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EXPANSION OF CONTINUUM FUNCTIONS ON RESONANCE WAVE FUNCTIONS AND AMPLITUDES. II.

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Разложение функций континуума по резонансным волновым функциям и амплитудам. II.

В качестве иллюстрации общих доказательств сходимости полюсных разложений (согласно теореме Миттаг-Леффлера) волновых функций, амплитуд рассеяния и функций Грина при положительных энергиях рассмотрены примеры дельта-потенциала и прямоугольной ямы. Численные результаты представлены в виде таблиц и графиков.

Работа выполнена в Лаборатории теоретической физики, ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1978

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Expansion of Continuum Functions on Resonance Wave Functions and Amplitudes. II.

The pole expansion (Mittag-Leffler expansion) of wave functions, scattering amplitudes and Green's functions at positive energies are discussed in a mathematically rigorous way. The general proofs of convergence are supplemented by numerical calculations, which, for simple examples, show the convergence to be fast. Applications of the method to nuclear structure calculations are discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1978

Introduction

In a recent publication /1/, henceforth reffered to as I, the Mittag-Leffler expansions (MLE) of wave functions, scattering amplitudes and Green's functions at positive energies are discussed in a mathematically rigorous way. In this paper, as an illustration of the obtained general results, some numerical investigations are discussed for two simple examples; δ -function potential and rectangular well potential, which were considered in detail by several authors (see, e.g., ref. /2,3,4,5/). It should be noticed that numerical investigations of the convergence of the MLE of the S-matrix are nearly absent in the literature (see, though, the article by Weidenmüller /6/) and for the wave functions, such investigations were never carried out.

We give here (IV) some formulae and numerical results which pertain to the above-mentioned potentials, restricting ourselves to the S-states (i.e. $\ell=0$).

In (Y) some conclusions are discussed.

IV. Numerical results

a) Potential $V(r) = V_0\, \delta(r-R)$

The exact expression for the S-matrix and wave function of the continuous spectrum for $r \leq R$ are

$$S(k) = \frac{1 + V_0(1 - e^{-2ikR})/2ik}{1 + V_0(e^{2ikR} - 1)/2ik}, \tag{4.1}$$

$$\Psi^G(k,r) = \frac{e^{ikR}}{k} \frac{\sin kr}{1 + V_0(e^{2ikR} - 1)/2ik}. \tag{4.2}$$

For the resonance wave function we have for $r \le R$

$$\varphi_n(r) = \left[\frac{2(V_0 - 2ik_n)}{1 + R(V_0 - 2ik_n)}\right]^{1/2} \sin k_n r \tag{4.3}$$

with k_n determined by

$$1 + V_0(e^{2ik_nR} - 1)/2ik_n = 0. \tag{4.3a}$$

The expansion of the S-matrix is

$$S^{(P)}(k) = e^{-2ikR}\left[S_R(0) + kS_R'(0) + \dots \frac{k^P}{P!}S_R^{(P)}(0) + \right.$$

$$\left. + \sum_{n=1}^{\infty}\left(\frac{k}{k_n}\right)^{P+1} \frac{r_n}{k - k_n}\right] \tag{4.4}$$

$$r_n = \frac{2k_n^2}{iV_0[1 + R(V_0 - 2ik_n)]} \tag{4.4a}$$

$$S_R(0) = 1, \quad S_R'(0) = \frac{2iR}{1 + V_0R}$$

$$S_R''(0) = -\frac{4R^2}{(1 + V_0R)^2} \tag{4.4b}$$

$$S_R'''(0) = -\frac{4iR^3(2 - V_0R)}{(1 + V_0R)^3}$$

and for the wave function, we have the expansions

$$\Psi^G(k,r)^P = \frac{r}{1 + V_0R} - \frac{1}{2}\sum_{n=1}^{\infty}\frac{k}{k_n}\frac{\varphi_n(r)\varphi_n(R)}{k_n(k - k_n)}, \text{ for } p = 0, \tag{4.5a}$$

$$\Psi^G(k,r)^P = \frac{r}{1 + V_0R} + \frac{ikrR}{(1 + V_0R)^2} - \frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{k}{k_n}\right)^2\frac{\varphi_n(r)\varphi_n(R)}{k_n(k - k_n)}, \text{ for } p = 1. \tag{4.5b}$$

So we can compare the exact expressions for the S-matrix and the wave functions with the corresponding MLE. In Table 1 the values of k_n are given for $V_0=40\,fm^{-1}$ and $-40\,fm^{-1}$ and in Table 2a and 2b the values of the exact S-matrix and the S-matrix, obtained from the MLE for $p=1,3$ with $n \le 1$ and $n \le 7$ ($V_0=40\,fm^{-1}$). Figure 1 shows the value of $\left|\frac{S^{exact} - S^P}{S^{exact}}\right|$ for $p=1,2,3$ and $n \le 7$ (dotted curve) and $n \le 1$ (full drawn). Tables 3a, 3b and 3c show the exact wave function and the MLE approximation for the wave functions

with $p=0,1$ and $n \le 1$, $n \le 7$ and $n \le 22$, for $V_0=40\,fm^{-1}$. In tables 4a, 4b and 4c the corresponding results for $V_0=-40\,fm^{-1}$ are shown.

b) Rectangular well

So, $V(r) = -V_0$ for $r \le R$, and $V(r) = 0$, for $r > R$. The exact expression for S-matrix and wave function for $r \le R$ are

$$S(k) = \frac{\varkappa\cos\varkappa R + ik\sin\varkappa R}{\varkappa\cos\varkappa R - ik\sin\varkappa R}\, e^{-2ikR}, \tag{4.6}$$

$$\Psi^G(k,r) = \frac{\sin\varkappa r}{\varkappa\cos\varkappa R - ik\sin\varkappa R}, \tag{4.7}$$

where

$$k^2 = \frac{2m}{\hbar^2}E, \quad \varkappa_0^2 = \frac{2m}{\hbar^2}V_0 \quad \text{and} \quad \varkappa^2 = k^2 + \varkappa_0^2.$$

