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# A MODEL FOR ELASTIC SCATTERING OF PIONS BY VERY LIGHT NUCLEI

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Модель упругого рассеяния $\pi$-мезонов на легчайших ядрах

Получена система одномерных интегральных уравнений для описания упругого рассеяния $\pi$-мезонов на $^3He$. Исследована зависимость угловых и энергетических распределений от параметров формфактора ядра $^3He$.

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A Model for Elastic Scattering of Pions by Very Light Nuclei

A model for scattering of pions by very light nuclei is proposed. The model takes strictly into account multiple scattering. For the elastic scattering amplitude and for the "optical potential" a system of one-dimensional integral equations is obtained. The dependence of differential and total cross sections on parameters characterizing the form factor is investigated.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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As is well known, the $\pi$ - nucleus interaction is most frequently described by means of optical potential. The quantity called optical potential is calculated in first order with respect to the $\pi N$ t -matrix, i.e., in the single scattering approximation. Obviously, such a description of the $\pi$ -nucleus interaction is in a sense not consistent. The reason is that, in the equation for the $\pi$ -nucleus scattering amplitude effects of multiple $\pi$ -scattering are taken into account while, in the calculation of the optical potential they are neglected.

Further, the first order optical potential is frequently calculated in the impulse approximation, i.e., the t -matrix describing the interaction of a pion with a bounded nucleon is replaced by the free $\pi N$ - t -matrix. Although such a replacement (approximation) at first sight seems to be justified it involves besides kinematical uncertainties still another approximation, which may lead to a non-adequate descritpion of the considered process. This is because the t -matrix, describing the scattering of the pion by a bound nucleon, is a many-body operator which has a well defined symmetry with respect to a permutation of the target particles. Obviously, the free t -matrix, being a two-body operator, will never obey such a symmetry. Therefore, as the ground state wave function of the nucleus is a superposition of components with different symmetry with respect to permutation of the target particles, the first order optical potential calculated in the impulse approximation may differ very much from the exact first order potential *. Below we consider a simple model for elastic scattering of pions at $^3$He, in which the above-mentioned difficulties have been overcome. For simplicity two assumptions have been made: 1) The $\pi N$ -interaction is chosen in a simplified manner. 2) The target nucleus is described only by the ground state wave function.

Of course, due to these assumptions we cannot expect a good quantitative agreement with the experimental data; rather in the present paper we want to study the sensitivity of the differential and total cross section to the chosen $\pi N$ -potential and to the properties of the ground state of the nucleus.

We start with the many-body Lippman-Schwinger equation for the transition operator $T_{\pi A}$

$$T_{\pi A}(Z) = V + V\,G(Z)\,T_{\pi A}(Z), \tag{1}$$

where

$$V = \sum_{i=1}^{A} V_i^{\pi A}\ ;\ G(Z) = (Z - K_\pi - H_N)^{-1};\ Z = E + i\epsilon,$$

$K_\pi$, kinetic energy of the pion, $H_N$, target Hamiltonian. As it is well known, eq. (1) is not of the Frendholm type. Therefore, following Faddeev, we introduce "channel" amplitudes $T^i_{\pi A}$ defined by

$$\begin{aligned} T^i_{\pi A}(Z) &= V_i^{\pi N} + V_i^{\pi N} G(Z)\,T_{\pi A}(Z), \\ T_{\pi A}(Z) &= \sum_{i=1}^{3} T^i_{\pi A}(Z). \end{aligned} \tag{2}$$

For the amplitudes $T^i_{\pi A}(Z)$ one obtains in the usual way a set of equations

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* In this respect the three-body model for the first order optical potential is more adequate /1/.

