - k_{01}j'_\ell(k_{01}a_1)f^{(1)}_\ell(k_1a_1)} & \text{for } r_1 < a_1 \\ \dfrac{i}{2k_1}[f^{(2)}_\ell(k_1r_1) - S_\ell(k_1,k_{01})f^{(1)}_\ell(k_1r_1)] & \text{for } r_1 > a_1. \end{cases} \qquad (A.2)$$

functions $j_\ell(x)$ equal:

$$j_\ell(x) = \sqrt{\frac{\pi x}{2}}\, J_{\ell+\frac{1}{2}}(x),$$

$f^{(1)}_\ell$, $f^{(2)}_\ell$ are combinations of the usual Coulomb functions:

$$f^{(1,2)}_\ell(x) = G_\ell(\eta, x) \pm i F_\ell(\eta, x).$$

The dot above the f_ℓ and prime near the j_ℓ mean the differentiation with respect to argument. The partial S-matrix in (A.2) is equal to:

$$S_\ell(k_1,k_{01}) = \frac{k_{01}f^{(2)}_\ell(k_1a_1)j'_\ell(k_{01}a_1) - k_1\dot{f}^{(2)}_\ell(k_1a_1)j_\ell(k_{01}a_1)}{k_{01}f^{(1)}_\ell(k_1a_1)j'_\ell(k_{01}a_1) - k_1\dot{f}^{(1)}_\ell(k_1a_1)j_\ell(k_{01}a_1)}$$

The wave numbers k_1, k_{01} are equal to

$$k_1 = \sqrt{\frac{2m_1}{h^2}E_1}, \qquad k_{01} = \sqrt{\frac{2m_1}{h^2}(E_1+V_{01})}$$

The asymptotic value of $\phi(r_1)$ is given by (2.2). The Coulomb and nuclear amplitudes which enter into (2.2) are equal to

$$f_c(\theta_1) = \frac{1}{2ik_1}\Sigma(2\ell+1)e^{2i\eta_\ell}P_\ell(\cos\theta) =$$

$$= \frac{Z_1Z_2e^2}{2m_1v_1^2}\,\frac{1}{\sin^2\theta_1}\exp[-i\alpha_1 \ln\sin^2\frac{\theta_1}{2} + i\pi + 2i\eta_0]$$

$$f_N(\theta_1) = \frac{1}{2ik_1}\Sigma(2\ell+1)e^{2i\eta_\ell}(S_\ell-1)P_\ell(\cos\theta_1).$$

Here η_ℓ is the Coulomb phase:

$$\eta_\ell = \arg\Gamma(\ell+1+i\alpha_1)$$

α_1 is Sommerfield's parameter:

$$\alpha_1 = \frac{Z_1Z_2e^2}{hv_1}.$$

The Green functions $G^\nu_\ell(r_1, r'_1)$ (see 2.4) are obtained using the method presented in [15]. The final formulas are:

$r, r' > a\ (\equiv a_1)$

$$G^\nu_\ell(r,r') = -\frac{i}{2k_\nu r r'}[f^{(2)}_\ell(k_\nu r_<) - S_\ell(k_{0\nu}, k_\nu)f^{(1)}_\ell(k_\nu r_<)]f^{(1)}_\ell(k_\nu r_>)$$

for $r, r' < a$.

$$G^\nu_\ell(r\,r') = \frac{1}{k_{0\nu}r\,r'}\, j_\ell(k_{0\nu}r_<)[n_\ell(k_{0\nu}r_>) -$$

$$- \frac{\dot{f}_\ell^{(1)}(k_\nu a)k_\nu n_\ell(k_{0\nu}a) - k_{0\nu}\dot{n}_\ell(k_{0\nu}a)f_\ell^{(1)}(k_\nu a)}{\dot{f}_\ell^{(1)}(k_\nu a)k_\nu j_\ell(k_{0\nu}a) - k_{0\nu}j_\ell'(k_{0\nu}a)f_\ell^{(1)}(k_\nu a)} \times$$

$$\times j_\ell(k_{0\nu}r_>)]$$

for $r < a < r'$ (or $r' < a < r$)

$$G_\ell^\nu(r\,r') = -\frac{j_\ell(k_{0\nu}r_<)}{rr'}f_\ell^{(1)}(k_\nu r_>)\times$$

$$\times \frac{1}{j_\ell'(k_{0\nu}a)f_\ell^{(1)}(k_\nu a)k_{0\nu} - k_\nu j_\ell(k_{0\nu}a)\dot{f}_\ell^{(1)}(k_\nu a)},$$

where as usual

$r_> = r$, $r_< = r'$ if $r > r'$

$r_> = r'$, $r_< = r$ if $r < r'$.

