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NON-R-MATRIX SHELL MODEL APPROACH TO α-DECAY OF SPHERICAL NUCLEI

I. Basic Features of α-Decay Width Calculations with Woods-Saxon Shell Basis

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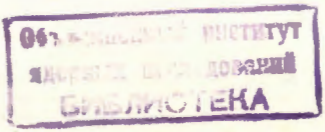

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Фурман В.И. и др. **E4 - 11286**

Не R-матричный оболочечный подход к α-распаду сферических ядер. I. Основные особенности расчета α-ширин с оболочечным базисом потенциала Вудса-Саксона

Сформулирован оболочечный подход к рассмотрению α-распада, основанный на использовании интегральной формулы для α-ширины. Описана методика и проведены расчеты α-ширин для оболочечного базиса с потенциалом Вудса-Саксона, учитывающие конечные размеры α-частицы. Дано критическое обсуждение использованных ранее приближений для подобных расчетов. С помощью расчитанных без свободных параметров α-ширин проведена классификация α-переходов. Из ее анализа следует, что остаточные взаимодействия главным образом ответственны за существование различных типов α-переходов.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1978

Furman V.I. et al. **E4 - 11286**

Non-R-Matrix Shell Model Approach to α-Decay of Spherical Nuclei. I. Basic Features of α-Decay Width Calculations with Woods-Saxon Shell Basis

A shell model approach to α-decay calculations for spherical nuclei based on the integral formula for α-decay widths is formulated. Calculating the overlap integrals with Woods-Saxon shell model basis and accounting for the finite size of the α-particle the earlier used approximations are discussed. A new procedure for the classification of α-transitions is proposed. From this classification it follows that the residual interactions are responsible for the existence of various types of α-transitions.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1978

1. Introduction

This paper is the first one from a serie of three papers (quated as refs. I, II and III) dedicated to a consistent analysis of α-decay transitions of heavy spherical nuclei.

In refs.[1,2] it was shown that the α-decay width of a spherical nucleus with mass number A can be expressed by the integral formula

$$\Gamma_\alpha = 2\pi \sum_L \left| \langle \mathcal{A}\{\mathcal{F}_L \mathcal{U}_{Lf}^{I_i M_i} \mid V_{\alpha A-4} \mid \mathcal{Y}_i^{I_i M_i}\} \rangle \right|^2 \quad (1)$$

where the wave function $\mathcal{Y}_i^{I_i M_i}$ describes the decaying nucleus and the potential $V_{\alpha A-4}$ stands for the nuclear part of the interaction between α-particle and final nucleus and can be expressed by the sums of nucleon-nucleon potentials

$$V_{\alpha A-4} = \sum_{i=1}^{4} \sum_{j=5}^{A} V_{ij} = \sum_{i>j=1}^{A} V_{ij} - \sum_{i>j=4}^{A} V_{ij} - \sum_{i>j=1}^{4} V_{ij}. \quad (2)$$

The final channel function $\mathcal{U}_{Lf}^{I_i M_i}$ has the form

$$\mathcal{U}_{Lf}^{I_i M_i} = \sum_{M_f M} C_{M_f M M_i}^{I_f L I_i} Y_{LM}(\Omega_{\vec{R}}) \mathcal{Y}_\alpha \mathcal{Y}_f^{I_f M_f}. \quad (3)$$

Here the wave functions ψ_α and $\psi_f^{I_f M_f}$ describe the internal motion of α -particle and of the final nucleus, while the quantum number L is related to the orbital momentum of their relative motion. The function $\mathcal{F}_L(R)$ in eq.(1) is expressed with the help of the regular coulombian function $F_L(R)$ as follows

$$\mathcal{F}_L(R) = \sqrt{\frac{K_\alpha}{\pi Q_\alpha}}\; F_L(R)/R \qquad (4)$$

since it is normalized to δ -function in energy. The variable R stands for the distance between the centres of mass of α -particle and final nucleus. The value of K_α is related to the α -decay energy Q_α by the formula $K_\alpha = \sqrt{Q_\alpha 2M_\alpha}/\hbar$. The operator $\mathcal{A}$ guarantees the antisymmetrization over the coordinates of all A nucleons. The calculation of α -widths based on formula (1) implies the use of some model representation for the functions $\psi_i^{I_i M_i}$, $\psi_f^{I_f M_f}$, ψ_α and for the potential $V_{\alpha A-4}$.

The shell model for the initial and final nuclei has to be used as it was already done in the frame of the R-matrix theory of α -decay[3,4]. The potential $V_{\alpha A-4}$ has to be taken consistent with the same shell model. In section 2 of the present paper the shell model approach to α -decay width calculations based on formula (1) is described and its new features are analysed in comparison with the R-matrix formula.

A recently developed method[5] has allowed us to calculate the α -widths taking correctly into account the finite size[6] of the α -particle, when the Woods - Saxon shell model basis is used, as having a correct behaviour at the nuclear surface. In section 3 the properties of basic matrix elements occuring in

α -decay widths calculations are presented, and the previous approximations used in similar calculations are discussed.

Comparing the α -widths calculated for a large number of spherical nuclei in the frame of the independent particle shell model with the corresponding experimental widths, evidence is given for the influence of residual nucleon-nucleon interactions on α -transition probabilities. This comparison also leads us to a classification of α -transitions which is discussed in section 4.