For resonance wave function we have for $r \le R$

$$\varphi_n(r) = \sqrt{\frac{2k_n}{i + k_nR}}\,\sin\varkappa_n r \tag{4.8}$$

with k_n determined by

$$\varkappa\,\mathrm{ctg}\,\varkappa R = ik. \tag{4.8a}$$

The MLE of the S-matrix is

$$S^P(k) = e^{-2ikR}\left[S_R(0) + kS_R'(0) + \dots + \frac{k^P}{P!}S_R^{(P)}(0) - \right.$$

$$\left. - i\sum_n\left(\frac{k}{k_n}\right)^{P+1}\frac{\varphi_n^2(R)}{k - k_n}\right] \tag{4.9}$$

$$S_R(0) = 1, \quad S_R'(0) = \frac{2i}{\varkappa_0}\,\mathrm{tg}\,\varkappa_0 R, \quad S_R''(0) = -\frac{4}{\varkappa_0^2}\,\mathrm{tg}^2\varkappa_0 R$$

$$S_R'''(0) = \frac{6i}{\varkappa_0^3}(\varkappa_0 R - \mathrm{tg}\,\varkappa_0 R)(1 + \mathrm{tg}^2\varkappa_0 R) - \frac{6i}{\varkappa_0^3}\,\mathrm{tg}^3\varkappa_0 R \tag{4.9a}$$

and for the wave function we have the expansion

$$\Psi^G(k,r)^P = \Psi^G(0,r) + k\Psi^{G\prime}(0,r) + \dots + \frac{k^P}{P!}\Psi^{G(P)}(0,r) -$$

$$- \frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{k}{k_n}\right)^{P+1}\frac{\varphi_n(R)\varphi_n(r)}{k_n(k - k_n)} \tag{4.10}$$

$$\Psi^G(0,r) = \frac{1}{\varkappa_0}\frac{\sin\varkappa_0 r}{\cos\varkappa_0 R}$$

$$\Psi^{G\prime}(0,r) = \frac{i}{\varkappa_0^2}\frac{\sin\varkappa_0 r\,\sin\varkappa_0 R}{\cos^2\varkappa_0 R} \tag{4.10a}$$

$$\Psi^{G\prime\prime}(0,r) = \frac{r\cos\varkappa_0 r}{\varkappa_0^2\cos\varkappa_0 R} - \frac{1}{\varkappa_0^3}\frac{\sin\varkappa_0 r}{\cos^3\varkappa_0 R}\left[1 - \varkappa_0 R\sin\varkappa_0 R\cos\varkappa_0 R + \sin^2\varkappa_0 R\right]$$

There is interesting relation

$$\Psi^{G}(\kappa,R)^{p} = \frac{S^{p+1}(\kappa)e^{2i\kappa R}-1}{2i\kappa} . \qquad (4.11)$$

In Tables 5 the values of k_n are given for V_0=48.72 Mev, β_0=4.6 and β_0=5.0 (where $\beta_0=\varkappa_0 R$). In Tables 7 the values of the exact S-matrix and the MLE of one are given for p =2,3 with $n \leq$ 1,7 and 22 (β_0 =4.6). Figures 2 show the values $\left| \frac{S^{exact}-S^{p}}{S^{exact}} \right|$ for p =2 (Fig.2a) and p =3 (Fig.2b), β_0=4.6. Figure 3 shows the same values for β_0 =5.0, p =2 (dotted curve) and p =3 (full drawn). Tables 6 show the exact wave function and its expansion with p =1,2 and $n \leq$ 1,7,22, for β_0 =4.6. In tables 8 the corresponding results with $n \leq 4$, for β_0=5.0 are shown.

From the tables and figures it is seen, that the convergence is strongly dependent on p and becomes faster for larger p-values. This is the advantage of the MLE , as p can be choosen so as to get a fast convergence. For these tables, it is seen, that for $n \leq 22$, the MLE expression for the wave function is correct with 3-4 digits for κ between 0 and 3 fm^{-1}, and in some cases for larger κ . A similar good convergence is seen for the Green functions.

It follows from the general relations given in I that the MLE of the wave function convergence for $0 \leq r < R$ however, it,is seen that the convergence is good also for $r = R$. If this point is regarded as the limit $r \to R-0$, this is not surprising.

V. Conclusions

Here we have shown, that the MLE method can be used to calculate simple, analytic expressions in κ for S-matrix, scattering states and Green's functions of the problem of one particle in a potential of a sort which includes, with reasonable accuracy the usual nuclear potentials.

It should be streessed, that although the expansion

$$\Psi = \sum_n \frac{c_n \varphi_n(r)}{k - k_n}$$

is valid only for $r \leq R$, an equally exact expression for Ψ^{+} is obtained for $r > R$, using the MLE of the S-matrix. These expansions converge uniformly in the whole κ -space, except at the poles, and in practice, the convergence can be improved by chosing the constant p sufficiently large. In the case, that

only one pole needs to be taken into account in the expansion of Ψ^{+} , we obtain the factorised expression $\Psi^{+} \approx \varphi_n(r) F(\kappa)$, which was earlier used by several authors in nuclear reaction calculations /7/.

As was shown, by some examples in I, the MLE highly facilitates the inclusion of continuum states in nuclear structure calculations, e.g.,of the shell model type. The same will be the cast with other nuclear structure calculations, e.g.,with RPA methods. However, the non-orthogonality of the wave functions means that some care is needed, when they are used in such problems.

Other discrete basis sets can be used in such calculations, e.g.,the above mentioned functions of Kapur and Peirls,etc., or the harmonic oscillator functions. Nevertheless, the expansion in resonance states has the advantage of giving a much simpler expression for $G^{+}(k)$ in the complex k-plane, thus facilitating the calculation of matrices like (3.18). Also, the particle emission properties of mixed states are in such a calculation determined in a simple way through S(k).

Such structure calculation will be the subject of a later publication. In conclusion, the authors want to thank collaborators of JINR, Dubna and the Niels Bohr Institute, Copenhagen, particularly B.Nilsson, for enlightening discussions.

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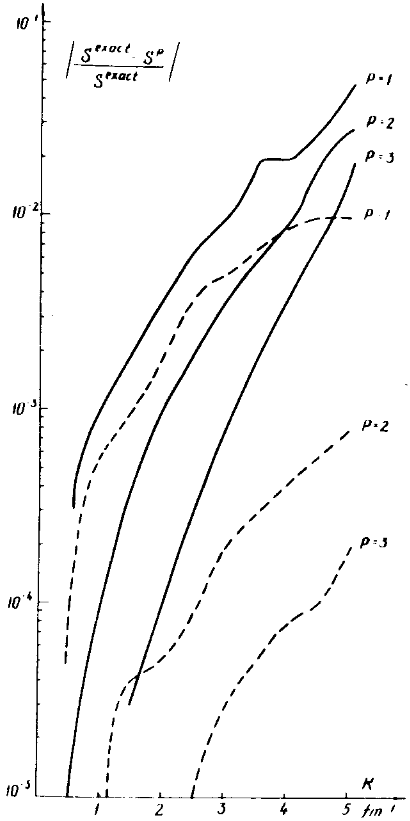

Fig.1. The value of $\left|\frac{S^{exact}-S^{p}}{S^{exact}}\right|$ for p=1,2,3 and n ≤7 (dotted curve) and n ≤1 (full drawn) for δ-function potential (V_o=40 fm^{-1}, R=1 fm).

