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NON-R-MATRIX SHELL MODEL APPROACH TO α-DECAY OF SPHERICAL NUCLEI

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NON-R-MATRIX SHELL MODEL APPROACH TO α-DECAY OF SPHERICAL NUCLEI

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Не R-матричный оболочечный подход к α-распаду сферических ядер. II. Необлегченные α-переходы в ^{210}Bi и ^{212}Po

Расчитаны абсолютные и относительные вероятности необлегченных α-переходов из изомерных состояний ^{212m}Po и ^{210m}Bi, а также из основного состояния ядра ^{210}Bi на основные и возбужденные состояния дочерних ядер. Расчеты проведены с использованием интегральной формулы для α-ширины в рамках сформулированного оболочечного подхода.

Сравнение с экспериментом показывает, что теория удовлетворительно воспроизводит относительные α-ширины, тогда как их абсолютные величины сказываются в 20-300 раз меньше экспериментальных. Эта разница уменьшается с ростом орбитального момента α-частиц.

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Non-R-matrix Shell Model Approach to α-Decay of Spherical Nuclei. II. Unfavoured α-Transitions in ^{210}Bi and ^{212}Po Nuclei

The probabilities of unfavoured α-transitions from isomeric states of ^{212}Po and ^{210}Bi nuclei and from the ground state of ^{212}Bi nucleus are calculated using the integral formula for α-widths in the frame of the shell model approach. The comparison with the experiment shows that the theory reproduces satisfactorily the relative α-widths, but the absolute values are 20-300 times smaller than the experimental ones. This difference decreases with increasing α-particle orbital momentum.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1978

1. Introduction

Each theory of α-decay has to reproduce the relative probabilities of α-transitions and then it can seriously pretend to obtain their absolute values. In this connection, the study of unfavoured α-decays of nuclei in the vicinity of the ^{208}Pb nucleus may serve as an important test of the integral formula for α-widths in the framework of the shell model approach presented in paper I.* We investigate here the possibilities of our theory [1] to describe the experimental α-widths using as an example the unfavoured α-decay from the ground state of ^{210}Bi nucleus and from isomeric states of ^{212}Po and ^{210}Bi nuclei. For the chosen nuclei the knowledge of initial and final states of α-decay is sufficient for calculating the probabilities of unfavoured α-transitions.

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* Here the references to formulae from paper I (JINR, E4-11286, Dubna, 1978) is done as follows: (I, the number of the formula).

In section 2 the calculation of relative and absolute α-widths for ^{210}Bi is undertaken. The probabilities of α-decay of a high-spin isomeric state of ^{212}Po nucleus to the ground state and 3^- and 5^- states of the ^{208}Pb daughter nucleus are calculated in sec. 3. Using the classification of α-transitions exposed in paper I the spin and parity assignation for the isomeric state ^{212m}Po is discussed. Finally, in section 4 we analyse the relation between the theoretical and experimental absolute α-widths.

2. Theoretical α-decay widths for ^{210}Bi nucleus

Let us consider the α-decay of ^{210}Bi nucleus, having one proton and one neutron over the closed shells of ^{208}Pb core. We study the unfavoured α-transitions from 1^- and 9^- states of ^{210}Bi nucleus leading to 1^- and 2^- states of ^{206}Tl nucleus (see fig. 1). The initial states can be described [1] by the

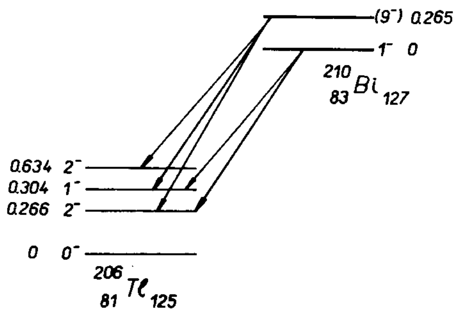

Fig. 1. Energy levels of ^{206}Tl and the alpha decay scheme of ^{210}Bi.