2. The shell model approach to the integral formula of α -decay widths

The basic idea of the shell model is that the interaction in a system of A nucleons may be represented by the selfconsistent field $V_i(x_i)$ and residual interactions $v_{ij}(x_i, x_j)$ Assuming that for heavy nuclei the potentials V_i and v_{ij} do not noticeably vary with the change of the mass number from A to A-4 , the interaction from eq.(2) may be rewritten as follows

$$V_{\alpha A-4} = \sum_{i=1}^{4} V_i + \sum_{i<j=1}^{4} (v_{ij} - V_{ij}) + \sum_{i=1}^{4}\sum_{j=5}^{A} v_{ij} . \qquad (5)$$

The wave functions for the initial and final nuclei are

$$\psi_i^{I_i M_i} = \sum_{P_i N_i} C^{P_i N_i} \,|\, (P_i N_i)\, I_i M_i \rangle \qquad (6)$$

and

$$\psi_f^{I_f M_f} = \sum_{P_f N_f} C^{P_f N_f} \,|\, (P_f N_f)\, I_f M_f \rangle , \qquad (7)$$

where the constants C^{PN} are the mixing coefficients for the pure configurations $|(PN)IM\rangle$. Symbol P(N) denotes the state with total angular momentum $J_p(J_N)$ of the proton (neutron) system and symbols IM are the spins of the nucleus and its projection.

Let us consider now only the first two sums in formula (5) assuming that the third one leads to a renormalization of potentials $V_i(r_i)$ only:

$$V_{\alpha A-4} = \sum_{i}^{4} V_i + \sum_{i<j}^{4} (v_{ij} - V_{ij}) . \qquad (8)$$

This is equivalent to the commonly adopted in the nuclear reaction theory "diagonal" (over (A-4) variables of the core) approximation. After the genealogic expansion of function $\psi_i^{I_i M_i}$ (A → (1,2,3,4 + (A-4)) and integration over the coordinates of particles 5,...,A, formula (1) for the α-decay widths yields

$$\Gamma_\alpha^{SM} = 2\pi \sum_{L} \Big| \sum_{P_i N_i P_f N_f} C^{P_i N_i} C^{P_f N_f} \sum_{j_{12} j_{34}} G^{P_\alpha N_\alpha L}_{P_i N_i P_f N_f} M_{P_\alpha N_\alpha L} \Big|^2 , \qquad (9)$$

where the "geometrical factor"

$$G^{P_\alpha N_\alpha L}_{P_i N_i P_f N_f} = \hat{J}_{P_i} \hat{J}_{N_i} \hat{I}_f \hat{L} \begin{Bmatrix} J_{P_f} & J_{N_f} & I_f \\ j_{12} & j_{34} & L \\ J_{P_i} & J_{N_i} & I_i \end{Bmatrix} \langle P_f P_\alpha \| P_i \rangle \langle N_f N_\alpha \| N_i \rangle \qquad (10)$$

includes the fractional parentage coefficients[4] $\langle P_f P_\alpha \| P_i \rangle$, $\langle N_f N_\alpha \| N_i \rangle$ for separating the proton pair in the state $P_\alpha = (n_1 \ell_1 j_1, n_2 \ell_2 j_2)_{j_{12}}$ with angular momentum j_{12} and the neutron pair in the state $N_\alpha = (n_3 \ell_3 j_3, n_4 \ell_4 j_4)_{j_{34}}$ with momentum j_{34}. The 9 j symbol yields the angular momentum selection rules. (The notation $\hat{L} = \sqrt{2L+1}$ was used).

The matrix element $M_{P_\alpha N_\alpha L}$ can be written in the form

$$M_{P_\alpha N_\alpha L} = \int \Theta_{P_\alpha N_\alpha L}(R)\, \mathcal{F}_L(R)\, R\, dR , \qquad (11)$$

where

$$\Theta_{P_\alpha N_\alpha L}(R) = S_{P_\alpha N_\alpha} \sqrt{8} \int \Phi^{LM}_{P_\alpha N_\alpha}(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4) \times$$
$$\times V_{\alpha A-4}(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4)\, \chi_\alpha(\vec{\xi}_1, \vec{\xi}_2, \vec{\xi}_3)\, Y_{LM}(\Omega_{\vec{R}})\, d\vec{\xi}_1 d\vec{\xi}_2 d\vec{\xi}_3 d\Omega_{\vec{R}} . \qquad (12)$$