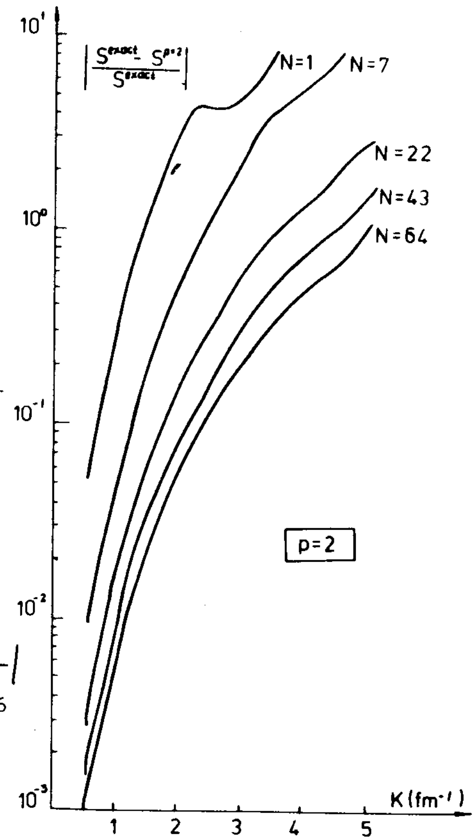

Fig.2a.The value of $\left|\frac{S^{exact}-S^{p}}{S^{exact}}\right|$ for rectangular well with β_0=4.6 for p=2 expansion.

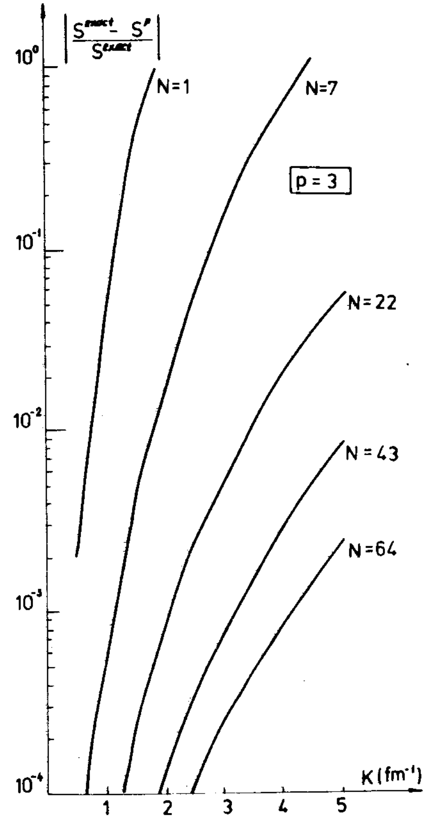

Fig.2b. The same, for p=3.

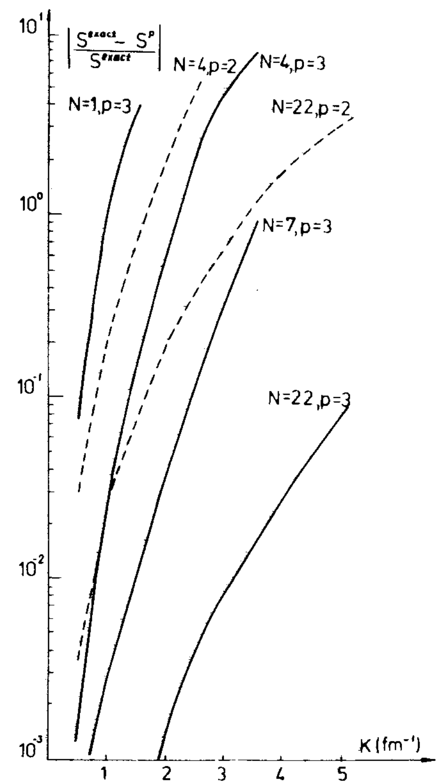

Fig.3. The value of $\left|\frac{S^{exact}-S^{p}}{S^{exact}}\right|$ for rectangular well with β_0=5.0 for p=2 (dotted curve) and p=3 (full drawn).

Table 1

Position of poles, $V = V_o \delta(r - R)$

n	V_o = +40., R =I		V_o =-40., R=I	
	Re(K_n)	Im(K_n)	Re(K_n)	Im(K_n)
I	.3066E+0I	-.5667E-02	0.	.2000E+02
2	.6I34E+0I	-.2I97E-0I	.322IE+0I	-.6563E-02
3	.9209E+0I	-.4709E-0I	.6439E+0I	-.2523E-02
4	.I229E+02	-.7866E-0I	.9649E+0I	-.5340E-0I
5	.I538E+02	-.II43E+00	.I285E+02	-.8796E-0I
6	.I847E+02	-.I522E+00	.I604E+02	-.I26IE+00
7	.2I58E+02	-.I908E+00	.I923E+02	-.I658E+00
8	.2468E+02	-.229IE+00	.224IE+02	-.2056E+00
9	.2780E+02	-.2666E+00	.2558E+02	-.2447E+00
I0	.309IE+02	-.3029E+00	.2875E+02	-.2825E+00
II	.3403E+02	-.3378E+00	.3I92E+02	-.3I88E+00
I2	.37I6E+02	-.37I2E+00	.3808E+02	-.3535E+00
I3	.4028E+02	-.4032E+00	.3824E+02	-.3866E+00
I4	.434IE+02	-.4337E+00	.4I40E+02	-.4I82E+00
I5	.4654E+02	-.4628E+00	.4455E+02	-.4483E+00
I6	.4967E+02	-.4907E+00	.477IE+02	-.4769E+00
I7	.5280E+02	-.5I73E+00	.5086E+02	-.5043E+00
I8	.5593E+02	-.5428E+00	.540IE+02	-.5305E+00
I9	.5906E+02	-.567IE+00	.57I6E+02	-.5555E+00
20	.6220E+02	-.5905E+00	.603IE+02	-.5794E+00
2I	.6533E+02	-.6I30E+00	.6346E+02	-.6024E+00
22	.6847E+02	-.6346E+00	.666IE+02	-.6244E+00

Table 2a

k	exact		p = 1		p = 3	
	Re(S)	Im(S)	Re(S)	Im(S)	Re(S)	Im(S)
I.00	-.3709	-.9287	-.37I5	-.9296	-.3709	-.9287
3.00	.8270	.5623	.8378	.56I6	.8277	.56I8
5.00	-.9502	.3II8	-.9628	.3563	-.9480	.3298
3.06	.22I4	.9752	.2325	.973I	.222I	.9746
3.07	-.9562	-.2926	-.945I	-.2949	-.9555	-.2932
3.08	.729I	-.6844	.7404	-.6869	.7299	-.6850
3.09	.9570	-.2900	.9682	-.2929	.9577	-.2907
3.I0	.9909	-.I345	I.0022	-.I376	.99I6	-.I352
3.II	.9980	-.0625	I.0093	-.0658	.9988	-.063I
3.I2	.9997	-.0262	I.0I09	-.0298	I.0004	-.0269
3.I3	I.0000	-.0083	I.0II2	-.0I22	I.0007	-.009I
3.I4	I.0000	-.00I0	I.0II3	-.0052	I.0007	-.0I8
3.I5	I.0000	-.0004	I.0II3	-.0048	I.0007	-.0I2
3.I6	I.0000	-.0042	I.0II2	-.0089	I.0007	-.0050
3.I7	.9999	-.0III	I.0II2	-.0I6I	I.0006	-.0I20

S-matrix expansion for different k, retaining only the first pole.