Table 1

The comparison between theoretical and experimental α-widths for unfavoured transitions of ^{210}Bi nucleus

$I_i^{\pi_i} \longrightarrow I_f^{\pi_f}$	Γ_α^{exp} (MeV)	L	With configuration mixing		Without configuration mixing	
			Γ_α^{SM} (MeV)	$\Gamma_\alpha^{exp}/\Gamma_\alpha^{SM}$	$\Gamma_\alpha^{pure}(L)$ (MeV)	$\Gamma_\alpha^{exp}/\Gamma_\alpha^{pure}$
1^- 1^-	7.86(-34)	0	1.65(-36)	365	1.59(-36)	328
		2	0.51(-36)		0.81(-36)	
1^- 2^-	5.24(-34)	2	1.7(-36)	308	2.94(-36)	178
9^- 1^-	1.48(-36)	8	0.28(-38)	250	0.035(-38)	242
		10	0.32(-38)		0.581(-38)	
9^- 2^-	2.37(-36)	8	2.75(-38)	82	5.20(-38)	43
		10	0.15(-38)		0.29(-38)	

multiplet $\left[1h_{\frac{9}{2}}\ 2g_{\frac{9}{2}}\right]_{I_i^{\pi_i}}$ with a weak admixture of other configurations. For the states of ^{206}Tl nucleus a more rich configuration mixing takes place[2,3], but the energy spectrum of excited levels is not very well reproduced by the theory. Using the wave functions from refs.[1-3] the α-decay widths Γ_α^{SM} were calculated by means of formula (I.9'). In table 1 the theoretical widths are compared with experimental data[4]. It can be seen that the relative probabilities of α-transitions from the ground-state of ^{210}Bi nucleus to 1^- and 2^- states of ^{206}Tl nucleus are well reproduced. For the transitions from isomeric state 9^- to the same final states the agreement between theory and experiment is not so good. This fact is connected with the accidental suppression of the contribution of dominating components of wave functions of initial and final states for angular momentum L =8 in the sum (I.9'). The above suppression which is due to the smallness of the geometrical factor (I,10') makes the partial (L =8) α-width small and dependent mainly on the weak components of 1^- state of ^{206}Tl, which are not so well determined. The role of configuration mixing for the studied α-transitions can be clarified by comparing the values of α -widths calculated without (Γ_α^{pure}) and with configuration mixing (Γ_α^{SM}) as it is done in table 1. The absolute values do not essentially change: they are somewhat lowered when configuration mixing is taken into account. However, the configuration mixing improves the relative α -widths. As it has been noted, the relative probabilities of α -transitions for the ground state and isomeric state of ^{210}Bi nucleus, when considered separately, are in acceptable agreement with the experiment.

On the other hand, the ratio $\omega = \Gamma_\alpha^{exp}/\Gamma_\alpha^{SM}$ turns out to be dependent on the angular momentum L , which varies from 0 to 10 for the α -transitions under consideration. The further discussion of this point shall be done in section 4.

3. Description of α -decay of the ^{212m}Po isomer

The nucleus ^{212}Po has two protons and two neutrons over the ^{208}Pb core and consequently its structure is more complicated than that of ^{210}Bi nucleus. The α -transition from the ground state of ^{212}Po nucleus to the ground state of ^{208}Pb nucleus is favoured, while all the transitions from the isomeric state ^{212m}Po, with excitation energy E^* =2.93 MeV (see fig.2) are unfavoured. The spin and parity of the isomeric state are not determined experimentally. The study of α -decay of the

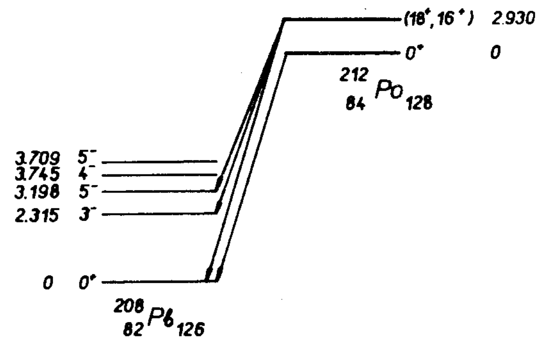

Fig. 2. Energy levels of ^{208}Pb and the alpha decay scheme of ^{212m}Po.

isomeric state ^{212m}Po is unique way to get information about its structure, since the deexcitation through α -transitions is strongly forbidden. The diagonalization of the shell model Hamiltonian leads[5] to the fact that the states with characteristics $I_i^{\pi_i}=18^+$, 16^+, 14^+ and 12^+ lie lower than the isomeric state ^{212m}Po. The lowest level with $I_i^{\pi_i}=18^+$ has the energy $E^*=2.26$ MeV. In table 2 the results are presented for α-decay widths of ^{212m}Po isomeric state, calculated with initial wave functions from ref.[5] and the final wave function of ^{208}Pb nucleus from ref.[6]. Two hypotheses for the isomeric state were verified, namely $I_i^{\pi_i}=18^+$ and 16^+.