Function $\Phi^{LM}_{P_\alpha N_\alpha}$ represents the spatial part of the four nucleon wave function

$$\Phi^{LM}_{P_\alpha N_\alpha} = \Big[\big[\psi_{n_1 \ell_1 j_1}(\vec{r}_1)\, \psi_{n_2 \ell_2 j_2}(\vec{r}_2) \big]_{j_{12}} \big[\psi_{n_3 \ell_3 j_3}(\vec{r}_3)\, \psi_{n_4 \ell_4 j_4}(\vec{r}_4) \big]_{j_{34}} \Big]_{LM} , \qquad (13)$$

where indices 1,2 stand for the proton, while 3,4 for the neutron states. Functions $\psi_{n_i \ell_i j_i}$ are the one-particle functions of the shell model basis corresponding to a nuclear potential $V_i(r_i)$. The function χ_α is the spatial part of the internal α-particle function Ψ_α. The factor $S_{P_\alpha N_\alpha}$

$$S_{P_\alpha N_\alpha} = \frac{1}{2} (2 - \delta_{n_1 n_2} \delta_{\ell_1 \ell_2} \delta_{j_1 j_2})^{1/2} (2 - \delta_{n_3 n_4} \delta_{\ell_3 \ell_4} \delta_{j_3 j_4})^{1/2} \times$$
$$\hat{j}_1 \hat{j}_2 \hat{j}_3 \hat{j}_4 (-1)^{\ell_1 + j_2 + \ell_3 + j_4 + 1} \begin{Bmatrix} \ell_2 & j_2 & 1/2 \\ j_1 & \ell_1 & j_{12} \end{Bmatrix} \begin{Bmatrix} \ell_4 & j_4 & 1/2 \\ j_3 & \ell_3 & j_{34} \end{Bmatrix} \qquad (14)$$

is the result of the transformation from the (jj) to (ℓs) coupling scheme, of the antisymmetrization in the two particle

states $P_\alpha(N_\alpha)$ and of the summation over the spin variables. The following spatial coordinates were used above

$$\vec{\xi}_1 = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2) \qquad \vec{\xi}_3 = \frac{1}{2}(\vec{r}_1 + \vec{r}_2 - \vec{r}_3 - \vec{r}_4)$$
$$\vec{\xi}_2 = \frac{1}{\sqrt{2}}(\vec{r}_3 - \vec{r}_4) \qquad \vec{R} = \frac{1}{4}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4). \qquad (15)$$

The factor $\sqrt{8}$ from eq.(11) is related to the replacement of integration variables $\{\vec{r}_i\}$ by $\{\vec{\xi}_i, \vec{R}\}$. The method[5] of separating the centre of mass motion of clusters in nuclei permits us to perform the integration in formula (12) with practically any form of functions $\psi_{n_i l_i j_i}(\vec{r}_i)$ and χ_α and of potentials (8).

However in this paper we shall use the conventional function χ_α :

$$\chi_\alpha(\xi_1, \xi_2, \xi_3) = \left(\frac{\beta}{\pi}\right)^{3/4} e^{-\frac{\beta}{2}(\xi_1^2 + \xi_2^2 + \xi_3^2)}$$
$$\beta = 0.434\ fm^{-2} \qquad (16)$$

though it does not fit very well the form factor of electron scattering on the ^{4}He nucleus [7]. For the possibility to work with an improved α -particle function[7] and for the effects which lead to using this function see ref.[8]

For some calculations (see paper II*)it is useful to separate in explicit form the dependence of matrix elements on quantum numbers j_{12} and j_{34} by means of an idea exposed by Mang[3].

* JINR, E4-11287, Dubna, 1978.

In this case we can write:

$$\Gamma_\alpha^{SM} = 2\pi \sum_L \Big| \sum_{P_i N_i P_f N_f} C^{P_i N_i} C^{P_f N_f} \tilde{G}^{l_1 l_2 l_3 l_4 L}_{P_i N_i P_f N_f} \tilde{M}^{n_1 l_1 n_2 l_2}_{n_3 l_3 n_4 l_4 L} \Big|^2, \qquad (9')$$

where

$$\tilde{G}^{l_1 l_2 l_3 l_4 L}_{P_i N_i P_f N_f} = \sum_{j_{12} j_{34}} G^{P_\alpha N_\alpha L}_{P_i N_i P_f N_f} S_{P_\alpha N_\alpha} C^{l_1 l_2 j_{12}}_{0\,0\,0} \times C^{l_3 l_4 j_{34}}_{0\,0\,0} C^{j_{12} j_{34} L}_{0\,0\,0} \qquad (10')$$

$$M_{P_\alpha N_\alpha L} = C^{l_1 l_2 j_{12}}_{0\,0\,0} C^{l_3 l_4 j_{34}}_{0\,0\,0} C^{j_{12} j_{34} L}_{0\,0\,0} \times S_{P_\alpha N_\alpha} \tilde{M}^{n_1 l_1 n_2 l_2}_{n_3 l_3 n_4 l_4 L}. \qquad (11')$$

However, further it is convenient to work with the previous forms of the matrix elements $M_{P_\alpha N_\alpha L}$ (11).

Since α -decay is a phenomenon which depends strongly on the surface properties of the nucleus[1,3], the asymptotic behaviour of the self consistent field and of the shell model functions may be important here. Below we shall use the Woods - Saxon potentials[10].

$$V_i(r_i) = V_{oi} / (1 + \exp((r_i - R_A)/a_i))$$
$$R_A = r_{oi} A^{1/3} \qquad (17)$$

Henceforth one uses functions $\psi_{n_i l_i j_i}$ which are the solutions of Schrödinger equation containing the nuclear potential (17) and a corresponding spin-orbit part.

Let us discuss some properties of the residual interaction v_{ij} which is important for describing the α-decay phenomenon. It is known[9] that the interaction v_{ij} transforms into the free particle interaction V_{ij} when the centre of mass of a pair (i,j) moves away from the centre of the nucleus.