$$T^{i}_{\pi A}(Z) = r_i(Z) + r_i(Z) G(Z) \sum_{j \neq i}^{3} T^{j}_{\pi A}(Z), \tag{3}$$

where $r_i$ satisfies the following equation:

$$r_i(Z) = V_i^{\pi N} + V_i^{\pi N} G(Z) r_i(Z). \tag{4}$$

Strictly speaking, the system of equations (3) and (4) is not yet of the Fredholm type and in the general case one would have to transform these equations once more by passing to the Yakubovsky [2] or AGS, ref.[3] equations. However, within the assumption 2) one can derive corresponding approximate equations which are of the Fredholm type. Indeed, within the aforesaid approximation we have for the target Green function

$$G(Z) \simeq \tilde{G}(Z) = \frac{|\Psi_0\rangle\langle\Psi_0|}{Z - K_\pi + |E_B|}, \tag{5}$$

where $|\Psi_0\rangle$ is the ground state vector and $E_B$ denotes the binding energy of the nucleus. Then, within the approximation (5) the equations (3) and (4) become a system of one-dimensional integral equations. Making use of the symmetry of the nucleus wave function, eq. (3) reads in the momentum representation:

$$\langle\vec{q}|T_{\pi A}|\vec{q}\,'\rangle = A\langle\vec{q}|r(i)|\vec{q}\,'\rangle + (A-1)\int\frac{d\vec{p}}{(2\pi)^3}\langle\vec{q}|r(i)|\vec{p}\rangle \times \tag{6}$$

$$\times\, G_0(p, Z+|E_B|)\,\langle\vec{p}|T_{\pi A}|\vec{q}\,'\rangle,$$

where

$$\langle\vec{q}|T_{\pi A}|\vec{q}\,'\rangle = \langle\vec{q}, \Psi_0|T^{i}_{\pi A}|\Psi_0\vec{q}\,'\rangle$$

is the needed $\pi$-nucleus scattering amplitude and

$$\langle\vec{q}|r(i)|\vec{q}\,'\rangle = \langle\vec{q},\Psi_0|r_i|\Psi_0,\vec{q}\,'\rangle$$

is the ground state average operator $r_i$. Here $\vec{q}$ denotes the $\pi$-nucleus relative momentum. $G_0$ stands for the free, energy shifted Green function of the pion. The amplitude $r$ satisfies the following equation:

$$<\vec{q}|r(i)|\vec{q}\,'> = <\vec{q}|v_i^{\pi N}|\vec{q}\,'> + \int \frac{d\vec{p}}{(2\pi)^3} <\vec{q}|v_i^{\pi N}|\vec{p}> G_0(p, Z + |E_B|) <\vec{p}|r(i)|\vec{q}\,'> . \tag{7}$$

For a local $\pi N$-potential the effective potential takes the form

$$<\vec{q}|v_i^{\pi N}|\vec{q}\,'> = V_i^{\pi N}(\vec{q}-\vec{q}\,')F(\vec{q}-\vec{q}\,'), \tag{8}$$

where $V^{\pi N}(\vec{p})$ is the elementary $\pi N$-potential and $F(\vec{p})$ denotes the nucleus form factor. Choosing the $\pi N$ potential as

$$V^{\pi N}(r) = -V_0 \delta(r-a) \tag{9}$$

after partial wave decomposition it reads

$$V_\ell^{\pi N}(q,q') = -V_0 4\pi a^2 j_\ell(qa) j_\ell(q'a). \tag{10}$$

Only the dominating P-wave potential has been included in the explicit calculations. The numerical calculations have been performed with the following parameters

$V_0 = 8.92$ MeV, $a = 0.61$ fm

which reproduce position and magnitude of the $\Delta_{33}$-resonance in P-wave $\pi$-nucleon scattering. However, these two parameters have proved to be not sufficient for a correct description of the width of the resonance.

The form factor $F(x), (x \equiv p^2)$ has been parametrized as

$$F(x) = e^{-cx}\left[1 - e^{\frac{\ln 2}{b}(b-x)}\right]^2.$$

Here the parameter b determibes the position of the minimum of the form factor, while c is defined by

