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Расчет низколежащих коллективных возбуждений для ^{168}Yb при больших угловых моментах

Рассчитана энергия возбуждения нижайших состояний с нечетными спинами (I) в ядре ^{168}Yb в рамках модели, предложенной ранее двумя из авторов (И.М. и Д.Я.). Это семейство состояний тесно связано с состояниями γ-вибрационной полосы при малых I и с однофононными прецессионными возбуждениями, когда $I \gg 0$.

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Calculations of Low-Lying Collective Excitation Energies in ^{168}Yb at High Angular Momenta

The excitation energy of the lowest I-odd states in ^{168}Yb is calculated in a wide range of spins (I) by using the microscopic model suggested earlier by two of the authors (D.J. and I.M.). This family of states has close relation to the γ-vibrational states at low spins and to the one-phonon precessional excitations when I is large.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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Experimental data and theoretical estimates show that the angular momenta of nuclear states whose structure may be investigated by the γ-spectroscopy methods may reach as much as $I_{cr} = 80 \div 100\,\hbar$ /1/. Many interesting effects are predicted theoretically in the region of high spins. The changes of a stationary nuclear shape with increasing spin was studied in refs. /2-5/. Such investigations predict that at $I \geq 20 \div 40$ most of nuclei lose axial symmetry and that the quadrupole deformation parameters ϵ and γ are complicated functions of spin. The momentum distribution of nuclear matter is also expected to change with spin leading to formation of yrast traps /1/ and to retardation of the intraband E2-transitions /6/.

These theoretical predictions may be tested studying the spectra of low-lying collective excitations at high spins. Mottelson /7/ considered a possibility to find in the nuclear spectra at large I the states typical for precessing three-axial bodies. In an even-even nucleus the precessional motion is expected to generate the states with energies

$$E(I,n) = E_{yr}(I) + \hbar\omega(I)\cdot 2n \qquad (1.a)$$

when I is even and

$$E(I,n) = E_{yr}(I) + \hbar\omega(I)\cdot(2n+1) \qquad (1.b)$$

when I is odd. Here $E_{yr}(I)$ is a smooth function of I and $n = 0,1,2,\ldots \ll I$. The energies of yrast states are given by (1.a) at $n = 0$. The excitation energy of the precessional phonon $\hbar\omega(I)$ is [8]

$$\hbar\omega(I) = \frac{I}{\mathcal{J}_x}\left[\frac{(\mathcal{J}_x-\mathcal{J}_y)(\mathcal{J}_x-\mathcal{J}_z)}{\mathcal{J}_y\mathcal{J}_z}\right]^{1/2}, \qquad (2)$$

where $\mathcal{J}_x > \mathcal{J}_y > \mathcal{J}_z$ are the three principal inertia moments.

In refs. [9,10,11] the excited states of rotating nuclei are treated microscopically by using the Hamiltonian H with schematic pairing plus quadrupole interactions. The technique of RPA is used for the "intrinsic" Hamiltonian $\tilde{H} = H - \vec{J}^2/2\mathcal{J}_x$, where J is the angular momentum operator in the laboratory frame and the inertia parameter $\mathcal{J}_x$ is found from the condition that the rotational energy is excluded from the RPA image of $\tilde{H}$. For the part of the energy $E_{yr}(I)$ in eqs. (1) the expressions analogous to those of the HFB+cranking model are obtained in the following form:

$$E_I = \langle\psi|\tilde{H}|\psi\rangle,$$
$$\sqrt{I(I+1)} \equiv J_x = \langle\psi|\hat{J}_x|\psi\rangle, \qquad (3)$$

where $|\psi\rangle$ is a deformed HFB vacuum state with respect to the quasiparticle operators $a_i, a_{\bar i}$. The states i and $\bar{i}$ transform differently when rotated through the angle π around the x-axis. They correspond to the creation operators

