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Определение константы связи π^3He^3H

Дисперсионные соотношения для реальной части антисимметричной амплитуды $\pi^{\pm}\,{}^3He$ рассеяния вперед использованы для вычисления константы π^3He^3H. Найденное значение больше элементарной πN константы связи и равняется $f^2_{\pi\,{}^3He\,{}^3H} = 0{,}12 \pm 0{,}01$.

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Determination of the π^3He^3H Coupling Constant

Dispersion relations for the real part of the antisymmetric amplitude of the $\pi^{\pm}\,{}^3He$ scattering have been used in order to determine the π^3He^3H coupling constant. The coupling constant value determined by this method is larger than the elementary pion-nucleon coupling constant, but is in good agreement with the value obtained by another method (Mach and Nichitiu, 1976). The obtained value is $f^2_{\pi\,{}^3He\,{}^3H} = 0.12 \pm 0.01$.

The investigation has been performed at the Laboratory of Nuclear Problems JINR.

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1. INTRODUCTION

Since forward dispersion relations for particle-nucleus scattering had been first extensively revised by Ericson and Locher [1], they were used in a wide variety of reactions, largely because new experimental total cross sections are now available. Some of nucleon-nucleus and pion-nucleus coupling constants have been determined using forward dispersion relations, a topic for which the reader is referred to an updated review of Locher [2].

The shadowing effect in pion scattering by nuclei is reflected in reducing the effective pion-nucleus coupling constant below the known value of the pion-nucleon coupling constant ($f^2_{\pi N} = 0.08$).

Thus, for ^7Li and ^9Be the pionic coupling constants obtained are:

$$f^2_{\pi\,{}^7Li} = f^2_{\pi\,{}^9Be} = 0.06$$

(Squier et al.[3], Osland[4], and Wilkin et al.[5]).

An interesting nucleus for such calculations is ^3He for which the theoretical predictions concerning the π^3He^3H coupling constant are within 0.08 - from the simple impulse approximation (Ericson and Locher[1]) - and 0.16 - from the dispersion relations for the pionic form factor of ^3He and Goldberger-Treiman relations (Kopeliovich [6]).

Mach and Nichitiu [7] have estimated the $\pi{}^3He{}^3H$ coupling constant using the Chew-Low equation and the semi-phenomenological analysis of the $\pi^{\pm}{}^3He$ elastic differential cross-sections in the energy region from 98 MeV to 156 MeV. They have found the value $f^2_{\pi{}^3He{}^3H} = 0.101 \pm 0.018$, which is larger than other pion-nucleon coupling constants but equal to Spencer's [8] upper limit ($f^2_{\pi{}^3He{}^3H} = 0.07 \pm 0.012$).

In this paper we use the forward dispersion relations for antisymmetric amplitude of $\pi{}^3He$, realistic scattering lengths and the Pade approximants for the total cross sections in order to obtain the $\pi{}^3He{}^3H$ coupling constant value.

Section 2 describes the usual method for the determination of the pion-nucleus coupling constant from the forward dispersion relations. Section 3 presents the analysis of the $\pi^{\pm}{}^3He$ total cross sections, and section 4 gives the values of the scattering lengths. The contributions to the coupling constant from the unphysical and asymptotic regions are given in section 5. Section 6 presents the results of these calculations and a few comments on the best energy regions for future $\pi{}^3He$ experiments.

2. THE METHOD

For the pion scattering by nuclei with isospin $I = 1/2$, the dispersion relation for the real part of the antisymmetric scattering amplitude

$$f^-(\omega) = \frac{1}{2}(f_{\pi^-}(\omega) - f_{\pi^+}(\omega)) \quad (1)$$

may be written in the form

$$\mathrm{Re}\, f^-(\omega) = \frac{2\omega f^2_{\pi{}^3He{}^3H}}{\omega^2 - \omega^2_{{}^3H}} + \frac{2\omega}{\pi} P \int_{\omega_0}^{\infty} \frac{\mathrm{Im}\, f^-(\omega')}{\omega'^2 - \omega^2} d\omega', \quad (2)$$

where ω is the pion total laboratory energy, $\omega_{{}^3H}$ is the energy of the ${}^3H$ pole, $f^2_{\pi{}^3He{}^3H}$ is the residue at this pole, and ω_0 is the beginning of the unphysical cut ($\pi^- + {}^3He \to d + n$). The values of $\omega_{{}^3H}$ and ω_0 are close to $\omega = 0$.