In principle the functions (6) and (7) have been calculated with the same residual interactions v_{ij} which appear in the potential (8). First such calculations using the finite-size interactions v_{ij} have been done in ref.[11] for the α-decay of some spherical nuclei in the vicinity of the ^{208}Pb nucleus.

In paper[12] it was shown that the contribution to the values of α-widths of the second sum of the potential (8) with the parametrization[11] for the density-dependent interaction v_{ij} does not exceed 30%. Thus we shall calculate α-decay widths using formulae (9) - (15) under approximation

$$V_{\alpha A-4} = \sum_{i=1}^{4} V_i(r_i) \qquad (18)$$

Now let us return to the form (11) of the matrix element $M_{\beta N L}$. While the function $F_L(R)$ grows rapidly with increasing radius R, the function $\Theta_{\beta N L}(R)$ decreases outside the nucleus (for $R \gtrsim R_A$), since it contains one particle functions $\psi_{n_i l_i j_i}(\vec{r}_i)$ and potentials $V_i(r_i)$. Hence, the integrand in formula (11) will have a sharp maximum in the vicinity of the nuclear surface and this maximum will determine the value of the matrix element $M_{\beta N L}$ [1]. The value $R = R_c^{\beta N L}$ for which this maximum occurs may be considered, getting some analogy with the case

of the R-matrix theory[13], as an effective channel radius. It is clear that in this case the "channel radius" is not an arbitrary parameter as in the R-matrix theory, but it is defined automatically. These considerations are also true when residual interactions in the potential $V_{\alpha A-4}$ (8) are taken into account[2]. It should be noted that the more deep analogy with the R - matrix theory can be got under some approximations (see section 3), since in the frame of the present theory, the amplitude of the probability for the α-particle preformation which is proportional to the function $\Theta_{\beta N L}(R)$, depends on nuclear structure and on concrete form of the potential $V_{\alpha A-4}$, i.e., on the dynamics of the α-decay process.

3. The investigation of the matrix elements $M_{\beta N L}$

In calculating the matrix elements $M_{\beta N L}$ as defined by formulae (11) - (14) and (16) we used the results for the four particle overlap integral, ref.[5] generalized here for the inclusion of potential (18). First we have to fix the parameters of shell model potential (17). As it can be seen from table 1 the influence of different choices of phenomenological sets of shell model parameters [14-16] on the values of matrix elements $M_{\beta N L}$ and consequently of α-decay widths is not essential. Throughout this paper the parameters from ref.[14] are used.

In order to understand qualitatively the properties of matrix elements $M_{\beta N L}$ the following form[6] of fun ction is useful

$$\Theta_{\beta N L}(R) = V(R)\,\psi_{\beta N L}(R) . \qquad (19)$$

Here the dependence of potential (18) on variables $\{\vec{r}_i\}$ is neglected, i.e.,

$$V_{\alpha A-4}(R) \approx \sum_{i=1}^{4} V_i(R) = V(R). \qquad (20)$$

The above defined function $\psi_{P_\alpha N_\alpha L}(R)$ is proportional to the amplitude of the probability for the α-particle preformation, as it is stated by the R-matrix theory of α-decay[3].

In tables 2 and 3 the values of ratio

$$\Delta_{P_\alpha N_\alpha L} = \left(M_{P_\alpha N_\alpha L} - M^{\circ}_{P_\alpha N_\alpha L} \right) / M_{P_\alpha N_\alpha L} \qquad (21)$$

are given for the various types of configuration $P_\alpha N_\alpha L$. In the calculation of matrix elements $M^{\circ}_{P_\alpha N_\alpha L}$ the approximation (20) is used. One may notice that the values of $\Delta_{P_\alpha N_\alpha L}$ are small except for some levels (4s1/2, 3d5/2, 2g9/2) far from the Fermi surface of heavy nuclei. But the contribution of these states to the wave function $\psi_i^{I_i M_i}$, formula (6), and consequently to α-decay widths Γ_α^{SM}, eq.(9) is not significant. So, the approximation (20) changes the widths Γ_α^{SM} not more than by 30%, and this fact justifies in some sense the use of alpha-nucleus potential as a function of R only. The dependence of matrix elements $M_{P_\alpha N_\alpha L}$ on the configurations $P_\alpha N_\alpha L$ can be understood through the dependence of function $\psi_{P_\alpha N_\alpha L}$ on the same quantum numbers $P_\alpha N_\alpha L$. The latter is mainly responsible for the variation of the "channel radius" which determines the values of matrix elements $M_{P_\alpha N_\alpha L}$ (see tables 2 and 3). Let us note that the values $M_{P_\alpha N_\alpha L}$ increase

strongly from the case when $(n_1 \ell_1 j_1) \equiv (n_2 \ell_2 j_2)$ and $(n_3 \ell_3 j_3) \equiv (n_4 \ell_4 j_4)$ to the case when the above configurations are different. Now we examine the approximations used previously in calculating the α-decay widths. The "point α-particle approximation"[17] consists in taking