$$a_i^+ = \sum_\alpha (A_\alpha^i c_\alpha^+ + B_{\bar\alpha}^i c_{\bar\alpha}),$$
$$a_{\bar i}^+ = \sum_\alpha (A_{\bar\alpha}^{\bar i} c_{\bar\alpha}^+ + B_\alpha^{\bar i} c_\alpha), \qquad (4)$$

where c_α^+, $c_\alpha (c_{\bar\alpha}^+, c_{\bar\alpha})$ are the creation and destruction operators for nucleons in the basic states, and A_α^i, $B_{\bar\alpha}^i$, $A_{\bar\alpha}^{\bar i}$, $B_\alpha^{\bar i}$ are the Bogolubov transformation coefficients.

For the excitation energy of states with the symmetry of the one-phonon excitations of precessional motion an expression is obtained in [10] which is analogous to eq.(2). A simple transformation of eq.(8) in ref. [10] gives

$$\hbar\omega_{(-)} = \frac{\langle\hat{J}_x\rangle}{\mathcal{J}_x}\left[\frac{(\mathcal{J}_x-\mathcal{J}_y(\omega_{(-)}))(\mathcal{J}_x-\mathcal{J}_z(\omega_{(-)}))}{\mathcal{J}_y(\omega_{(-)})\mathcal{J}_z(\omega_{(-)})+2\langle\hat{J}_x\rangle S(\omega_{(-)})+S(\omega_{(-)})\left(\frac{\langle\hat{J}_x\rangle^2}{\mathcal{J}_x^2}-\omega_{(-)}^2\right)}\right]^{1/2}. \qquad (5)$$

In the Hamiltonian $\tilde{H}$ and in eq.(5) the inertia parameter $\mathcal{J}_x$ may be found from the equation

$$\mathcal{J}_x^{-1} = \frac{d^2E(I)}{dI^2} = \frac{d\Omega}{dI}. \qquad (6)$$

The explicit expressions for the quantities $\mathcal{J}^{y,z}(\omega_{(-)})$ and $S(\omega_{(-)})$ are given is ref. [10] (see eq. (9) in [10]).

In this paper we present the results of calculations of the quadrupole deformation parameters, the lowest two-quasi-particle excitation energies (E_{ik} , $E_{\bar i\bar k}$) and also the function $\hbar\omega_{(-)}(I)$ in eq.(1) at different I. The calculations are performed for the isotope ^{168}Yb .

The quasiparticle energies E_i , $E_{\bar i}$ and the vectors $(A_\alpha^i, B_{\bar\alpha}^i)$, $(A_{\bar\alpha}^{\bar i}, B_\alpha^{\bar i})$ are determined by diagonalizing the rotating Nilsson Hamiltonian plus monopole pairing. That is, we solve numerically the equations

$$\begin{pmatrix} (e_\alpha-\lambda)\delta_{\alpha\beta} - \Omega j^x_{\alpha\alpha}\delta_{\alpha\beta} & \Delta\delta_{\alpha\bar\beta} \\ \Delta\delta_{\bar\alpha\beta} & (-e_{\bar\alpha}+\lambda)\delta_{\bar\alpha\bar\beta} + \Omega j^x_{\bar\alpha\bar\alpha}\delta_{\bar\alpha\bar\beta} \end{pmatrix}\begin{pmatrix} A_\alpha^i \\ B_{\bar\alpha}^i \end{pmatrix} = E_i \begin{pmatrix} A_\alpha^i \\ B_{\bar\alpha}^i \end{pmatrix} \qquad (7)$$

where e_α are the single particle energies corresponding to the Nilsson Hamiltonian, λ and Δ are the chemical potential and the gap parameter and Ω is the angular frequency of rotation related to the spin by an equation $I \equiv \sqrt{I(I+1)} = \langle\psi|\hat{J}_x|\psi\rangle$. The basic states $\alpha, \bar{\alpha}$ are taken as the states in the Nilsson potential with the x-axis being the quantization axis, assuming $\hat{J}_x|\alpha\rangle = m_\alpha|\alpha\rangle$ ($N - m_\alpha - \frac{1}{2}$ - even number) and $|\bar{\alpha}\rangle = T|\alpha\rangle$ (T being the time reversion). The following relations hold /11-13/

$$A^i_\alpha = B^{\bar{i}}_\alpha, \quad A^{\bar{i}}_{\bar{\alpha}} = B^i_{\bar{\alpha}}, \quad E_i = -E_{\bar{i}}. \qquad (8)$$

