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NUCLEAR FIELD THEORY OF FERMI SYSTEMS IN AN EXTERNAL FIELD

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Ядерная теория полей для систем взаимодействующих фермионов во внешнем поле

Ядерная теория полей развита для системы взаимодействующих фермионов во внешнем поле при помощи метода функционального интегрирования. Получено ядерно-полевое представление оператора внешнего поля и соответствующие диаграммные правила.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Reinhardt H. E4 - 11402

Nuclear Field Theory of Fermi Systems in an External Field

Nuclear field theory (NFT) is developed for interacting Fermi systems in an external field by using path integral techniques. The NFT representation of the external field operator together with the corresponding diagrammatic rules is strictly derived.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

Recently we have formulated a field theory of collective nuclear motion using path integral methods [1]. We have been able to give a strict foundation of the so-called "Nuclear Field Theory", (NFT) [1-4]. The NFT Lagrangian together with the corresponding graphical rules have been rigorously derived in a non-perturbation theoretical way. It is the aim of the present paper to extend the derivation of the NFT given in ref. [1] to interacting Fermi systems in an external field. In practical application of the NFT a correct treatment of an external field is needed, as from its action on the nucleus much information about the nuclear structure can be obtained.

In principle, the interaction of the system with an external field, which is given by a one-body operator

$$\hat{Q} = \sum_{\alpha\gamma} Q_{\alpha\gamma} a^{+}_{\alpha} a_{\gamma}, \tag{1}$$

would not require any special treatment within the nuclear field approach as we could always include the external field operator $\hat{Q}$ into the s.p. Hamiltonian. However, sometimes the physical situation itself suggests a separate treatment of the external field, e.g., of an "external" electromagnetic field. Then we have to find the correct representation of the external field operator [2-4] within the NFT. However, the nuclear field representation of such an operator has previously been derived only in a heuristic way [2-4]: to the full fermion operator $\hat{Q}$ one adds a "collective" part $\hat{Q}_c$

$$\hat{Q}_{nf} = \hat{Q} + \hat{Q}_c , \tag{2}$$

where $\hat{Q}_c$ is that part of Q which can be expressed by the collective modes, i.e., by the corresponding phonon operators. Obviously, by the introduction of the collective part $\hat{Q}_c$, the action of the external field is overestimated in $\hat{Q}_{nf}$. To remove this double counting one postulates the exclusion of definite diagrams (bubble graphs) involving the vertex of the external field operator $\hat{Q}$ from the diagrammatic nuclear field expansion. Clearly, such a "heuristic" formulation of the nuclear field treatment of external field operators, although it may prove correct, is not satisfactorily from the theoretical point of view. It is the aim of the present paper to give a rigorous derivation of the NFT representation of such (one-body) operators together with the graphical rules for the calculation of their matrix elements within the NFT.

2. *THE NUCLEAR FIELD TREATMENT OF FERMI SYSTEMS IN AN EXTERNAL FIELD*

2.1. *Derivation of the NFT Representation of the External Field Operator*

An interacting Fermi system in an external field (see eq. (1)) is described by a Lagrangian

$$\mathfrak{L}^{Q} = \mathfrak{L} + \hat{Q}, \tag{3}$$

where

$$\mathfrak{L}(t) = \sum_{\alpha\gamma} a^{+}_{\alpha}(t)(i\partial_t \delta_{\alpha\gamma} - e_{\alpha\gamma})a_{\gamma}(t) - \frac{1}{2}\sum_{\alpha\beta\gamma\delta} V_{\alpha\gamma,\beta\delta}\, a^{+}_{\alpha}(t)a^{+}_{\beta}(t)a_{\delta}(t)a_{\gamma}(t) \tag{4}$$

is the Lagrangian in the absence of the external field.

The Green functions of the nucleons in the external field $\hat{Q}$ are obtained from the generating functional

$$Z[\eta,\eta^{+},Q] = N\int da\,da^{+} \exp[\,i\int dt\{\mathfrak{L}^{Q}(t) + \sum_{\alpha}(\eta^{+}_{\alpha}(t)a_{\alpha}(t) + a^{+}_{\alpha}(t)\eta_{\alpha}(t))\}] \tag{5}$$

by functional differentiation with respect to the fermion sources η,η^{+}. Linearizing the residual interaction V by means of a hermitian Bose field Φ (see ref./1/) the generating functional can be written as *

$$Z[\eta,\eta^{+},Q] = N\int da\,da^{+}d\Phi\, e^{\,i\{\mathfrak{L}^{Q}_{c} + \eta^{+}a + a^{+}\eta\}} \tag{6}$$

with the new effective Lagrangian

$$\mathfrak{L}^{Q}_{c}(t) = \sum a^{+}_{\alpha}(t)(i\partial_t\delta_{\alpha\gamma} - e_{\alpha\gamma} + \Phi_{\alpha\gamma}(t) + Q_{\alpha\gamma})a_{\gamma}(t) + \frac{1}{2}\sum_{\alpha\gamma\beta\delta}\Phi_{\alpha\gamma}(t)(V^{-1})_{\alpha\gamma,\beta\delta}\Phi_{\beta\delta}(t). \tag{7}$$