We can evaluate equation (2) with $\omega = m_\pi$ (elastic threshold) and ignoring terms of the order of $\omega^2_{{}^3H}/m^2_\pi$ we obtain:

$$f^2_{\pi{}^3He{}^3H} = \frac{m_\pi}{2} \mathrm{Re}\, f^-(m_\pi) - \frac{m^2_\pi}{\pi} P \int_0^{\infty} \frac{\mathrm{Im}\, f^-(\omega')}{\omega'^2 - m^2_\pi} d\omega'. \quad (3)$$

The values of the total cross section for $\pi^+{}^3He$ and $\pi^-{}^3He$ in the physical region ($m_\pi < \omega < \infty$) , the value of $\mathrm{Re}\, f^-(m_\pi)$ and the analytical extrapolation of $\mathrm{Im}\, f^-(\omega)$ in the unphysical region ($0 < \omega < m_\pi$) are required to estimate the coupling constant $f^2_{\pi{}^3He{}^3H}$.

3. THE TOTAL CROSS SECTION

For the total cross section we have used:

a) the values calculated by the Glauber model by Mach et al. [9] for the energy region of $250 \le T \le 3000$ MeV;

b) the experimental values for π^+ and $\pi^-{}^3He$ scattering of Spencer, Jr., [8] in the energy region of the first baryonic resonance;

c) some of the lower and upper limit values from Mach et al. [10] for $70 \le T \le 110$ MeV, evaluated by the optical model.

These data have been interpolated using the Pade approximants. The best representation of the $\pi^{\pm}{}^3He$ total cross section data is achieved by the $[\frac{3}{4}]$ approximant:

$$\sigma_{tot} = \frac{4\pi}{k}\left[\frac{3}{4}\right] F^{n/2} = \frac{4\pi}{k} \frac{\sum_{i=0}^{3} A_i T^i}{\sum_{i=0}^{4} B_i T^i} F^{n/2} \tag{4}$$

with $A_0 = \mathrm{Im}\, a_{\pm}$(see eq.(7)) and $B_0 = 1$, and where T is the laboratory kinetic energy, k is the laboratory momentum, and $a_{\mp}$ is the scattering length for $\pi^- {}^3He$ and $\pi^+ {}^3He$ (or $\pi^- {}^3H$), respectively. The factor F is introduced to fulfil Adler's self-consistency condition requiring that $\mathrm{Im} f(0) = 0$ in the soft pion limit (Ericson and Locher,[1]) for n=1 and 3

$$F = \frac{T + m_\pi}{m_\pi}. \tag{5}$$

The best χ^2 value is obtained for n=3 (see Table 1).

Table 1

	σ_{tot}	
Without Coulomb Corrections	$\frac{4\pi}{K}\left[\frac{3}{4}\right]F^{1/2}$	$\frac{4\pi}{K}\left[\frac{3}{4}\right]F^{3/2}$
Number of points	χ^2	χ^2
50	55.6	26.5
57	63.1	43.0
$f^2_{\pi\, {}^3He\, {}^3H}$	0.116 ± 0.024	0.104 ± 0.008
With eq.(7) for $\mathrm{Re} f^-(m_\pi)$	statistical errors	

Figures 1a and 1b give the experimental data for the total cross section together with the curves obtained using eq.(4) for n=1 and n=3 .