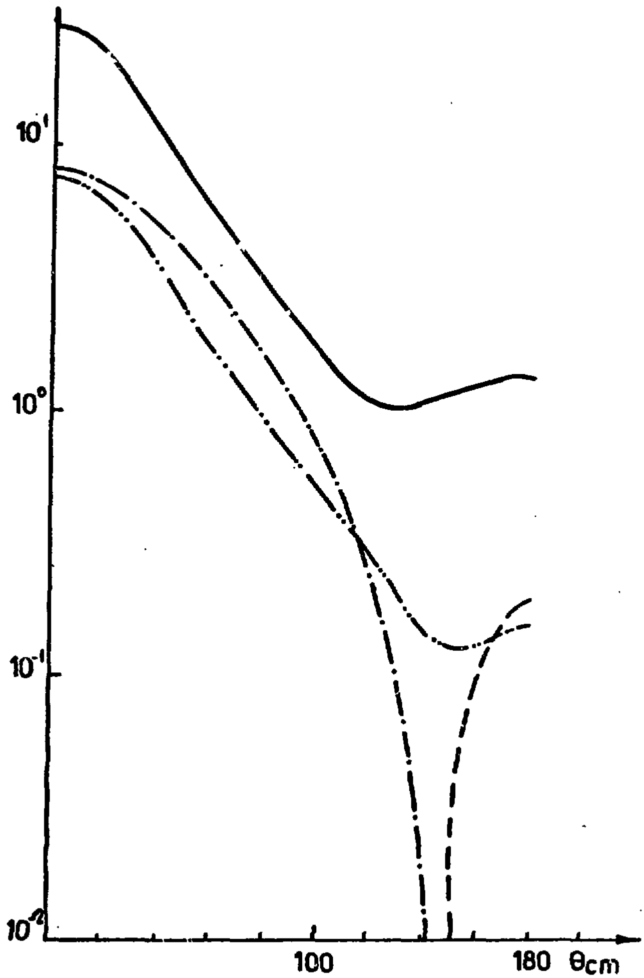


Fig. 1. Differential cross section of the $\pi^+ {}^3He$ elastic scattering at E = 290 MeV, —— $\sqrt{\overline{r^2}}$ = 1,92 fm; —··— $\sqrt{\overline{r^2}}$ = 1,82 fm; —·—·— $\sqrt{\overline{r^2}}$ = 1,65 fm.

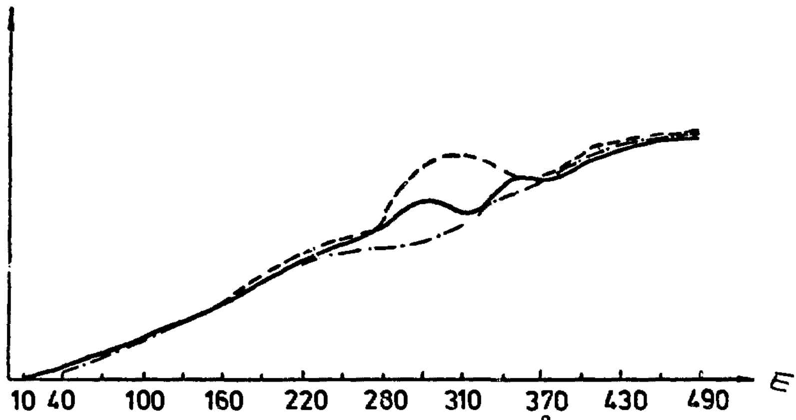


Fig. 2. Total cross section of the $\pi^{+3}$He scattering as a function of the pion energy for different parameters of the form factor $^3$He: – – – (b = 12 fm$^{-1}$, c = 0.3 fm), —·—·— (b = 15 fm$^{-1}$, c = 0.45 fm); ——— (b = 12 fm$^{-1}$, c = 0.45 fm).

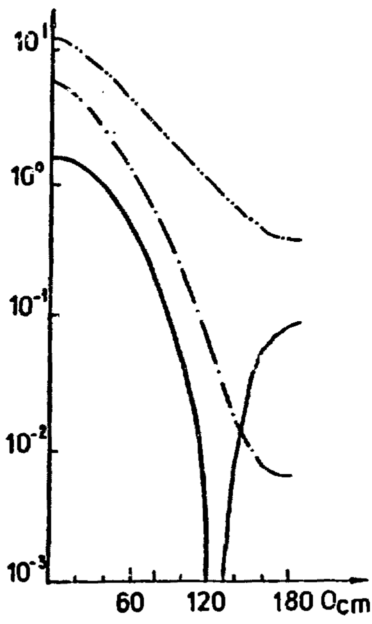


Fig. 3. Dependence of the angular distributions on energy at experimental values of the parameters of the form factor —··—··— E = 135 MeV; —·—·— 90 MeV; ——— 68 MeV.

$$c = \frac{1}{6}\langle r^2\rangle - 4\,\frac{\ln 2}{b}\,.$$

The values $b = 12\,\mathrm{fm}^{-2}$ and $c = 0.321\,\mathrm{fm}^2$, $(\sqrt{\langle r^2\rangle} = 1.82\ \mathrm{fm})$. correspond to the experimentally observed form factor of $^3$He.