We could now absorb the "source" Q into redefined s.p. matrix elements $e'_{\alpha\gamma} = e_{\alpha\gamma} + Q_{\alpha\gamma}$. However, as mentioned in the introduction it is sometimes more con-

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*In this paper we use the same conventions adopted in ref. /1/. If confusion is excluded, we drop the fermion indices α,β,γ, etc., which have to be summed over. Further the matrix multiplication is understood in the functional sense which implies integration over intermediate times. In this convention we have $V(t,t') = V\delta(t,t')$. In addition, if in an exponent a time variable is not explicitly indicated it has to be integrated over, e.g., $\exp(i\mathfrak{L}) \equiv \exp(i\int\mathfrak{L}(t)dt)$.

venient to treat the external field separately. For this purpose we eliminate the source Q from the s.p. Lagrangian by Changing the integration variable Φ to

$$\Phi \to \Phi + Q. \tag{8}$$

Defining the s.p. Green function in the collective field Φ via

$$G^{-1}_{\alpha\gamma}[\Phi](t,t') = (i\partial_t \delta_{\alpha\gamma} - e_{\alpha\gamma} + \Phi_{\alpha\gamma}(t))\delta(t,t') \tag{9}$$

the integration over the fermion variables yields

$$Z[\eta,\eta^+,Q] = N\int d\Phi \exp\{i(S[\Phi] + + \frac{1}{2}QV^{-1}Q - QV^{-1}\Phi - \eta^+ G[\Phi]\eta)\}, \tag{10}$$

where the collective action in the absence of the external field Q, $S[\Phi]$, is given by

$$S[\Phi] = \frac{1}{2}\int \Phi(t)V^{-1}\Phi(t)dt - i\,\mathrm{tr}(\log G^{-1}[\Phi]). \tag{11}$$

As is explicitly shown in ref. /1/ from the least action principle $\delta S[\Phi]/\delta\Phi = 0$ one finds the equation of motion of the collective field $\Phi(t)$, which in the static approximation $\Phi(t) \to \Phi^\circ$ reduces to the Hartree-Fock (HF) equation. At first sight it may seem that due to the change in the integration variable Φ, eq. (8), the external field Q contributes also to the HF self-consistent field $e_{\alpha\gamma} + \Phi^\circ_{\alpha\gamma}$. However, the resulting HF equation is the same one as in the absence of the external field (see eq. (2.10) of ref. /1/). Therefore, in the present approach the external field does not contribute to the HF s.p. energies.

In order ro introduce the collective degrees of freedom one expands the collective action $S[\Phi]$ (eq. (11)) around the static solution $\Phi = \Phi^\circ$. In second order this yields the familiar RPA equations. Then, finally the generating functional (10) can exactly be transformed into

the generating functional of the NFT (for processes connecting intermediate states, see eq. (3.16) of ref. /1/).

$$Z[\eta,\eta^+,Q] = N\int d\phi\, dc\, dc^+ e^{-i\sum_{n=1}^{2} L_n[\phi - Vc^+c + Q]} \times$$

$$\times \exp[i\{(\mathcal{L}_{nf} - QV^{-1}\phi + Qc^+c) + \eta^+c + c^+\eta\}], \tag{12}$$

where $\mathcal{L}_{nf}$ is the NFT Lagrangian in the absence of the external field Q. Further ϕ denotes the field of the RPA phonons and the Grassman variables c^+, c correspond to the fermion operators in the Hartree-Fock s.p. basis. The quantity

$$L_n[\Phi'] = -i\,\mathrm{tr}\{\frac{(-)^{n+1}}{n}(G^\circ\Phi')^n\} \tag{13}$$

gives the n-th order fermion loop, where $G^\circ = G[\Phi^\circ]$ is the s.p. Green function in the Hartree-Fock approximation.