The standard parameters for the single particle Hamiltonian are used (set " A = 165" in ref. /14/)and the pairing strength [$g = (19.2 \pm 7.4 \times N(Z)/A)/A$ MeV]. Axial and nonaxial deformations are taken into account. All the shells up to N=7 are included in the calculations. The summation in HFB equations runs over $2\sqrt{15 \cdot Z(N)}$ lowest quasiparticle states. The equilibrium deformation parameters ϵ, γ are determined by minimizing the energy /4/

$$E(\epsilon, \gamma) = E_{LD} + \delta E_{strut}$$

which includes the rotated liquid drop component (E_{LD})and the shell-correction $\delta E_{strut} = E_{HFC} - \tilde{E}_{strut}$. The calculations are performed for 10 different values of Ω in the interval from 0.05 to 0.7 MeV.

The solution to eqs. (3), (5), (6) is found by using the energies E_i and vectors ($A^i_\alpha, B^i_{\bar{\alpha}}$) in eq.(7) calculated at the equilibrium values of ϵ, γ, Δ and λ. The quantity $\mathcal{J}_x$ is calculated approximating eq. (6) by the formula

$$\mathcal{J}_x(\Omega_i) = \frac{1}{2}\left[\frac{\langle\hat{J}_x\rangle_i - \langle\hat{J}_x\rangle_{i-1}}{\Omega_i - \Omega_{i-1}} + \frac{\langle\hat{J}_x\rangle_{i+1} - \langle\hat{J}_x\rangle_i}{\Omega_{i+1} - \Omega_i}\right]. \qquad (9)$$

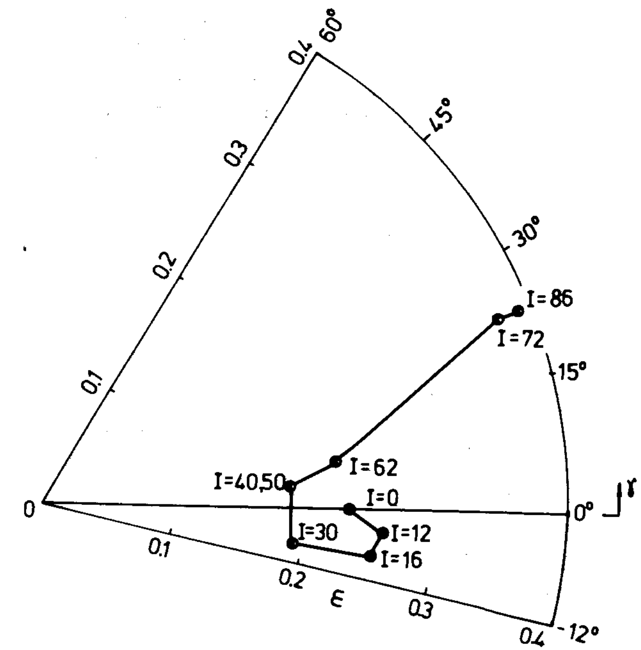

Fig.1. The equilibrium quadrupole deformation in ^{168}Yb at different spins. The values of spin corresponding to the calculated points are indicated in the plot.