The total cross section values used have been corrected for Coulomb barrier effects and Coulomb trajectory distortion in a similar way as in the paper by Wilkin et al.[5] using:

$$\frac{\Delta\sigma}{\sigma} = \frac{2 Z \alpha T}{k^2 R}\left(2 + k \frac{\partial \log \sigma}{\partial k}\right) \tag{6}$$

with the ${}^3He$ radius R = 1.88 fm.

At low energies, where neither data, nor good theory exist, the corrected total cross section was extrapolated by means of eq.(4). This correction is about 5% (4%) for the $\pi^+(\pi^-)$ total cross section at 150 MeV and vanish at 240 MeV, but anyway the Coulomb corrections seem to be very important for the dispersion relation calculations because the value of the $\pi\, {}^3He\, {}^3H$ coupling constant obtained using corrected total cross sections is larger by about 0.014 than the value obtained without these corrections.

4. THE SCATTERING LENGTH

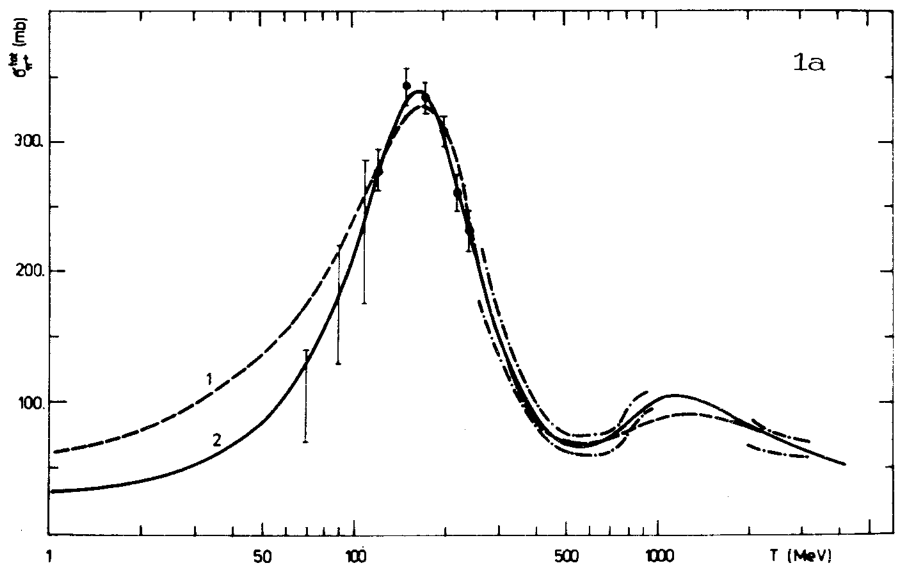
The real part of the antisymmetric amplitude at the elastic threshold is given by $\pi^{\pm}\, {}^3He$ scattering lengths

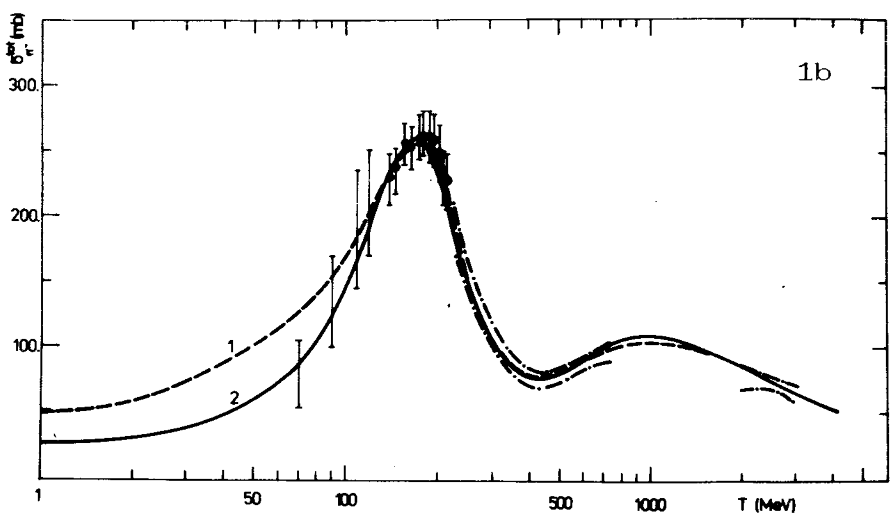
$$f^-(m_\pi) = \frac{1}{2}(a_- + a_+).$$