Table 2b

k	exact		p = 1		p = 3	
	Re(S)	Im(S)	Re(S)	Im(S)	Re(S)	Im(S)
I.00	-.3709	-.9287	-.37I2	-.9292	-.3709	-.9287
3.00	.8270	.5623	.83I9	.5636	.8270	.5623
5.00	-.9502	.3II8	-.96I9	.3203	-.9503	.3II9
3.06	.22I4	.9752	.2267	.9759	.22I4	.9752
3.07	.9562	-.2926	-.9509	-.29I9	-.9562	-.2926
3.08	.729I	-.6844	.7345	-.6838	.7292	-.6844
3.09	.9570	-.2900	.9625	-.2896	.9570	-.2900
3.I0	.9909	-.I345	.9964	-.I342	.9909	-.I345
3.II	.9980	-.0625	I.0036	-.0622	.998I	-.0625
3.I2	.9997	-.0262	I.0052	-.026I	.9997	-.0262
3.I3	I.0000	-.0083	I.0056	-.0083	I.0000	-.0083
3.I4	I.0000	-.00I0	I.0056	-.00II	I.0000	-.00I0
3.I5	I.0000	-.0004	I.0057	-.0006	I.0000	-.0004
3.I6	I.0000	-.0042	I.0057	-.0045	I.0000	-.0042
3.I7	.9999	-.0III	I.0057	-.0II6	I.0000	-.0III

S-matrix expansion for different k with the first seven poles in the expansion

Table 3a

		exact		p = 0		p = 1	
k	r	Re(Ψ)	Im(Ψ)	Re(Ψ)	Im(Ψ)	Re(Ψ)	Im(Ψ)
I.0	.2	.5806E-02	.I428E-03	.5960E-02	.2490E-03	.5906E-02	.I468E-03
	.4	.II38E-0I	.2800E-03	.II53E-0I	.4II6E-03	.II53E-0I	.2839E-03
	.6	.I650E-0I	.4060E-03	.I645E-0I	.43I3E-03	.I645E-0I	.4050E-03
	.8	.2096E-0I	.5I58E-03	.207IE-0I	.30I2E-03	.207IE-0I	.5094E-03
	I.0	.2459E-0I	.6050E-03	.2453E-0I	.66I5E-04	.2453E-0I	.602IE-03
3.0	.2	.2059E+00	.3259E-0I	.2078E+00	.3304E-0I	.2078E+00	.3274E-0I
	.4	.3399E+00	.5380E-0I	.34I6E+00	.5432E-0I	.34I6E+00	.5394E-0I
	.6	.3552E+00	.5622E-0I	.3545E+00	.5625E-0I	.3545E+00	.56I7E-0I
	.8	.2464E+00	.3899E-0I	.2435E+00	.38I5E-0I	.2435E+00	.3877E-0I
	I.0	.5I47E-0I	.8I46E-02	.5086E-0I	.6444E-02	.5086E-0I	.8052E-02
5.0	.2	-.2240E-0I	-.2908E-02	-.9682E-02	-.6I32E-03	-.9682E-02	-.II24E-02
	.4	-.242IE-0I	-.3I42E-02	-.I406E-0I	-.I0I5E-02	-.I406E-0I	-.I653E-02
	.6	-.3757E-02	-.4877E-03	-.9758E-02	-.I066E-02	-.9758E-02	-.II97E-02
	.8	.20I5E-0I	.26I5E-02	.3430E-02	-.7478E-03	.3430E-02	.293IE-03
	I.0	.2553E-0I	.33I4E-02	.2248E-0I	-.I705E-03	.2248E-0I	.2509E-02

Here E+02 10^2 etc.

Wave functions expanded for $r \leqslant R$ and different k-values retaining only the first pole in the expansion.

Table 3b

		exact		p = 0		p = 1	
k	r	Re(Ψ)	Im(Ψ)	Re(Ψ)	Im(Ψ)	Re(Ψ)	Im(Ψ)
I.0	.2	.5806E-02	.I428E-03	.5804E-02	.I264E-03	.5804E-02	.I428E-03
	.4	.II38E-0I	.2800E-03	.II38E-0I	.2853E-03	.II38E-0I	.2800E-03
	.6	.I650E-0I	.4060E-03	.I650E-0I	.4308E-03	.I650E-0I	.4060E-03
	.8	.2096E-0I	.5I58E-03	.2096E-0I	.496IE-03	.2096E-0I	.5I58E-03
	I.0	.2459E-0I	.6050E-03	.2459E-0I	.3I98E-03	.2459E-0I	.6049E-03
3.0	.2	.2059E+00	.3259E-0I	.2059E+00	.3254E-0I	.2059E+00	.3259E-0I
	.4	.3399E+00	.5380E-0I	.3399E+00	.5382E-0I	.3399E+00	.5380E-0I
	.6	.3552E+00	.5622E-0I	.3552E+00	.5629E-0I	.3552E+00	.5622E-0I
	.8	.2464E+00	.3899E-0I	.2464E+00	.3893E-0I	.2464E+00	.3899E-0I
	I.0	.5I47E-0I	.8I46E-02	.5I44E-0	.7287E-02	.5I44E-0I	.8I42E-02
5.0	.2	-.2240E-0I	-.2908E-02	-.2244E-0I	-.2994E-02	-.2244E-0I	-.29I2E-02
	.4	-.242IE-0I	-.3I42E-02	-.242IE-0I	-.3II5E-02	-.242IE-0I	-.3I4IE-02
	.6	-.3757E-02	-.3877E-03	-.3697E-02	-.3575E-03	-.3697E-02	-.48I5E-03
	.8	.20I5E-0I	.26I5E-02	.20I7E-0I	.25I5E-02	.20I7E-0I	.26I4E-02
	I.0	.2553E-0I	.33I4E-02	.2545E-0I	.I869E-02	.2545E-0I	.3294E-02

Here E+02 10^2 etc.

Wave functions expanded for $r \leqslant R$ and different k-values with the first seven poles in the expansion.