As it can be seen from table 2 the calculated width for the α-transition $16^+ \rightarrow 0^+$ is greater than the experimental one. But the results for ^{210}Bi nucleus (section 2) and the theoretical arguments presented in section 4 give some evidence that the widths Γ_α^{SM} cannot be greater than the experimental ones. Thus, the assignation $I_i^{\pi_i}=16^+$ with wave function from ref.[5] for the isomeric state ^{212m}Po may be rejected, as well as the smaller values of the spin for this state. By assigning $I_i^{\pi_i}=18^+$ we reproduce the relative α-decay probabilities (see table 2). Meanwhile, the values of the ratios $\omega=\Gamma_\alpha^{exp}/\Gamma_\alpha^{SM}$ are approximately the same as for α-decay of ^{210}Bi nucleus.

Table 2

The comparison between theoretical and experimental α-widths for the isomeric state ^{212m}Po, calculated with wave function from ref.[5]

$I_i^{\pi_i} \rightarrow I_f^{\pi_f}$	L	$\Gamma_\alpha^{SM}(L)$	$\Gamma_\alpha^{exp}/\Gamma_\alpha^{SM}$
16^+ 0^+	16	3.03(-23)	0.32
18^+ 0^+	18	3.8 (-25)	25.5
18^+ 3^-	15	2.72(-27)	36.8
18^+ 5^-	13	5.85(-27)	34.2
	15	2.98(-30)	

Another way to describe the structure of isomeric state ^{212m}Po is proposed in ref.[7]. In the frame of simple model of four nucleons above the inert core, one derives, using the Green function technique the energy $E^*=2.92$ MeV and the spin $I_i^{\pi_i}=16^+$ for the isomeric state. In table 3 the results of our calculation using the wave function[7] $|(P_iN_i)I_iM_i\rangle = [(1h_{\frac{9}{2}})^2_8(2g_{\frac{9}{2}})^2_8]_{16^+}$ are presented. As can be seen the experimental relative widths are reproduced and the values of ratio ω are in agreement with the same values obtained in section 2. Thus, it is difficult to make a choice between the two possibilities of describing the structure of ^{212m}Po isomeric state: $I_i^{\pi_i}=16^+$ with the wave function from ref.[7] and $I_i^{\pi_i}=18^+$, with the wave function from ref.[5] This situation appears as a consequence of a specific property of α-particle preformation probability, as it was stated in paper I. Namely, the amplitude of α-performation probability is about 10 times larger for α-configurations

$$P_\alpha N_\alpha L = [[(n_1l_1j_1)\neq(n_2l_2j_2)]_{J_{12}}[(n_3l_3j_3)\neq(n_4l_4j_4)]_{J_{34}}]_L$$

with protons as well as neutrons in different states, than

for the configurations $P_\alpha N_\alpha L = [(n_1 l_1 j_1)^2_{J_{12}} (n_3 l_3 j_3)^2_{J_{34}}]_L$.
From this point of view, the main configuration of 18^+ state gives the first type of the α -configuration, while the wave function[7] of 16^+ state gives the second one. In terms of the conventional R-matrix theory we can say that the decrease of the penetrability when one goes from angular momentum L =16 to L =18 is compensated by the increase of the α -performation probability.

Finally, let us note that the previous attempt[8] to describe the relative α -widths for the isomer ^{212m}Po in the framework of the R-matrix theory and under the point α -particle approximation has been unsuccesfull although various values for the channel radius were taken.