$$\vec{r}_1 = \vec{r}_2 = \vec{r}_3 = \vec{r}_4 = \vec{R} \qquad (22)$$

in formulae (12) and (13). In this case the factorization (20) takes place automatically and the functions $\psi^{Point}_{P_\alpha N_\alpha L}(R)$ are obtained without integration over coordinates $\{\vec{r}_i\}$. As it can be seen from fig.1 the functions $\psi^{Point}_{P_\alpha N_\alpha L}(R)$ differ from functions $\psi_{P_\alpha N_\alpha L}(R)$ both in the form and in the amplitude. Henceforth the matrix elements $M^{Point}_{P_\alpha N_\alpha L}$ are also different from the exact values of $M_{P_\alpha N_\alpha L}$. In order to make the point approximation useful the correcting factor

$$B_{P_\alpha N_\alpha L} = \left[M_{P_\alpha N_\alpha L} / M^{Point}_{P_\alpha N_\alpha L} \right]^2 \qquad (23)$$

has been introduced[17,6]. A convenient interpolating formula fitting the dependence of factors $B_{P_\alpha N_\alpha L}$ on quantum numbers n_i, ℓ_i, j_i has been found[17,6] using a harmonic oscillator shell model basis. Let us study the factors $B_{P_\alpha N_\alpha L}$ in the framework of Woods - Saxon shell model. In figs. 2, 3 and 4 typical values of factors $B_{P_\alpha N_\alpha L}$ are plotted (point connected with solid lines) together with the families of the dotted lines resulting from interpolating formulae of the same type as in refs.[17,6]. We tried to choose the parameters of these formulae for fitting as properly as possible the dependence of factors

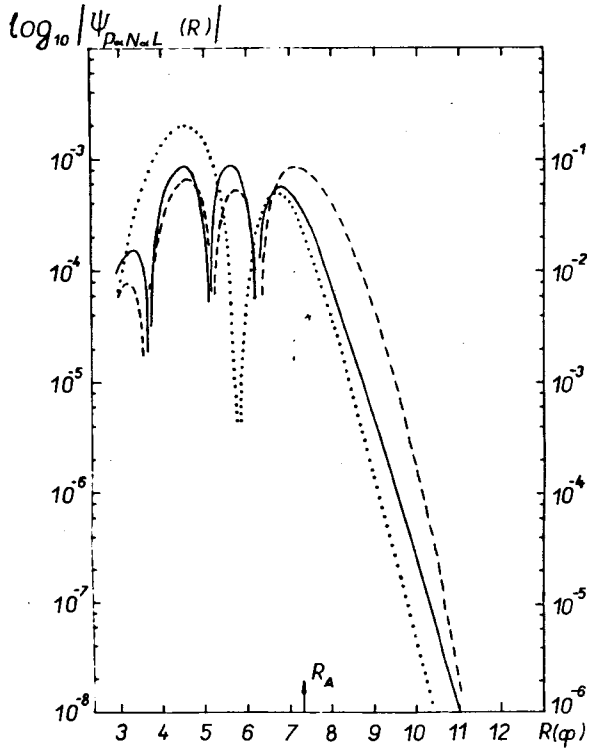

Fig. 1. Functions $\psi_{P_\alpha N_\alpha L}(R)$ for the ^{212}Po nucleus with configuration $P_\alpha N_\alpha L = [(1h_{9/2})^2_8 (2g_{9/2})^2_8]^2_{16}$. The solid line stands for the function $\psi_{P_\alpha N_\alpha L}(R)$, the dashed one, for the function $\psi^{H.O.}_{P_\alpha N_\alpha L}(R)$ and dotted, for the function $\psi^{Point}_{P_\alpha N_\alpha L}(R)$. The ordinate for the latter is given on the right-hand side.

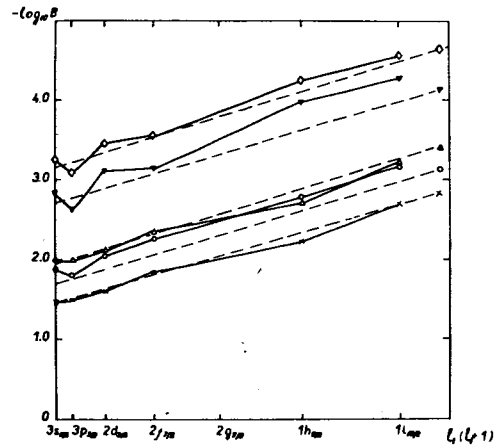

Fig. 2. The dependence of the correcting factors $B_{P_\alpha N_\alpha 0}$ for the ^{210}Po nucleus on proton pair states $P_\alpha = (n_1 l_1 j_1)^2_0$ for different neutron pair states N_α is represented by the following symbols: $\times$ – $N_\alpha = (3p_{1/2})^2_0$; o – $N_\alpha = (2f_{5/2})^2_0$; Δ – $N_\alpha = (2g_{9/2})^2_0$; ∇ – $N_\alpha = (1i_{11/2})^2_0$ and $\Diamond$ – $N_\alpha = (1j_{15/2})^2_0$ connected by continuous lines. The dashed lines labelled with the corresponding symbols represent the results obtained with the interpolating formula described in the text.

$B_{P_\alpha N_\alpha L}$ on quantum numbers n_i, l_i, j_i . Unsuccess of our fitting proves that it is impossible to obtain a useful interpolating formula for the correcting factor $B_{P_\alpha N_\alpha L}$.