From eq. (12) we can now immediately read off the NFT representation of the external field operator $\hat{Q}$. Comparing eqs. (5) and (12) we find the NFT Lagrangian in the presence of an external field Q to be

$$\mathcal{L}^Q_{nf} = \mathcal{L}_{nf} - QV^{-1}\phi + Qc^+c, \tag{14}$$

where $\mathcal{L}_{nf}$ is the usual NFT Lagrangian (in the absence of the external field) and

$$\hat{Q}_{nf} = Qc^+c - QV^{-1}\phi \tag{15}$$

is nothing else but the external field operator in the NFT representation (see *fig. 1*). We observe, within the NFT an external field is represented by the full one-body operator of the fermion treatment (see eq. (1)) in the Hartree-Fock basis $\{\lambda,\mu,\nu,\ldots\}$

$$\hat{Q} = Qc^+c = \sum_{\lambda\mu} Q_{\lambda\mu} c^+_\lambda c_\mu \tag{16}$$

plus a collective part

$$\hat{Q}_c = -QV^{-1}\phi \tag{17}$$

acting on the phonon field ϕ. After quantization of the field ϕ (see eq. (3.10) of ref. /1/) the collective part can be expressed by the RPA phonon operators B^+_n, B_n as

$$\hat{Q}_c = \Sigma (e_n B_n + e^*_n B^+_n), \tag{18}$$

where the so-called phonon charges are given by

$$e_n = \sum_{\lambda\mu} Q_{\lambda\mu} \chi^n_{\lambda\mu}$$

with $\chi^n_{\lambda\mu}$ being the usual RPA amplitudes defined in eq. (3.13) of ref. /1/.

The functional approach to collective nuclear motion developed in ref. /1/, yields not only the NFT represen-

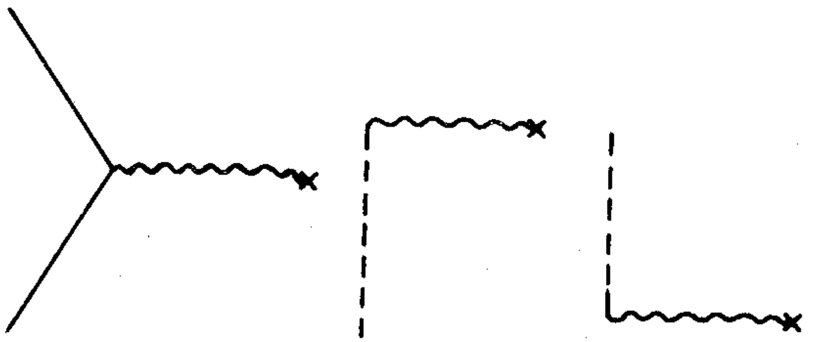

Fig. 1. Diagrammatic representation of the external field operator $\hat{Q}$ *denoted by a wavy line with an asterisk in the NFT (see eq. (15)). A full line represents a fermion and a dashed line a phonon. A fat dot stands for the two-body interaction.*

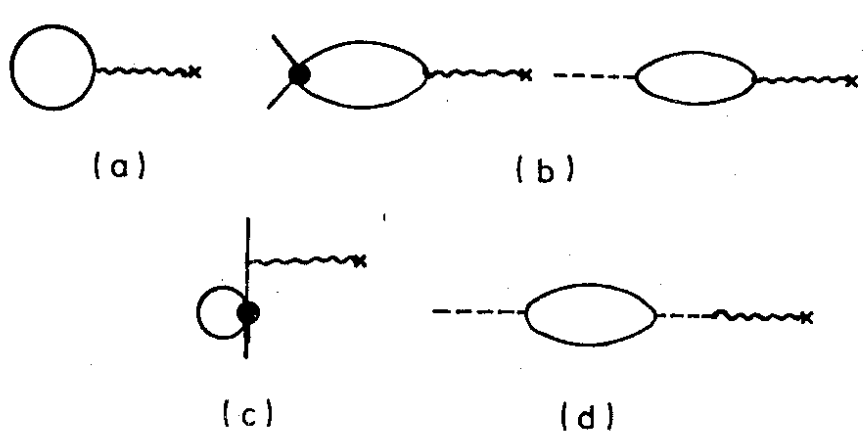

Fig. 2. Diagrams of the external field operator $\hat{Q}$ *with a first or second order closed fermion loop. Such diagrams are excluded from the nuclear field expansion. More precisely: in the generating functional (20) first and second order fermion loops involving the external field operator* $\hat{Q}$, *shown in (a) and (b), are eliminated by the linear term* $(-L_1 - \tilde{L}_2)$, *(c) and (d) shows diagrams with a usual HF insertion and a bubble graph, respectively, which are projected out from the NFT expansion by the exponent* $\exp[-i(L_1 + L_2)]$ *in eq. (20).*

tation of the operators but simultaneously the corresponding diagrammatic rules for its treatment within the NFT: the first exponent in eq. (12), $\exp[-iL_1 - iL_2]$, which projects out from the perturbation expansion the first order and second order closed fermion loops, contains in the argument of L_1 and L_2 also the external field Q. Therefore, not only the ordinary Hartree-Fock insertions and bubble graphs are excluded but in addition, no first order or second order closed fermion loop involving the vertex of the external field operator $\hat{Q}$ appears in the diagrammatic nuclear field treatment (see *fig. 2*). The first order loop with the vertex of $\hat{Q}$ (see *fig. 2a*) contributes to the ground state energy, that is in irrelevant constant in field theory. The second order loops (*fig. 2b*) are eliminated because they have already been included in the collective part of the external field operator, $\hat{Q}_c$.