Table 3c

Wave function with $n \leq 22$ (V_o=40)

K	τ	exact		p = 0		p = 1	
		Re (Ψ)	Im (Ψ)	Re (Ψ)	Im (Ψ)	Re (Ψ)	Im (Ψ)
I.0	0.2	.5806E-02	.I428E-03	.5806E-02	.I4I2E-03	.5806E-02	.I428E-03
	0.4	.II38E-0I	.2800E-03	.II38E-0I	.2789E-03	.II38E-0I	.2800E-03
	0.6	.I650E-0I	.4060E-03	.I650E-0I	.4084E-03	.I650E-0I	.4060E-03
	0.8	.2096E-0I	.5I58E-03	.2096E-0I	.52I3E-03	.2096E-0I	.5I58E-03
	I.0	.2459E-0I	.6050E-03	.2459E-0I	.494IE-03	.2459E-0I	.6050E-03
3.0	0.2	.2059E-+00	.3259E-0I	.2059E+00	.3259E-0I	.2059E+00	.3259E-0I
	0.4	.3399E+00	.5380E-0I	.3399E+00	.5380E-0I	.3399E+00	.5380E-0I
	0.6	.3552E+00	.5622E-0I	.3552E+00	.5622E-0I	.3552E+00	.5622E-0I
	0.8	.2464E+00	.3899E-0I	.2464E+00	.390IE-0I	.2464E+00	.3899E-0I
	I.0	.5I47E-0I	.8I46E-02	.5I47E-0I	.78I3E-02	.5I47E-0I	.8I46E-02
5.0	0.2	-.2240E-0I	-.2908E-02	-.2240E-0I	-.29I6E-02	-.2240E-0I	-.2908E-02
	0.4	-.242IE-0I	-.3I42E-02	-.242IE-0I	-.3I48E-02	-.242IE-0I	-.3I42E-0I
	0.6	-.3757E-02	-.4877E-03	-.3756E-02	-.4758E-03	-.3756E-02	-.4876E-03
	0.8	.20I5E-0I	.26I5E-02	.20I5E-0I	.2643E-02	.20I5E-0I	.26I5E-02
	I.0	.2553E-0I	.33I4E-02	.2553E-0I	.2758E-02	.2553E-0I	.33I3E-02

Table 4a

Wave function with $n \leq 1$ (V_o = -40)

K	τ	exact		p = 0		p = 1	
		Re (Ψ)	Im (Ψ)	Re (Ψ)	Im (Ψ)	Re (Ψ)	Im (Ψ)
I.0	0.2	-.5995E-02	-.I523E-03	-.5I28E-02	.I403E-09	-.5I28E-02	.I3I5E-03
	0.4	-.II75E-0I	.2986E-03	-.I026E-0I	.766IE-08	-.I026E-0I	.2630E-03
	0.6	-.I704E-0I	.4329E-03	-.I538E-0I	.4I83E-06	-.I538E-0I	.3945E-03
	0.8	-.2I65E-0I	.5500E-03	-.205IE-0I	.2284E-04	-.205IE-0I	.5259E-03
	I.0	-.2539E-0I	.645IE-03	-.2558E-0I	.I247E-02	-.2558E-0I	.6543E-03
3.0	0.2	-.6539E-0I	.32I3E-02	-.5I28E-02	.4I26E-09	-.5I28E-02	.3945E-03
	0.4	-.I079E+00	.5304E-02	-.I026E-0I	.2253E-07	-.I026E-0I	.7890E-03
	0.6	-.II28E+00	.5542E-02	-.I538E-0I	.I230E-05	-.I538E-0I	.II83E-02
	0.8	-.7822E-0I	.3844E-02	-.2050E-0I	.67I7E-04	-.2050E-0I	.I576E-02
	I.0	-.I634E-0I	.803IE-03	-.2509E-0I	.3667E-02	-.2509E-0I	.I890E-02
5.0	0.2	.2085E-0I	-.25I4E-02	-.5I28E-02	.66I7E-09	-.5I28E-02	.6575E-03
	0.4	.2253E-0I	-.27I6E-02	-.I026E-0I	.36I4E-07	-.I026E-0I	.I3I5E-02
	0.6	.3497E-02	-.42I6E-03	-.I538E-0I	.I973E-05	-.I538E-0I	.I972E-02
	0.8	-.I875E-0I	.226IE-02	-.2049E-0I	.I077E-03	-.2049E-0I	.2623E-02
	1.0	-.2376E-0I	.2864E-02	-.24I7E-0I	.5882E-02	-.24I7E-0I	.2920E-02

Table 4b

Wave function with $n \leq 7$ (V_o = -40)

K	τ	exact Re(Ψ)	exact Im(Ψ)	p = 0 Re(Ψ)	p = 0 Im(Ψ)	p = 1 Re(Ψ)	p = 1 Im(Ψ)
I.0	0.2	-.5995E-02	.I523E-03	-.5997E-02	.I702E-03	-.5997E-02	.I524E-03
	0.4	-.II75E-0I	.2986E-03	-.II75E-0I	.28I4E-03	-.II75E-0I	.2985E-03
	0.6	-.I704E-0I	.4329E-03	-.I704E-0I	.42I2E-03	-.I704E-0I	.4329E-03
	0.8	-.2I65E-0I	.5500E-03	-.2I65E-0I	.6I28E-03	-.2I65E-0I	.550IE-03
	I.0	-.2539E-0I	.645IE-03	-.2540E-0I	.9479E-03	-.2540E-0I	.6453E-03
3.0	0.2	-.6539E-0I	.32I3E-02	-.6540E-0I	.3268E-02	-.6540E-0I	.32I4E-02
	0.4	-.I079E+00	.5304E-02	-.I079E+00	.5252E-02	-.I079E+00	.5303E-02
	0.6	-.II28E+00	.5542E-02	-.II28E+00	.5506E-02	-.II28E+00	.554IE-02
	0.8	-.7822E-0I	.3844E-02	-.7827E-0I	.4035E-02	-.7827E-0I	.3847E-02
	I.0	-.I634E-0I	.803IE-03	-.I638E-0I	.I7I6E-02	-.I638E-0I	.8082E-03
5.0	0.2	.2085E-0I	-.25I4E-02	.2080E-0I	-.24I9E-02	.2080E-0I	-.2509E-02
	0.4	.2253E-0I	-.27I6E-02	.2259E-0I	-.2806E-02	.2259E-0I	-.2720E-02
	0.6	.3497E-02	-.42I6E-03	.35I4E-02	-.4843E-03	.35I4E-02	-.4262E-03
	0.8	-.I875E-0I	.226IE-02	-.I889E-0I	.2588E-02	-.I889E-0I	.2274E-02
	I.0	-.2376E-0I	.2864E-02	-.2387E-0I	.4402E-02	-.2387E-0I	.2889E-02

Table 4c

Wave function with $n \leq 22$ (V_o = -40)