Up to now almost all calculations on α-decay have been performed using the harmonical oscillator(H.O.)shell basis. As

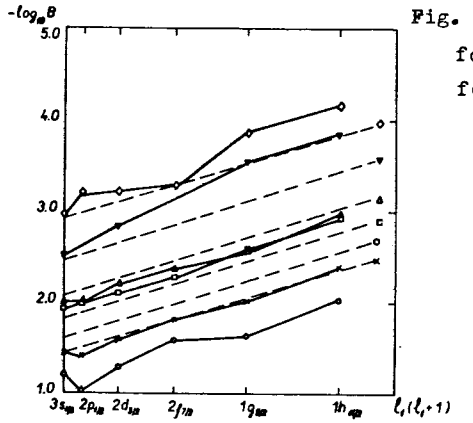

Fig. 3. The same as in fig. 2 for the ^{174}Pt nucleus with the following symbols:

$x - N_\alpha = (3p_{1/2})^2_0$;

$o - N_\alpha = (3d_{5/2})^2_0$;

$\square - N_\alpha = (2f_{7/2})^2_0$;

$\Delta - N_\alpha = (2g_{9/2})^2_0$;

$\nabla - (1h_{9/2})^2_0 = N_\alpha$;

$\Diamond - N_\alpha = (1i_{13/2})^2_0$

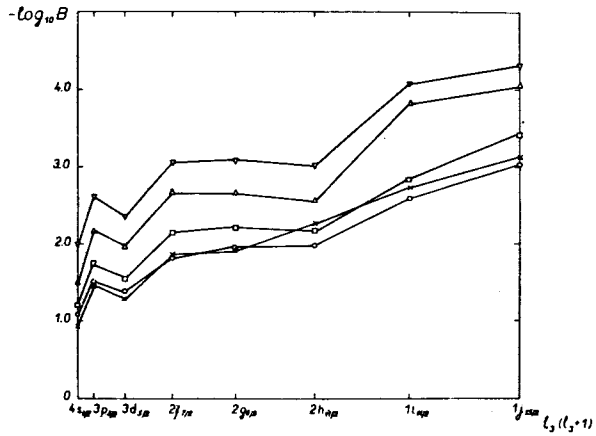

Fig. 4. The dependence of the correcting factors $B_{P_\alpha N_\alpha 0}$ for the ^{230}Th nucleus on neutron configurations $N_\alpha = (n_3 l_{3/3})^2_0$. Proton configurations are labelled with the following symbols: $x - P_\alpha = (3s_{1/2})^2_0$;

$o - P_\alpha = (3p_{3/2})^2_0$; $\square - P_\alpha = (2f_{5/2})^2_0$;

$\Delta - P_\alpha = (1h_{9/2})^2_0$; $\nabla - P_\alpha = (1i_{13/2})^2_0$.

the H.O. shell model functions differ within the surface region from the Woods - Saxon ones, the functions $\psi^{H.O.}_{P_\alpha N_\alpha L}(R)$ and $\psi_{P_\alpha N_\alpha L}(R)$ differ mainly within the same region, as it can be seen from fig.1.

The founded difference between the functions $\psi_{P_\alpha N_\alpha L}(R)$ and $\psi^{H.O.}_{P_\alpha N_\alpha L}$ can influence the value of the correction connected with the antisymmetrization in the final channels, as obtained in ref.[18].

As it follows from the calculations the number of nodes N of function $\psi_{P_\alpha N_\alpha L}(R)$ satisfies the same conditions as in the case of function $\psi^{H.O.}_{P_\alpha N_\alpha L}(R)$:

$$\sum_{i=1}^{4} (2n_i + l_i) = 2N + L . \qquad (24)$$

The values of ratio $M^{H.O.}_{P_\alpha N_\alpha L} / M_{P_\alpha N_\alpha L}$ depend on configuration $P_\alpha N_\alpha L$ (see table 4) so that the ratios of corresponding theoretical α -decay widths calculated without configuration mixing vary from 6 to 30.

Also from the results presented in table 4 it is easy to evaluate the limitations of the so-called "hybrid approximation"[6] when the matrix elements $M_{P_\alpha N_\alpha L}$ are calculated with H.O. one-particle wave functions and the potentials $V_i(R_i)$ are of the Woods - Saxon type. We can conclude that this approximation[6] can pretend only to a qualitative description of α -decay phenomena.

4. On the classification of α -transitions

We investigate the experimental α -widths in terms of α -energy independent ratios[1,6]

$$K_{exp} = \Gamma_\alpha^{exp} / \Gamma_\alpha^{pure} . \qquad (25)$$

Here the theoretical width Γ_α^{pure} is defined by formula (9) accounting only for the main configurations i_o and f_o in the initial and final states (i.e., when $C^{P_i N_i} C^{P_f N_f} = C^{i_o} C^{f_o} \delta_{i,i_o} \delta_{f,f_o}$).

The values of the ratio K_{exp} were calculated for a large number of even-even, odd-A and odd-odd spherical nuclei and plotted against the neutron number N in fig.5. The experimental α -widths Γ_α^{exp} are taken from refs.[19-21]. The consistency of a simultaneous analysis of the whole set of ratios K_{exp} is guaranteed by the absence of the free parameters in calculation of α -widths Γ_α^{pure}.