2.2. *Calculation of the Matrix Elements of the External Field Operator in the NFT*

In the practical application of the NFT we are interested in the matrix elements of the external field operator $\hat{Q}$ (having in mind, e.g., the electromagnetic field) between the nuclear states. These can be obtained from the generating functional $Z[\eta,\eta^+, jQ]$ defined by eq. (12) by differentiating with respect to the auxiliary variable $j(t)$ and equating then $j(t)$ to zero

$$Z_{Q(t)}[\eta,\eta^+] = \left[\frac{\delta Z[\eta,\eta^+, jQ]}{i\delta j(t)}\right]_{j=0} . \qquad (19)$$

After introducing in view of later application also external phonon sources q, q^+ this yields $(\phi = \phi(B^+,B))$

$$Z_{Q(t)}[\eta,\eta^+,q,q^+] = \int d\phi\, dc\, dc^+ \{\hat{Q}_{nf} - L_1[Q] - \tilde{L}_2[Q,\phi - Vc^+c]\} e^{-i\sum_{n=1}^{2} L_n[\phi - Vc^+c]} \times$$

$$\times\, e^{i\{\mathcal{L}_{nf} + \eta^+ c + c^+\eta + q^+ B + B^+ q\}} , \qquad (20)$$

where the term

$$\tilde{L}_2[Q,\phi - Vc^+c] = \left(\frac{\delta}{\delta j(t)} L_2[\phi - Vc^+c + jQ]\right)_{j=0} =$$

$$= i\int dt' G^{\circ}(t't) Q(t) G^{\circ}(t,t')[\phi(t') - Vc^+(t)c(t)]$$

eliminates the bubble diagrams involving the vertex of the operator Q (see *fig. 2b*) while $L_1[Q]$ removes the contractions of the external field operator Q (see *fig. 2a*). The remaining part of the generating functional (20) coincides with that for the NFT Green functions in the absence of the external field (see ref. /1/). Now from the generating functional (20) the needed matrix elements of the external field operator $\hat{Q}$ between the nuclear states can be immediately calculated by differentiating with respect to the external sources η, η^+, q, q^+. For the sake of illustration let Q denote the electric quadrupole operator. Then, e.g., the quadrupole moment in the one-phonon state $|n\rangle$ is given by

$$\langle n|\hat{Q}|n\rangle = \left[\frac{\delta Z[0,0,q,q^+]}{i\delta q_n^+(t_2)\, i\delta q_n(t_1)}\right]_{\substack{q = q^+ = 0,\\ t = 0,\\ t_1 \to -\infty,\ t_2 \to \infty}}$$

Analogously other required matrix elements of the operator $\hat{Q}$ are calculated from eq. (20).

The generating functional for the matrix elements of the external field operator Q, eq. (20), together with the generating functional for the NFT Green functions defined by eq. (4.7) of ref. /1/ plays the central role in the nuclear field theory. From these functionals all the quantities characterizing the nuclear structure as, e.g., excitation energies, static moments, transition probabilities, can be calculated in the framework of the NFT in a unique way. In so doing we need not worry about the specific diagrammatic rules of the nuclear field treatment (exclusion of bubble diagrams, etc., see refs. /1-4/). Even these rules are already involved in the generating functionals.

3. *SUMMARY AND CONCLUSIONS*

By extending the field theory of collective nuclear excitations developed in ref. /1/ to interacting Fermi systems in an external field we have given a rigorous, nonperturbation theoretical derivation of the NFT representation of the external field operators (eq. (15)). Further, for the calculation of the corresponding matrix elements of such operators in the framework of the nuclear field theory, we have derived a generating functional (eq. (20)) which contains all the specific NFT diagram rules.

The present paper completes the foundation of the nuclear field theory. The NFT Lagrangian (in ref. /1/) as well as the NFT representation of an external field operator (in the present paper) have now been strictly derived in a throuhout field theoretical approach together with the corresponding rules of the diagrammatic nuclear field treatment.

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