K	τ	exact Re(Ψ)	exact Im(Ψ)	p = 0 Re(Ψ)	p = 0 Im(Ψ)	p = 1 Re(Ψ)	p = 1 Im(Ψ)
I.00	0.2	-.5995E-02	.I523E-03	-.5995E-02	.I504E-03	-.5995E-02	.I523E-03
	0.4	-.II75E-0I	.2986E-03	-.II75E-0I	.2999E-03	-.II75E-0I	.2986E-03
	0.6	-.I704E-0I	.4329E-03	-.I704E-0I	.43I3E-03	-.I704E-0I	.4329E-03
	0.8	-.2I65E-0I	.5500E-03	-.2I65E-0I	.5430E-03	-.2I65E-0I	.5500E-03
	I.0	-.2539E-0I	.645IE-03	-.2539E-0I	.7584E-03	-.2539E-0I	.6452E-03
3.0	0.2	-.6539E-0I	.32I3E-02	-.6539E-0I	.32I8E-02	-.6539E-0I	.32I3E-02
	0.4	-.I079E+00	.5304E-02	-.I079E+00	.5308E-02	-.I079E+00	.5304E-02
	0.6	-.II28E+00	.5542E-02	-.II28E+00	.5536E-02	-.II28E+00	.5542E-02
	0.8	-.7822E-0I	.3844E-02	-.7822E-0I	.3823E-02	-.7822E-0I	.3844E-02
	I.0	-.I634E-0I	.803IE-03	-.I634E-0I	.II43E-02	-.I634E-0I	.8033E-03
5.0	0.2	.2085E-0I	-.25I4E-02	.2085E-0I	-.2505E-02	.2085E-0I	-.25I4E-02
	0.4	.2253E-0I	-.27I6E-02	.2253E-0I	-.27I0E-02	.2253E-0I	-.27I6E-02
	0.6	.3497E-02	-.42I6E-03	.3497E-02	-.43I3E-03	.3497E-02	-.42I6E-03
	0.8	-.I875E-0I	.226IE-02	-.I875E-0I	.2226E-02	-.I875E-0I	.226IE-02
	I.0	-.2376E-0I	.2864E-02	-.2377E-0I	.3432E-02	-.2377E-0I	.2865E-02

Table 5

N	$\beta_o = 4.6$ Re(K_n)	Im(K_n)	$\beta_o = 5.0$ Re(K_n)	Im(K_n)
I.	.5703E-0I	-.3335E+00	0.	.2955E+00
2	0.	.I275E+0I	0.	-.8756E+00
3	.2066E+0I	-.4623E+00	0.	.I3IIE+0I
4	.328IE+0I	-.5563E+00	.I808E+0I	-.405IE+00
5	.44I3E+0I	-.6303E+00	.296IE+0I	-.4900E+00
6	.55I3E+0I	-.69IIE+00	.40I8E+0I	-.5570E+00
7	.6597E+0I	-.7428E+00	.5038E+0I	-.6I24E+00
8	.7670E+0I	-.7878E+00	.6040E+0I	-.6596E+00
9	.8738E+0I	-.8274E+00	.7032E+0I	-.7006E+00
I0	.980IE+0I	-.8630E+00	.80I7E+0I	.7368E+00
II	.I086E+02	-.895IE+00	.8998E+0I	-.7694E+00
I2	.II92E+02	-.9245E+00	.9975E+0I	-.7988E+00
I3	.I298E+02	-.95I5E+00	.I095E+02	-.8257E+00
I4	.I403E+02	-.9765E+00	.II92E+02	-.8505E+00
I5	.I509E+02	-.9998E+00	.I290E+02	-.8734E+00
I6	.I6I4E+02	-.I02IE+0I	.I387E+02	-.8948E+00
I7	.I7I9E+02	-.I042E+0I	.I484E+02	-.9I48E+00
I8	.I824E+02	-.I06IE+0I	.I580E+02	-.9336E+00
I9	.I929E+02	.I080E+0I	.I677E+02	-.95I3E+00
20	.2034E+02	-.I097E+0I	.I774E+02	-.9680E+00
2I	.2I40E+02	-.III3E+0I	.I87IE+02	-.9839E+00
22	.2245E+02	-.II29E+0I	.I968E+02	-.9990E+00

Positions of the poles for the rectangular potential wells.

Table 6a

K	r	exact Re (Ψ)	Im (Ψ)	p = 1 Re (Ψ)	Im (Ψ)	p = 2 Re (Ψ)	Im (Ψ)
I.0	.50	.530IE+00	-.2930E+00	.4654E+00	-.3047E+00	.5340E+00	-.3047E+00
	I.20	.4824E+00	-.2667E+00	.458IE+00	-.2564E+00	.5055E+00	-.2564E+00
	I.80	-.9I0IE+0I	.503IE+0I	-.6225E-0I	.I046E+00	-.5970E-0I	.I046E+00
	2.40	-.5652E+00	.3I25E+00	-.6048E+00	-.3705E+00	-.5589E+00	.3705E+00
	3.00	-.4234E+00	.234IE+00	-.5480E+00	.20I7E+00	-.4302E+00	.20I7E+00
3.0	.60	-.2259E+00	-.I636E+00	.I807E+00	.I5I7E+00	.7978E+00	.I5I7E+00
	I.20	.I968E+00	.I425E+00	.75I2E-0I	.3053E+00	.50I4E+00	.3053E+00
	I.80	.5446E-0I	.3943E-0I	-.2797E+00	.2708E+00	-.3467E+00	.2708E+00
	2.40	-.2443E+00	-.I768E+00	-.5206E+00	-.I560E+00	-.I073E+00	-.I560E+00
	3.00	.I583E+00	.II46E+00	-.2998E+00	-.7895E+00	I66IE+0I	-.7895E+00
5.0	.50	-.7065E-03	.I248E-04	.I497E+00	.3072E+00	.I862E+0I	.3072E+00
	I.20	.I4I3E-02	-.2496E-04	.3275E-0I	.5393E+00	.I2I6E+0I	.5392E+00
	I.80	-.2II9E-02	.3744E-04	-.3036E+00	.3685E+00	-.4896E+00	.3885E+00
	2.40	.2825E-02	-.4992E-04	-.5093E+00	-.3893E+00	.6387E+00	-.3893E+00
	3.00	-.3532E-02	.6240E-04	-.2604E+00	-.I393E+0I	.5I85E+0I	-.I393E+0I

Wave functions expanded for $r \leq R$ and different κ-values retaining only the first pole in the expansion (rectangular well: V_o=48.72 MeV, R=3 fm i.e. $\beta_o = \kappa R = 4.6$)