The minimal values K_{exp} in fig.5 correspond to the unfavoured α -transitions, for example, to α -decay of the ground state of the ^{210}Bi nucleus and of the isomeric states of the ^{212}Po and ^{210}Bi nuclei. It is important to note that the wave functions of initial and final states for these transitions are known. Owing to the fact that the configuration mixing is small, the values of the widths Γ_α^{pure} turn out to be a good approximation for the widths Γ_α^{SM} (9) (see for more details paper II). Thus the difference between the theoretical and experimental α -widths ($K_{exp} \approx 10^2$) is mainly caused by the limitations of the above formulated shell model approach for the integral formula of α -widths. The maximal values of K_{exp} correspond to the numerous group of α -transitions consisting from the ground state - ground state transitions in even-even nuclei and from the "strong" α -transitions (with the hindrance factors[20] less than 5) of odd-A and odd-odd nuc-

lei. For such transitions the initial and final states have, or may be assumed to have the same spin. The configurations i_o, f_o are chosen following the filling of the levels of the Woods - Saxon shell model potential[10]. The values of K_{exp} for this group of α -transitions are of about 10^4-10^5 and are situated in a narrow band not wider than 0.6 in $\log K_{exp}$.

The difference between the values of K_{exp} for the favoured and unfavoured α -transitions has to be considered as a phenomenological indication to the contribution of the configuration mixing into the absolute values of α -widths. This is a unique possibility to explain the above-mentioned difference in the framework of our shell model approach to α -decay. As it can be seen from fig.5 the effect of configuration mixing on favoured α -transitions is practically independent of the number of nucleons outside the closed shell core. (Compare, for example, the values K_{exp} for the chain of polonium isotopes, Z = 84, with the corresponding chains of Ra, Rn, etc.).

It is appropriate to note that the jump in values of $\log K_{exp}$ (see fig. 5) near to N = 126 shows that it is impossible in the framework of a simple Woods - Saxon shell model to explain the relative widths in this region. This conclusion is in contradiction with the affirmation from ref.[22] where a harmonical oscillator basis is used for the relative α -widths calculation.

We have not included into our analysis the values of ratios K_{exp} for lead neutrondeficient isotopes as having a different behaviour from the transitions of other even-even nuclei. Namely, when the number of neutrons diminishes, the values K_{exp} for light Pb isotopes falls down from the band of favoured α -transitions (fig.5). We intend to consider this phenomenon elsewhere.

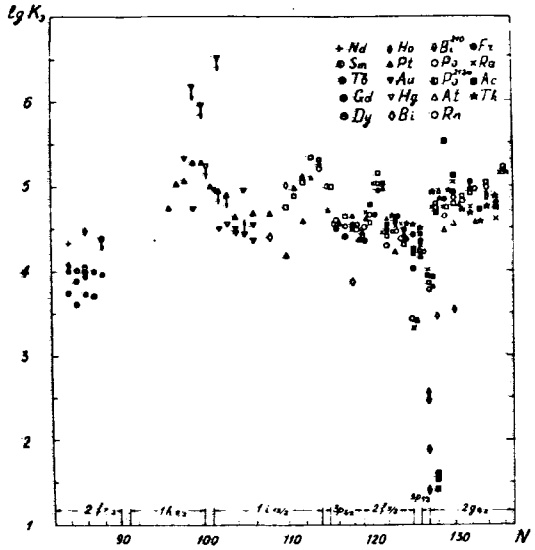

Fig. 5. The dependence of enhancement coefficients K_{exp} on neutron number N. On the absciss the neutron shell configurations are indicated.

Let us note now that for the two lightest isotopes of Pb giving the largest difference from the general tendency of values K_{exp} the experimental data[21] are only preliminary. In fig. 5 the ratios K_{exp} are also shown for the α-decay of nuclei with N = 125,127 of neutrons, and of some odd-mass Bi isotopes. We classify them as semifavoured α-transitions using the fact that their values K_{exp} are situated between the values K_{exp} belonging to the favoured and unfavoured transitions.

Thus, the three groups of α-transitions can be distinguished in terms of the values of K_{exp} : favoured, semifavoured and unfavoured. The explanation of the splitting between these

three groups, related to the different influence of residual interactions on each type of α-transitions shall be given in papers II and III.

Table 1.

The dependence of matrix elements $M_{P_\alpha N_\alpha L}$ on the shell model parameters. Calculations are performed for the ^{212}Po nucleus (Q = 11.87 MeV).