Cheon and von Egidy[11] have proposed an empirical formula for the energy shifts of pionic atoms, which shows a remarkably good agreement with experimental data. From their equation the predicted values for the scattering lengths for $\pi^- {}^3He$ and $\pi^+ {}^3He$(or $\pi^- {}^3H$) are:

$$a_- = 0.067 + i \cdot 0.035 \text{ fm},$$

$$a_+ = -0.287 + i \cdot 0.028 \text{ fm},$$





and thus

$$f^{-}(m_{\pi}) = 0.177 + i \cdot 0.0035 \text{ fm} \tag{7}$$

seems to be a realistic value for the antisymmetric amplitude at the elastic threshold.

The formula for the pion-nucleus scattering lengths frequently used in dispersion calculations derived from the pion-nucleon scattering lengths is:

$$f_{\pm}(m_{\pi}) = (Z a_{\pi^{\pm} p} + N a_{\pi^{\pm} n}), \tag{8a}$$

and thus

$$f^{-}(m_{\pi}) = \frac{1}{3}(\alpha_1 - \alpha_3) + i \cdot 0, \tag{8b}$$

where α_1 and α_3 are the pion-nucleon scattering lengths for isospin states $T=1/2$ and $T=3/2$, respectively.

Squier et al. [3] have fitted the values at pion-nucleus scattering length by the least squares method (with eq.8a) to obtain the best values for α_1 and α_3 (for eq.8b). The antisymmetric amplitude at $\omega = m_{\pi}$ is then given by the unique value (independent of nuclei)

$$f^{-}(m_{\pi}) = 0.068 + i \cdot 0 \text{ . fm} . \tag{9}$$

Fig. 1. Total cross section for $\pi^{+\,3}$He (Fig.1a) and for $\pi^{-\,3}$He (Fig.1b) scattering. Curves 1 and 2 correspond to the $[3/4]F^{1/2}$ and $[3/4]F^{3/2}$ approximants, respectively, (see eq. (4)). The dashed-dotted line represents the error corridor from the Glauber calculations and vertical error bars represent the limits for the total cross section from the optical model calculations. Experimental points are taken from Spencer, Jr., [8].

Using eqs. (8) and the value of a_1 and a_3 from free pion-nucleus scattering, the value of the antisymmetric amplitude at the elastic threshold is

$$f^-(m_\pi) = 0.097 + i \cdot 0 . \text{fm}. \qquad (10)$$

5. THE UNPHYSICAL AND ASYMPTOTIC REGIONS

For the unphysical region we have extrapolated the imaginary part of the forward scattering amplitude using eq.(4) for $T < 0$. Figure 2 shows the energy behaviour of $\text{Im} f_+(\omega)$ (for $\pi^+ \, ^3\text{He}$) in the unphysical region from eq. (4) and $n = 0,1$ and 3. The contribution of the principal value integral (eq.3) from this energy interval to the coupling constant is 0.017, larger than it had been expected for other nuclei (0.01) and the uncertainty in $f^2_{\pi \, ^3\text{He} \, ^3\text{H}}$ due to different extrapolation at $T < 0$ is 0.012. The imaginary part of the forward scattering amplitude crosses the axis at $T = 420$-450 MeV (see Figure 3) and goes to zero for high energy. The contribution to the coupling constant from the asymptotic region (from the $T > 2500$ MeV region) is smaller than 0.0005 and thus is sufficient to take the upper limit of the integral in eq. (3) at $T \sim 3000$ MeV.