Figure 1 shows the equilibrium deformations in ^{168}Yb for different spins (in the plot the convention of ref. /4/ is used for γ). At $\Omega \lesssim 0.3$ MeV these results can be compared to the calculations in refs./4,13/. All calculations predict similar changes with spin in the shape parameters: the nonaxial deformation developes as is expected in the body with the moments of inertia depending on γ according to the hydrodynamic model; some stretching takes place for low spins but gives way to the opposite tendency at somewhat higher spins. At still higher spins corresponding to $\Omega > 0.3$ MeV the nonaxiality angle γ changes sign showing that the inertia properties are close here to those of a rigid body. The stretching is quite prominent here again. Concerning the description of the energy of yrast states shown in Fig.2 one may say that at

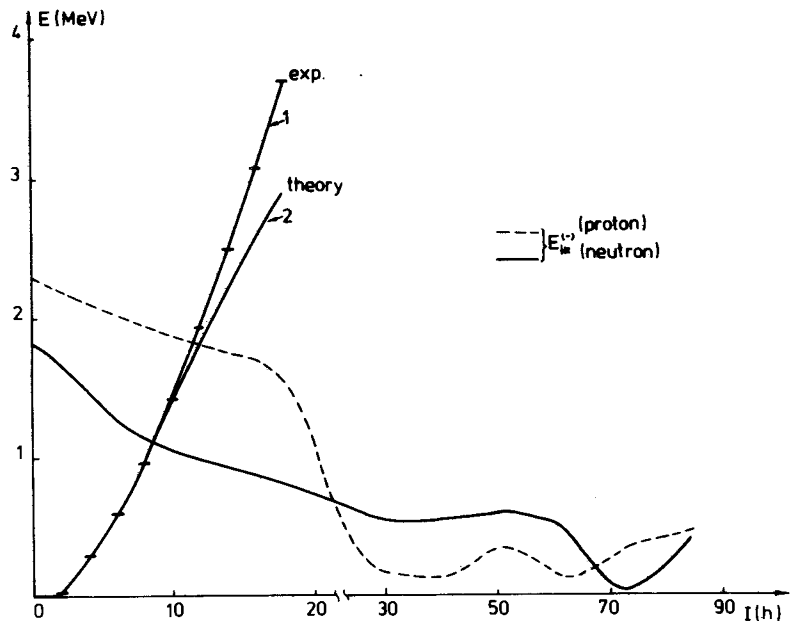

Fig.2. Lowest two-quasiparticle energies $E_{ik}^{(-)}$ as functions of spin and the experimental (1) and calculated (2) energies of the yrast states in ^{168}Yb.

$I \leq 12$ the agreement between theory and experiment is quite good. At larger spins the model overestimates the nonadiabatic effects. The calculated moment of inertia $\theta = \langle \hat{J}_x \rangle / \Omega$ increases much faster than derived from the experimental data [4]. Similar results are reported elsewhere (see, e.g., ref. [4,12]). The lowest two-quasiparticle energies $E_{ik}^{(-)} = \min\{E_{ik}, E_{\bar{i}\bar{k}}\}$ for protons and neutrons lie at about 1 MeV or higher at $I \leq 20$. The neutron two-quasiparticle energies remain large at much higher spins. No other regularities are evident for the two-quasiparticle energies when $\Delta = 0$.

In Figure 3 the estimates for the excitation energy of the lowest states with odd values of I are given (i.e., for the function $\hbar\omega$ in eq. (1)). The solid line with crosses is obtained using eq.(2) with the rigid body moments of inertia. The same formula for $\hbar\omega$ with the moments of inertia calculated microscopically gives different results (the solid line with triangles). The values of $\mathcal{J}_{y,z}$ are found by using the cranking formula and $\theta = \langle \hat{J}_x \rangle / \Omega$ stands in these calculations for $\mathcal{J}_x$.

The lowest solutions to eq. (5) at the nine values of I, corresponding to rotational frequencies in the interval $0.14 \leq \Omega \leq 0.7$ MeV are indicated in Fig.3 by circles connected by the broken line. At low spins $\hbar\omega_{(-)}$ lies well below the two-quasiparticle states and represents a collectivized mode of excitation. When $I > 30$ in vicinity of the first root of eq.(5) there are several two-quasiparticle states and eq.(5) may have more than one solution with comparable $\hbar\omega_{(-)}$. For example, at $I = 40$ the two lowest roots are $\hbar\omega_{(-)} = 64$ keV, $\hbar\omega_{(-)} = 223$ keV. The collectivity of different modes may be studied by calculating the intraband B(E2) factors. Such calculations are in progress.