The set of equations (6) and (7) was solved by expanding each partial wave of the form factor $F(\vec{q}-\vec{q}\,')$ in a series of 10 separable terms. Further, into the $\pi$-nucleus relative motion, four partial waves were included.

The numerical results are displayed in figs. 1-5.

Figure 1 shows the differential cross section at an energy of $E_\pi = 290$ MeV for different values of the root-mean-square radius of the three-body nucleus. With decreasing radius, i.e., with increasing nuclear density, the shape of the differential cross section changes drastically, what may indicate to a great importance of multiple scattering effects at this energy. Further we should mention that the $\pi N$-potential (10) shows a wrong behaviour for large $q, q'$, which is characteristic for all rapidly changing in space potentials (e.g., hard core potential, square well potential), namely it only slowly decreases at large $q$ or $q'$. In the present calculations this circumstance has caused an unnaturally strong dependence of the $\pi$-nucleus scattering on energy. This can be read off, for instance, fig. 2 where the total elastic cross section again slowly increases upon having passed the resonance energy. Moreover, this figure shows that, if the position of the minimum in the form factor, or (equivalently) the root mean square radius, is varied the total cross section changes only in the resonance region.

A further consequence of the wrong behaviour of the choosen $\pi N$ potential is the energy dependence of the position of the minimum in the differential cross section (see fig. 3). By including a cut off factor into the potential (10), the position of the minimum is immediately stabilized (see fig. 4). On the basis of this fact and of the effective po-

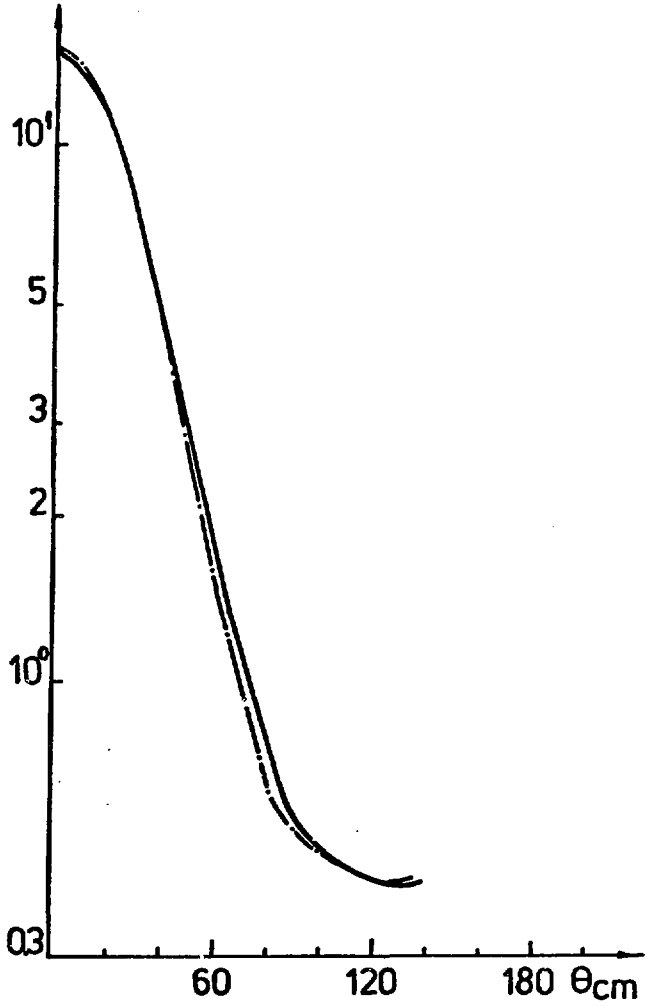


Fig. 4. Differential cross section of $\pi^{+3}$He scattering for an additional cut-off in the form factor: ——E = =290 MeV, —·—·— E=320 MeV.