6. RESULTS AND COMMENTS

The first term from eq. (3) the real part of the forward antisymmetric amplitude at $\omega = m_\pi$ contributes about 50% from the value of $f^2_{\pi \, ^3\text{He} \, ^3\text{H}}$. The values

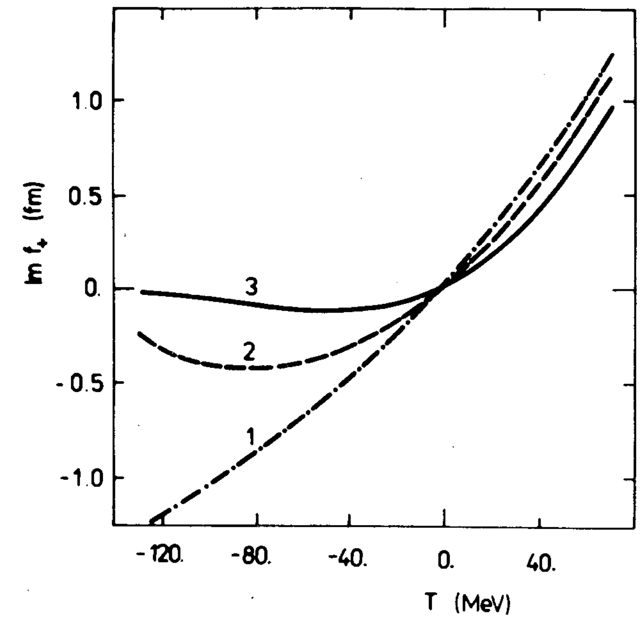


Fig. 2. The imaginary part of the forward $\pi^+ \, ^3\text{He}$ scattering amplitude in the physical region from the extrapolation of eq.(4) with $n = 0, 1$ and 3 - curves 1,2 and 3, respectively.

obtained for $\pi \, ^3\text{He} \, ^3\text{H}$ coupling constant using the Coulomb corrected total cross sections, eq.(4) with $n = 3$ for different values of $\text{Re} f^-(m_\pi)$ are given in Table II.

Table II

$\text{Re} f^-(m_\pi)$ (fm)		$f^2_{\pi \, ^3\text{He} \, ^3\text{H}}$
0.068	eq. /9/	0.079
0.097	eq. /10/	0.089
0.177	eq. /7/	0.118

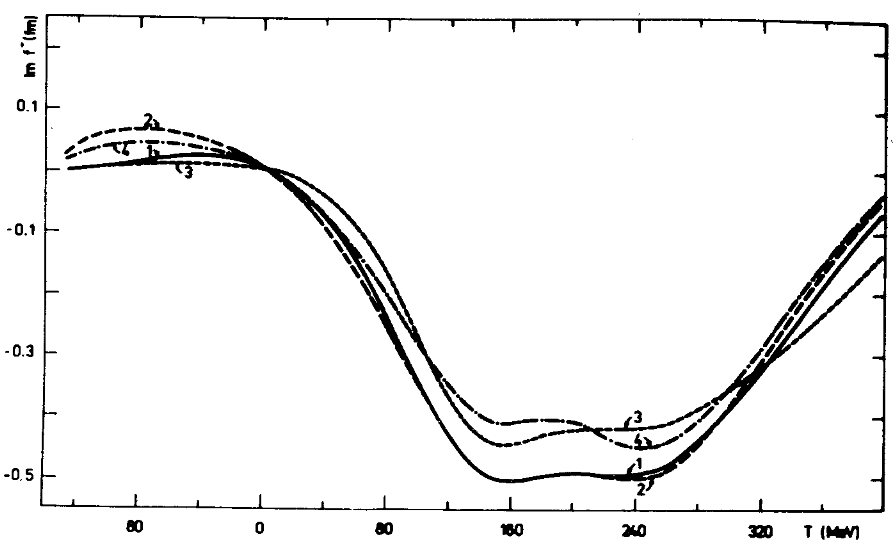


Fig. 3. The imaginary part of the antisymmetric scattering amplitude as a function of kinetic laboratory energy:

Approximant	Curve number	
$[\frac{3}{4}]\,F^{1/2}$	1	3
$[\frac{3}{4}]\,F^{3/2}$	2	4
	Coulomb corrections	No Coulomb corrections

Spencer, Jr.,[8] obtained $f^2_{\pi\,{}^3He\,{}^3H} = 0.07 \pm 0.012$ by using the dispersion relation method and Re $f^-(m_\pi) = 0.068$ fm (eq.9) including a contribution of 0.01 from the unphysical region.