As is proved in ref. [10] $\hbar\omega_{(-)} \to \hbar\omega_\gamma$ when $\Omega \to 0$, $\hbar\omega_\gamma$ being the excitation energy of the γ-vibrational band-head state. The extrapolation of the known energies of the 3^+ and 5^+ γ-vibrational states is represented in Fig.3 by the line with open squares. In the extrapolation we use the formula

$$\hbar\omega_\gamma(I) = (E_\gamma^{odd}(I) - E_{yr}(I)) = 910.3 - 1.1 \cdot I(I+1).$$

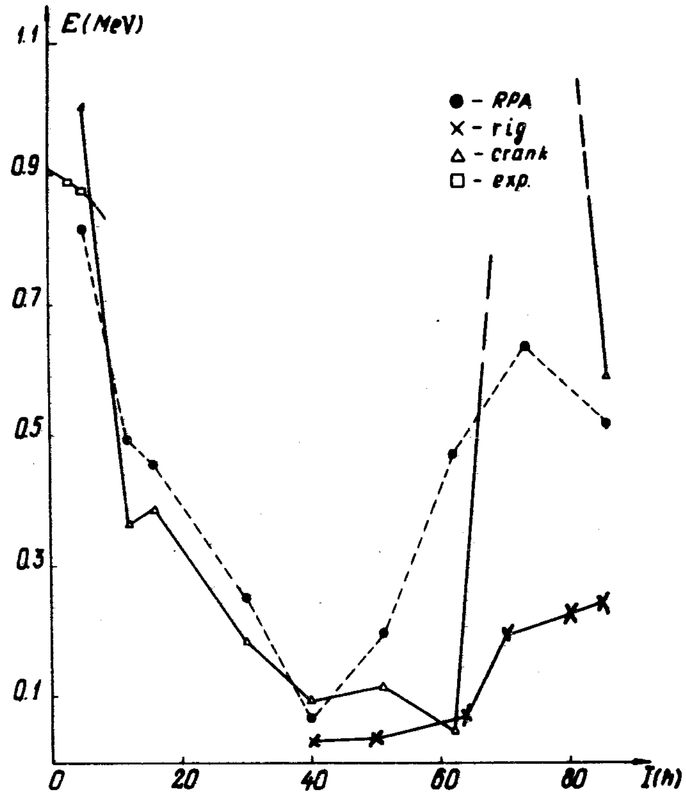

Fig.3. Estimates of the excitation energy of the lowest states with odd I-values in the nucleus ^{168}Yb. The explanation is given in the text.

As is seen in Fig.3, the results of microscopic calculations are rather close to the latter formula when $I < 11$.

In the interval $15 < I < 50$ the predictions of the adiabatic formula (2) with the microscopically calculated moments of inertia are close to solutions of eq. (5). At higher spins the three estimates for $\hbar\omega_{(-)}$ described above are rather different from each other. In particular, no tendency is seen for $\hbar\omega_{(-)}$ from eq.(5) to come close to the predictions of the adiabatic formula with the rigid-body moments of inertia.

On the basis of a formal analysis given in refs./10,11/ and of the numerical calculations reported here the following tentative conclusions might be drawn:

i) The position of low-lying states of even deformed nuclei with odd values of I and positive parity depends strongly on the shape and inertia properties of nuclei. Thus, the investigation of these states may yield a valuable information on nuclear structure and models.

ii) The spin dependence of excitation energy of such states may be understood qualitatively from their relation to the precessional motion at large values of I and to the γ-vibrational states when I is small.

iii) Equation (2) derived from the three-axial rigid rotator model may lead to substantial errors in estimation of the excitation energy of precessional states.

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