Function $n_\ell(x)$ may be expressed through the standard Neumann function

$$n_\ell(x) = \sqrt{\frac{\pi x}{2}}\,Y_{\ell+\frac{1}{2}}(x).$$

Functions w_ν, $w_{\nu\mu}$ (rel. (3.3)) are defined in the following way:

$$w_\nu(\vec{r}_1) = \phi(\vec{r}_1)\int \bar{u}_\nu(\vec{r}_2)V_{12}(|\vec{r}_1 - \vec{r}_2|)u_0(\vec{r}_2)d\vec{r}_2$$

$$w_{\nu\mu}(\vec{r}_1) = \int \bar{u}_\nu(\vec{r}_2)V_{12}(|\vec{r}_1 - \vec{r}_2|)u_\mu(\vec{r}_2)d\vec{r}_2 .$$

The functions $w^\nu_{\ell m}$ (see 3.4) are the radial components of w_ν:

$$w^\nu_{\ell m}(r_1) = \int \bar{Y}_{\ell m}(\vec{n}_1)w_\nu(\vec{r}_1)d\Omega_1 .$$

Note that symbol ν marks the oscillator basis ($\nu = n_2\ell_2 m_2$). As the motion always takes place in plane, the $w^{n_2\ell_2 m_2}_{\ell m}$ is different from zero only, when $m = -m_2$. This non-vanishing component is equal to:

$$w^{n_2\ell_2 m_2}_{\ell,-m_2} = \frac{1}{r_1}\sqrt{\frac{(2\ell_2+1)(2\ell+1)}{4\pi}}\,V_{n_2\ell_2}(r_1)\times$$

$$\times \sum_{\ell_1}\phi_{\ell_1}(r_1)(2\ell_1+1)i^{\ell_1}e^{i\eta_{\ell_1}}\begin{pmatrix}\ell_2 & \ell_1 & \ell\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}\ell_2 & \ell_1 & \ell\\ -m_2 & 0 & m_2\end{pmatrix}.$$

$V_{n_2\ell_2}$ is obtained by folding the two oscillator functions with a partial component of the potential

$$V_{n_2\ell_2}(r_1) = \int r_2^2 \cdot R^{osc}_{n_2\ell_2}(r_2)R_{00}(r_2)v^{\ell_2}_{12}(r_1, r_2)dr_2 ,$$

$$V_{12}(|\vec{r}_1 - \vec{r}_2|) = \sum v^\ell_{12}(r_1, r_2)P_\ell(\cos\theta_{12})\frac{2\ell+1}{4\pi}.$$

For the simplest potentials the $V_{n_2\ell_2}$ are obtained explicitly. For the delta potential:

$$V_{12} = V_0 \cdot \delta(\vec{r}_1 - \vec{r}_2)$$

one has:

$$V_{n_2\ell_2} = V_0\,R_{n_2\ell_2}(r_1)R_{00}(r_1).$$

For the square well potential:

$$V_{12} = \begin{cases} -V_0 & r_{12} < a_{12} \\ 0 & r_{12} > a_{12} \end{cases}$$

One easily finds

$$v_{12}^{\ell} = -\frac{4\pi}{2\ell+1} V_0 \delta_{\ell,0} \qquad \text{if} \quad r_1 + r_2 < a_{12}$$

$$v_{12}^{\ell} = 0 \qquad \text{if} \quad |\vec{r}_1 - \vec{r}_2| \geq a_{12}$$

$$v_{12}^{\ell} = \frac{2\pi V_0}{2\ell+1} \left[P_{\ell+1}\left(\frac{r_1^2 + r_2^2 - a_{12}^2}{2 r_1 r_2}\right) - P_{\ell-1}\left(\frac{r_1^2 + r_2^2 - a_{12}^2}{2 r_1 r_2}\right)\right]$$

if $|\vec{r}_1 - \vec{r}_2| < a_{12}$.

Finally, the coefficient $C(\ell_1 \ell ; n_2 \ell_2)$ (2.5) equals

$$C(\ell_1 \ell ; n_2 \ell_2) = -\frac{1}{j'_{\ell}(k_{0\nu}a) f_{\ell}^{(1)}(k_{\nu}a) k_{0\nu} - k_{\nu} j_{\ell}(k_{0\nu}a) \dot{f}_{\ell}^{(1)}(k_{\nu}a)} \times$$

$$\times \frac{1}{j'_{\ell}(k_{01}a) f_{\ell}^{(1)}(k_1 a) k_{01} - k_1 j_{\ell}(k_{01}a) \dot{f}_{\ell}^{(1)}(k_1 a)} \times$$

$$\times \int_0^a j_{\ell}(k_{0\nu} r) j_{\ell_1}(k_{01} r) V_{n_2 \ell_2}(r) dr +$$

$$+ \frac{1}{4 k_1 k_{\nu}} \int_a^{\infty} [f_{\ell}^{(2)}(k_{\nu} r) - S_{\ell}(k_{0\nu}, k_{\nu}) f_{\ell}^{(1)}(k_{\nu} r)] \times$$

$$\times [f_{\ell_1}^{(2)}(k_1 r) - S_{\ell_1}(k_{01}, k_1) f_{\ell_1}^{(1)}(k_1 r)] \times V_{n_2 \ell_2}(r) dr$$

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