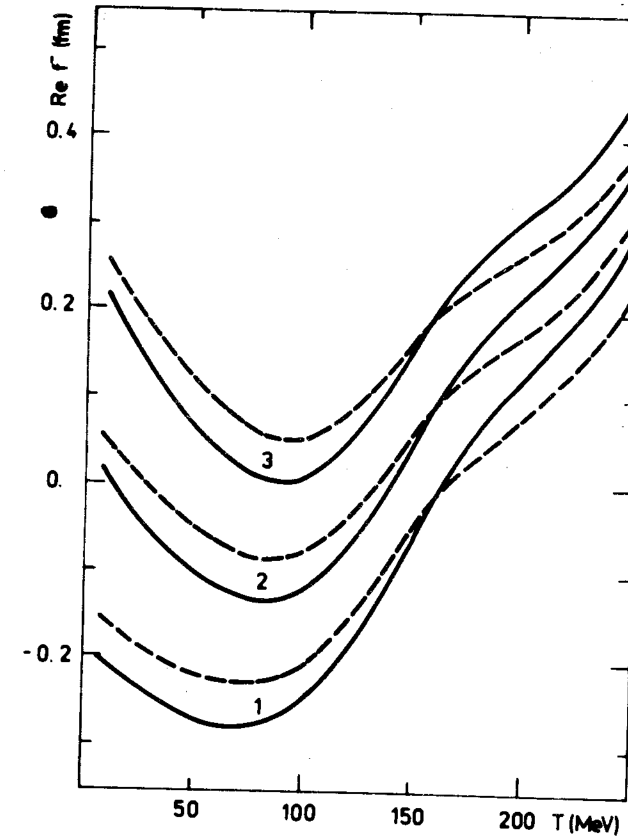


Fig.4. The real part of the antisymmetric scattering amplitude as a function of the kinetic laboratory energy for $f^2_{\pi\,{}^3He\,{}^3H}$ = 0., 0.08 and 0.16 - curves 1,2 and 3, respectively, with (solid lines) and without (broken lines) Coulomb corrections.

Our recommended value for $f^2_{\pi\,{}^3He\,{}^3H}$ obtained by the best fit to the experimental data by eq.(4) with n = 3, with Coulomb corrections to the total cross sections and realistic scattering lengths, given by eq.(7) is follows:

$$f^2_{\pi\,{}^3He\,{}^3H} = 0.12 \pm 0.01\,,$$

which is in good agreement with our previous determination (R.Mach and F.Nichitiu [7]): $f^2_{\pi\, ^3He\, ^3H}$ = =0.101± 0.018). This value shows that the shadowing effect discussed by Squier et al.[3] seems to not work for the case of the ^3He nucleus.

The uncertainty in the coupling constant value due to the uncertainty in the scattering length can be removed by future experiments for $\pi^{\pm}\, ^3He$ forward elastic scattering. The sensibility of $Re\, f^-(\omega)$ for $f^2_{\pi\, ^3He\, ^3H}$ determinations is given in Figure 4, where $Re\, f^-(\omega)$ is plotted against energy for $f^2_{\pi\, ^3He\, ^3H}$ = =0, 0.08 and 0.16, respectively, with and without Coulomb corrections for the total cross sections. The best region for such experiments seems to be at low energy, but the uncertainty due to Coulomb corrections still exists, or at 160 MeV, where Coulomb corrections are very small.

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