Table 6b

κ	r	exact		p = 1		p = 2	
		Re(Ψ)	Im (Ψ)	Re (Ψ)	Im (Ψ)	Re (Ψ)	Im (Ψ)
I.0	.60	.530IE+00	-.2930E+00	.5294E+00	-.293IE+00	.5300E+00	-.293IE+00
	I.20	.4824E+00	-.2667E+00	.4827E+00	-.2666E+00	.4824E+00	-.2666E+00
	I.80	-.9I0IE-0I	.503IE-0I	-.893IE-0I	.5025E-0I	-.9098E-0I	-.5025E-0I
	2.40	-.5652E+00	.3I25E+00	-.5684E+00	.3II9E+00	-.5653E+00	.3II9E+00
	3.00	-.4234E+00	.234IE+00	-.46I4E+00	.2335E+00	-.4237E+00	.2335E+00
3.0	.60	-.2259E+00	-.I636E+00	-.2328E+00	-.I645E+00	-.2270E+00	-.I645E+00
	I.20	.I968E+00	.I425E+00	.I998E+00	.I458E+00	.I969E+00	.I458E+00
	I.80	.5446E-0I	.3943E-0I	.7262E-0I	.3786E-0I	.5759E-0I	.3786E-0I
	2.40	-.2443E+00	-.I768E+00	-.2739E+00	-.I945E+00	-.2455E+00	-.I945E+00
	3.00	.I583E+00	.II46E+00	-.20I9E+00	.9629E-0I	.I376E+00	.9628E-0I
5.0	.60	-.7065E-03	.I248E-04	-.2832E-0I	-.8597E-02	-.I2I4E-0I	-.8597E-02
	I.20	.I4I3E-02	-.2496E-04	.7930E-02	.2268E-0I	-.2II5E-03	.2268E-0I
	I.80	-.2II9E-02	.3744E-04	.7374E-0I	-.I6I5E-0I	.3I99E-0I	-.I6I5E-02
	2.40	.2826E-02	-.4992E-04	-.7762E-0I	-.II53E+00	.II44E-02	-.II53E+00
	3.00	-.3532E-02	.6240E-04	-.II44E+0I	-.I306E+00	-.20I5E+00	-.I306E+00

Wave functions expanded for $r \leq R$ and different κ-values with the first 7 poles in the expansion (rectangular well: V_0=47.72 MeV, R=3 fm i.e. $\beta_0 = \varkappa_0 R$=4.6)

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Table 6c

κ	r	exact		p = 1		p = 2	
		Re (Ψ)	Im (Ψ)	Re (Ψ)	Im (Ψ)	Re (Ψ)	Im (Ψ)
I.0	.60	.530IE+00	-.2930E+00	.5300E+00	-.2930E+00	.530IE+00	-.2930E+00
	I.20	.4824E+00	-.2667E+00	.4824E+00	-.2667E+00	.4824E+00	-.2667E+00
	I.80	-.9I0IE-0I	.503IE-0I	-.9093E-0I	.503IE-0I	-.9I0IE-0I	.503IE-0I
	2.40	-.5652E+00	.3I25E+00	-.5649E+00	.3I25E+00	-.5652E+00	.3I25E+00
	3.00	-.4234E+00	.234IE+00	-.4352E+00	.234IE+00	-.4234E+00	.234IE+00
3.0	.60	-.2259E+00	-.I636E+00	-.226IE+00	-.I636E+00	-.2259E+00	-.I636E+00
	I.20	.I968E+00	.I425E+00	.I966E+00	.I424E+00	.I968E+00	.I424E+00
	I.80	.5446E-0I	.3943E-0I	.5524E-0I	.3934E-0I	.5448E-0I	.3934E-0I
	2.40	-.2443E+00	-.I768E+00	-.2409E+00	-.I765E+00	-.2442E+00	-.I765E+00
	3.00	.I583E+00	.II46E+00	.5I84E-0I	.II39E+00	.I577E+00	.II39E+00
5.0	.60	-.7065E-03	.I248E-04	-.I299E-02	.II27E-04	-.7344E-03	.II27E-04
	I.20	.I4I3E-02	-.2496E-04	.8585E-03	-.238IE-03	.I393E-02	-.238IE-03
	I.80	-.2II9E-02	.3744E-04	.I258E-03	-.3653E-03	-.200IE-02	-.3653E-03
	2.40	.2826E-02	-.4992E-04	.I234E-0I	.I450E-02	.3I86E-02	.I450E-02
	3.00	-.3532E-02	.6240E-04	-.3024E+00	-.3256E-02	-.825IE-02	-.3256E-02

Wave functions expanded for $r \leq R$ and different κ-value with the first 22 poles in the expansion (rectangular well: V_0=48.72 MeV, R=3 fm (β_0=4.6))

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Table 7a

n	k	exact ReS	exact ImS	p = 2 ReS	p = 2 ImS	p = 3 ReS	p = 3 ImS
I	.50	.6079E+00	.7940E+00	.5982E+00	.8477E+00	.6059E+00	.7938E+00
	I.00	.7473E+00	-.6645E+00	.9350E+00	-.I078E+0I	.8I33E+00	-.6594E+00
	I.50	-.8033E+00	-.5956E+00	-.2063E+0I	.579IE+00	-.I457E+0I	-.7607E+00
	2.00	-.9870E+00	.I608E+00	.3I39E+0I	.2376E+00	.I269E+0I	.3I79E+0I
	2.50	-.9220E+00	.3872E+00	-.4I96E+0I	.I448E+0I	.23I3E+00	-.66I9E+0I
	3.00	-.5073E+00	.86I8E+00	.5I39E+0I	.3I2IE+0I	-.3695E+0I	.I089E+02
	3.50	-.4I55E+00	.9096E+00	-.5833E+0I	-.5285E+0I	.9795E+0I	-.I552E+02
	4.00	-.9603E-0I	.9954E+00	.6I24E+0I	.7924E+0I	-.I9I3E+02	.I976E+02
	4.50	.3246E-0I	.9995E+00	-.5855E+0I	-.I098E+02	.32IIE+02	-.2258E+02
	5.00	.I89IE+00	.9820E+00	.4876E+0I	.I435E+02	-.4893E+02	.2275E+02
4	.50	.6079E+00	.7940E+00	.605IE+00	.8II3E+00	.6076E+00	.7940E+00
	I.00	.7473E+00	-.6645E+00	.7927E+00	-.7994E+00	.7537E+00	-.6656E+00
	I.50	-.8033E+00	-.5956E+00	-.I036E+0I	-.I603E+00	-.8420E+00	-.5888E+00
	2.00	-.9870E+00	.I608E+00	-.2376E+00	-.80IIE+00	-.8358E+00	.I397E+00
	2.50	-.9220E+00	.3872E+00	-.2808E+0I	.2068E+0I	-.I393E+0I	.4I37E+00
	3.00	-.5073E+00	.86I8E+00	.3593E+0I	-.I52IE+0I	.7672E+00	.9635E+00
	3.50	-.4I55E+00	.9096E+00	-.85I3E+0I	.323IE+0I	-.35I4E+0I	-.4I08E+0I
	4.00	-.9603E-0I	.9954E+00	.I396E+02	.2I39E+0I	.5882E+0I	.5922E+0I
	4.50	.3246E-0I	.9995E+00	-.I552E+02	-.8644E+0I	-.3373E+0I	-.I235E+02
	5.00	.I89IE+00	.9820E+00	.I527E+02	.I5I4E+02	-.I94IE+0I	I773+02