P_α	N_α	L	$M_{P_\alpha N_\alpha L}$		
			14)	16)	15)
$(1p_{1/2})^2_0$	$(2g_{9/2})^2_0$	0	0,716(-7)	0,540(-7)	0,960(-7)
$(2p_{1/2})^2_0$	$(2g_{9/2})^2_0$	0	0,188(-6)	0,194(-6)	0,255(-6)
$(3p_{1/2})^2_0$	$(2g_{9/2})^2_0$	0	0,501(-6)	0,827(-6)	0,613(-6)
$(1f_{5/2})^2_0$	$(2g_{9/2})^2_0$	0	0,167(-6)	0,144(-6)	0,231(-6)
$(1h_{9/2})^2_0$	$(2g_{9/2})^2_0$	0	0,202(-6)	0,222(-6)	0,283(-6)
$(1h_{9/2})^2_8$	$(2g_{9/2})^2_8$	0	0,375(-7)	0,414(-7)	0,539(-7)
$(1h_{9/2})^2_8$	$(2g_{9/2})^2_0$	8	0,696(-9)	0,785(-9)	1,01(-9)
$(1h_{9/2})^2_8$	$(2g_{9/2})^2_8$	16	0,177(-11)	0,147(-11)	0,177(-11)
$(1h_{9/2}2f_{7/2})_8$	$(2g_{9/2}1i_{13/2})_8$	16	0,642(-11)	0,845(-11)	0,943(-11)

Table 2

The results obtained for the nucleus ^{212}Po, Q =11.87 MeV and configuration $P_\alpha N_\alpha L = [(n_1 \ell_1 j_1)^2_0 (2g_{9/2})^2_0]_0$

$n_1 \ell_1 j_1$	$\Delta_{P_\alpha N_\alpha L}$	$\lvert M_{P_\alpha N_\alpha L} \rvert$	$R_c^{P_\alpha N_\alpha}$ fm	$\lvert \psi_{P_\alpha N_\alpha L}(R_c) \rvert$
2 $s_{1/2}$	2,8	0.14(-6)	7,2	0.15(-2)
3 $s_{1/2}$	12,1	0.33(-6)	7,6	0.20(-2)
4 $s_{1/2}$	25,4	0.65(-6)	8,2	0.18(-2)
1 $p_{1/2}$	-2,8	0.71(-7)	7,0	0.10(-2)
2 $p_{1/2}$	6,4	0.18(-6)	7,3	0.17(-2)
3 $p_{1/2}$	19,3	0.50(-6)	7,85	0.57(-6)
1 $p_{3/2}$	-2,5	0.12(-6)	7,0	0.17(-2)
2 $p_{3/2}$	7,0	0.28(-6)	7,3	0.26(-2)
3 $p_{3/2}$	19,1	0.68(-6)	7,85	0.29(-2)
1 $d_{5/2}$	1,03	0.19(-6)	7,1	0.24(-2)
2 $d_{5/2}$	11,3	0.43(-6)	7,5	0.29(-2)
3 $d_{5/2}$	35,3	0.13(-5)	8,2	0.28(-2)
1 $f_{5/2}$	3,0	0.17(-6)	7,1	0.20(-2)
2 $f_{5/2}$	16,8	0.50(-6)	7,7	0.24(-2)
1 $g_{9/2}$	8,3	0.30(-6)	7,3	0.26(-2)
2 $g_{9/2}$	24,3	0.72(-6)	7,9	0.26(-2)
1 $h_{9/2}$	11,9	0.20(-6)	7,4	0.14(-2)

Table 3

The results obtained for the nucleus ^{212}Po, Q_α =11.87 MeV.

L	$\Delta_{P_\alpha N_\alpha L}$	$\lvert M_{P_\alpha N_\alpha L} \rvert$	$R_c^{P_\alpha N_\alpha L}$ fm	$\lvert \psi_{P_\alpha N_\alpha L}(R_c^{P_\alpha N_\alpha}) \rvert$
$P_\alpha N_\alpha L = [(1h_{9/2})^2_8 (2g_{9/2})^2_8]_L$				
0	12,4	2.42(-8)	7,4	0.24(-3)
4	12,4	4.26(-9)	7,5	0.10(-3)
8	13,2	4.76(-10)	7,5	0.84(-4)
12	14,5	2.41(-11)	7,6	0.95(-4)
16	16,4	8.15(-13)	7,7	0.17(-3)
$P_\alpha N_\alpha L = [(1h_{9/2}\, 2f_{7/2})_8 (2g_{9/2}\, 1i_{11/2})_8]_L$				
0	10.0	1.31(-7)	7,4	0.13(-2)
4	9,6	2.30(-8)	7,45	0.51(-3)
8	10,6	2.55(-9)	7,50	0.46(-3)
12	12,8	1.28(-10)	7,6	0.51(-3)
16	13,7	4.30(-12)	7,7	0.92(-3)

Table 4

The dependence of ratio $M^{H.O.}_{2N,L}/M_{2N,L}$ on $P_\alpha=(n_1\ell_1 j_1)^2_0$ and $N_\alpha=(n_3\ell_3 j_3)^2_0$. The results are obtained for the ^{210}Po nucleus, Q_α = 5.33 MeV.

P_α \ N_α	$1j_{15/2}$	$1i_{11/2}$	$2g_{9/2}$	$3p_{1/2}$	$2f_{5/2}$
$3p_{3/2}$	3.56	3.62	2.56	2.46	2.86
$1i_{13/2}$	3.65	3.68	3.44	3.35	3.60
$2f_{7/2}$	3.80	4.36	3.02	2.88	3.34
$3d_{1/2}$	3.69	4.20	3.00	2.86	3.18
$2d_{3/2}$	3.89	4.36	3.18	3.08	3.44
$1h_{9/2}$	4.85	5.25	4.17	3.86	4.22

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