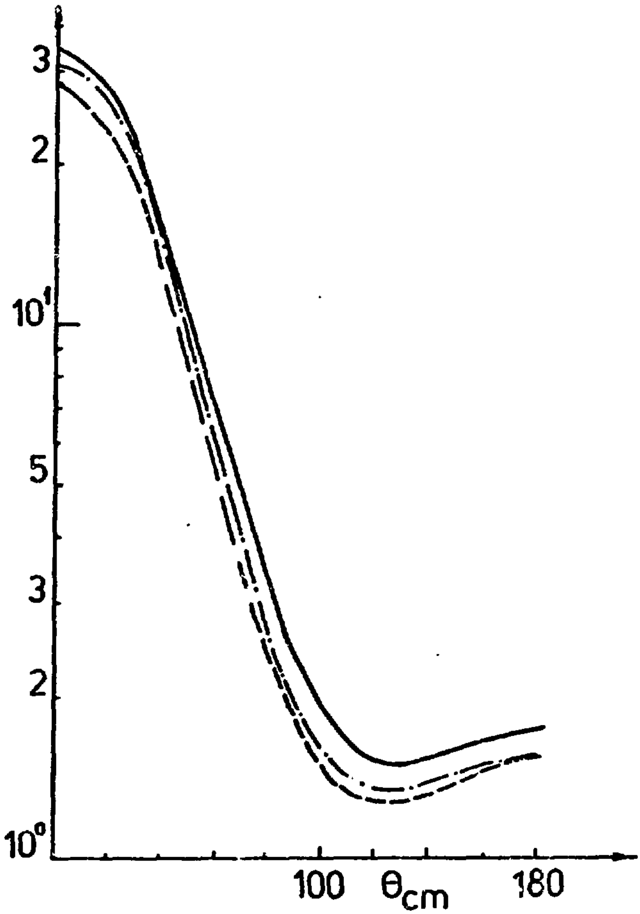


Fig. 5. Angular distributions of the $\pi^+ {}^3He$ scattering for different values of min in the form factor $^3He$ ($E = 290 MeV$): - - - $b = 15 fm^{-1}$ ; —·—·— $b = 8 fm^{-1}$ ; $b = 12 fm$
——— $b = 12 fm^{-1}$ .

tential (8) we suppose that the "mobility" of the angular distribution with energy observed for heavier nuclei is produced by the "extension" of their nuclear form factors in momentum space. Finally, fig. 5 shows how the position of the minimum in the differential cross section depends on the position of the minimum in the form factor in the resonance region. As is seen, only the depth of the minimum changes while its position remains fixed.

In this way, the presented model allows one to investigate easily the dependence of the observable quantities on any parameter of the problem, whereat care is strictly taken of the multiple scattering character of the considered process. A special feature of eqs. (6) and (7) is that they do not depend directly on the binding energy of the target nucleus. So, for small transferred momenta (small angle scattering) where one can expect that the contributions from the neglected part of the three-body Green's function are small and that the model works well, the scattering is determined by the density distribution in the nucleus and by the number of particles. Let us now consider in more detail the approximation (1). As the $\pi N$-potential has been fitted to the $P_{33}$ phase our model, strictly speaking describes the scattering of pions by three bounded protons rather than by the nucleus $^3He$. However, this does not influence the position of the minimum in the differential cross section. This result was obtained by studying the other limiting case where the pion does not interact at all with the neutron but only with two protons. Indeed, the differential cross section has not changed.

As the proposed model does not include approximation except (1) and (2), we are led to the conclusion that the shift of the minimum in the differential cross section toward large angles relative to the experiment /4/ results either from the unrealistic form of the chosen $\pi N$-potential or from contributions from absorption and emission of the pion in the multiple scattering processes. In the conventional optical model[5] and in the fixed scatterer approximation[6] the minimum in the angular distribution is turned back into the region of "small" angles (80°- 90°) by including the nuclear recoil. In our above considered model the few-body kinematic is exactly taken into account. Therefore, there is no necessity for an additional shift in the nucleon momenta.

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