S-matrix expansion for different k with the first 1 and 4 poles in the expansion (rectangular well: V_o=48.72 MeV, R=3 fm (β_o =4.6))

Table 7b

n	k	exact ReS	exact ImS	p = 2 ReS	p = 2 ImS	p = 3 ReS	p = 3 ImS
7	.50	.6079E+00	.7940E+00	.6065E+00	.8033E+00	.6078E+00	.7940E+00
	I.00	.7473E+00	-.6645E+00	.7696E+00	-.737IE+00	.7485E+00	-.6646E+00
	I.50	-.8033E+00	-.5956E+00	-.9I5IE+00	-.3628E+00	-.8I02E+00	-.5948E+00
	2.00	-.9870E+00	.I608E+00	-.6383E+00	-.35IIE+00	-.9622E+00	.I582E+00
	2.50	-.9220E+00	.3872E+00	-.I758E+0I	.I287E+0I	-.99I5E+00	.39I6E+00
	3.00	-.5073E+00	.86I8E+00	.II89E+0I	-.4828E+00	-.34I0E+00	.862IE+00
	3.50	-.4I55E+00	.9096E+00	-.3476E+0I	.265IE+0I	-.7703E+00	.8792E+00
	4.00	-.9603E-0I	.9954E+00	.4969E+0I	-.9229E+00	.5972E+00	.II25E+0I
	4.50	.3246E-0I	.9995E+00	-.7803E+0I	.26I9E+0I	-.I229E+0I	.6I04E+00
	5.00	.I89IE+00	.9820E+00	.II66E+02	.5I27E+00	.2346E+0I	.I967E+0I
22	.50	.6079E+00	.7940E+00	.6075E+00	.7969E+00	.6079E+00	.7940E+00
	I.00	.7473E+00	-.6645E+00	.7539E+00	-.687IE+00	.7473E+00	-.6645E+00
	I.50	-.8033E+00	-.5956E+00	-.8363E+00	-.5233E+00	-.8036E+00	-.5956E+00
	2.00	-.9870E+00	.I608E+00	-.8850E+00	.I997E-02	-.9860E+00	.I609E+00
	2.50	-.9220E+00	.3872E+00	-.II64E+0I	.6663E+00	-.9244E+00	.3870E+00
	3.00	-.5073E+00	.86I8E+00	-.2468E-0I	.4430E+00	-.50I8E+00	.8625E+00
	3.50	-.4I55E+00	.9096E+00	-.I270E+0I	.I460E+0I	-.4263E+00	.9074E+00
	4.00	-.9603E-0I	.9954E+00	.I287E+0I	.3622E+00	-.7650E-0I	.I00IE+0I
	4.50	.3246E-0I	.9995E+00	-.205IE+0I	.I6I3E+0I	-.3490E-03	.9869E+00
	5.00	.I89IE+00	.9820E+00	.3I47E+0I	.5538E+00	.2408E+0I	.I007E+0I

S-matrix expansion for different k with the first 7 and 22 poles in the expansion (rectangular well: V_o=48.72 MeV, R=3 fm (i.e. β_o =4.6))

Table 8

K	r	exact Re (Ψ)	exact Im (Ψ)	p = 1 Re (Ψ)	p = 1 Im (Ψ)	p = 2 Re (Ψ)	p = 2 Im (Ψ)
0.5	.60	.7799E+00	-.3973E+00	.7927E+00	-.3969E+00	.7799E+00	-.3969E+00
	I.20	.8847E+00	-.4507E+00	.882IE+00	-.4507E+00	-.8346E+00	-.4507E+00
	I.80	.2238E+00	-.II40E+00	.22I8E+00	-.II46E+00	.2238E+00	-.II46E+00
	2.40	-6309E+00	.32I3E+00	-.6229E+00	-.32I9E+00	-.6307E+00	.32I9E+00
	3.00	-.9394E+00	.4786E+00	-.9470E+00	.4798E+00	-.9397E+00	.4798E+00
I.0	.60	.4958E+00	-.879IE-0I	.5079E+00	-.8459E-0I	.4969E+00	-.8459E-0I
	I.20	.45I3E+00	-.8000E-0I	.4396E+00	-.804IE-0I	.4498E+00	-.804IE-0I
	I.80	-.85I3E-0I	.I509E-0I	-.930IE-0I	.9279E-02	-.8500E-0I	.9279E-02
	2.40	-.5287E+00	.9374E-0I	-.4949E+00	.9848E-0I	-.5259E+00	.9848E-0I
	3.00	-.396IE+00	.7022E-0I	-.4300E+00	.80I8E-0I	-.4008E+00	.80I8E-0I
I.5	.60	.4334E+00	.26I2E+00	.4640E+00	.2755E+00	.4393E+00	.2755E+00
	I.20	.2427E+00	.I463E+00	.2II3E+00	.I432E+00	.234IE+00	.I432E+00
	I.80	-.2975E+00	-.I793E+00	-.3I38E+00	-.202IE+00	-.2958E+00	-.202IE+00
	2.40	-.4093E+00	-.2467E+00	-.3240E+00	-.2252E+00	-.3938E+00	-.2252E+00
	3.00	.6830E-0I	.4II7E-0I	-.2488E-0I	.7492E-0I	.4084E-0I	.7492E-0I
2.0	.60	-.2083E+00	.434IE+00	-.I455E+00	.4844E+00	-.I894E+00	.4844E+00
	I.20	-.2445E-0I	.5095E-0I	-.9787E-0I	.3203E-0I	-.5736E-0I	.3203E-0I
	I.80	.2055E+00	-.4282E+00	.I875E+00	-.4955E+00	.2I95E+00	-.4955E+00
	2.40	.4856E-0I	-.I0I2E+00	.2243E+00	-.206IE-0I	.I002E+00	-.206IE-0I
	3.00	-.I998E+00	.4I63E+00	-.4262E+00	.4889E+00	-.3093E+00	.4889E+00

Wave functions expanded for $r \leq R$ and different K-value with the 4 poles in the expansion (rectangular well: V_0=48.72 MeV, R=3.26 fm, i.e. β_0=5.0)

References

1. J.Bang, F.A.Gareev, M.H.Gizzatkulov and S.A.Goncharov. JINR, E4-11377, Dubna, 1978.
2. W.Romo, Nucl.Phys. A116, 618 (1968).
3. R.M.More, E.Gerjuoy. Phys. Rev. A7, 1188 (1975).
4. H.M.Nussenzveig, Nucl.Phys. 11, 499 (1959).
5. H.M.Nussenzveig, "Causality and Dispersion Relations",Academic Press, New York, 1972.
6. H.A.Weidenmüller, Ann. of Phys. 28, 60 (1964).
7. J.Bang and J.Zimanyi, Nucl.Phys. A139, 534 (1969).

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