4. The absolute α -widths, the possibilities and limitations of theory

At first we analyse the influence of used approximations on the values of ratio $\omega = \Gamma_\alpha^{exp}/\Gamma_\alpha^{SM}$. In our calculations of α -widths Γ_α^{SM} the contribution of residual interactions in the potential $V_{\alpha A-4}$, eq. (I,8) has been neglected. Taking into account these interactions one obtains[9] that the theoretical α -widths decrease by a factor less than 2 and consequently the values of ω become larger. As it was shown in ref.[10] the replacement of the conventional internal wave function of α -particle (I,16) by the function[11] giving the correct form factor of ^4He nucleus is followed by the diminish of α -widths. Besides that, the calculations from ref.[10]

Table 3

The same as for table 2, but with the wave function from ref.[7]

$\Gamma_i^{\pi_i} \to \Gamma_f^{\pi_f}$	L	$\Gamma_\alpha^{SM}(L)$	$\Gamma_\alpha^{exp}/\Gamma_\alpha^{SM}$
$16^+ \to 0^+$	16	6.1(-25)	15.9
$16^+ \to 3^-$	13	2.59(-27)	37.5
	15	8.22(-31)	
$16^+ \to 5^-$	11	3.94(-27)	49.8
	13	8.56(-29)	
	15	1.77(-32)	

indicate an attenuation of L -dependence of ratio ω, as obtained in section 2. We did not give the numerical results from ref.[10] provided that there the oscillator shell model basis as well as the approximation (I.20) have been used.

Finally, the values of α -widths Γ_α^{SM} are rather stable[I] against the variations of shell model parameters in range of their phenomenological uncertainties. Thus, the disagreement between the absolute theoretical and experimental α -widths can not be eliminated, being an essential feature of shell model approach to integral formula of α -decay.

We may understand better this conclusion using the approximation (I,19) for α -widths as being sufficiently accurate for our discussion.

In this approximation we introduce the radial function of the final channel $\mathcal{U}^{SM}_{P_\alpha N_\alpha L}(R)$ which is the result of overlapping of the wave function of the parent nucleus (I,6) and the final channel wave function (I,3). On the other hand, we may obtain the so-called "experimental" radial channel function $\psi_c(R)$, having a correct asymptotic behaviour in the barrier region. The function $\psi_c(R)$ is obtained[12-13] by integrating the Schrödinger equation with the potential $V_{\alpha A-4}(R)$, eq. (I,20), starting from the correct boundary conditions[12-13]. The function $\psi_c(R)$ is defined up to the point $R = R_{ce} = R_A + 1.5\,fm$ (see ref.[12]) lying near to the surface of parent nucleus, where the α-particle and the daughter nucleus may be yet considered as non overlapping.

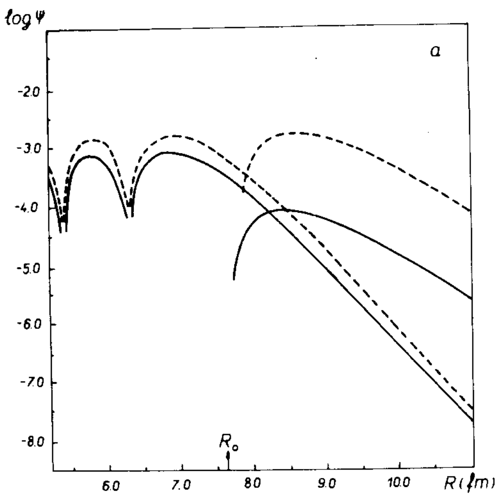

Fig. 3. The comparison of functions $\psi^{SM}_{R_\alpha N_\alpha L}(R)$ and $\psi_c(R)$ for α-transitions of ^{210}Bi nucleus: $1^- \longrightarrow 1^-$ (L =0)-dotted lines and $9^- \longrightarrow 2^-$ (L =8)-solid lines. The functions $\psi^{SM}_{R_\alpha N_\alpha L}$ are labeled by number 1, the functions ψ_c - by number 2.

In fig. 3 the functions $\psi^{SM}_{R_\alpha N_\alpha L}(R)$ and $\psi_c(R)$ are represented for the case of unfavoured α-decay from the ground and isomeric states of ^{210}Bi nucleus. The distinct behaviour of these two functions in the surface region of parent nucleus permits one to understand qualitatively the above obtained difference between the theoretical and experimental α-widths. It is interesting to note that with increasing orbital momentum L the difference between the functions $\psi^{SM}_{R_\alpha N_\alpha L}(R)$ and $\psi_c(R)$ is attenuated. This fact is responsible for the α-dependence of ratios $\omega = \Gamma_\alpha^{exp} / \Gamma_\alpha^{SM}$.

Keeping in mind the results for the unfavoured α-transitions, it is interesting to perform the consistent calculations for the favoured α-transitions to clear out in what extent the obtained situation is general.

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