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Ядерная теория поля

При помощи функционального интегрирования развита ядерная теория поля для ферми-систем с двухчастичным взаимодействием. Получен эффективный лагранжиан ядерной полевой теории. Кроме того, получены соответствующие графические правила для модифицированной теории возмущения. Установлено соответствие между ядерной полевой теорией и обычной графической теорией возмущения.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Nuclear Field Theory

By using path integral techniques the Nuclear Field Theory (NFT) is developed for Fermi systems interacting via a general two-body force. The NFT Lagrangian is strictly derived. As a byproduct, the corresponding graphical rules are obtained. The relation between the NFT and the conventional Feynman diagrammatic many-body perturbation theory is established for processes connecting initial and final states, too.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

In recent years the Nuclear Field Theory (NFT)[1-4] has become an attractive theory for treating the different anharmonic and coupling effects of nuclear structure in a unified manner [5,6]. The NFT has proved most successfully in the lead region where its systematic application by the Copenhagen group has led to an overall understanding of the low-energy spectroscopic data [5,7].

The most striking advantage of the NFT consists in that it allows a simultaneous treatment of single particle and collective degrees of freedom whereat care is taken of the overcompleteness of the basis and the Pauli principle. The interplay between the different excitation modes (single particle excitations, shape oscillations and pairing vibrations) can be studied in a sophisticated way. The rules for the diagrammatic nuclear field treatment have been found empirically in ref.[1]. As it has been shown in refs.[2,3], for processes connecting intermediate states there exists a one-to-one correspondence between the Mottelson diagrams of the nuclear field treatment and the Feynman diagrams of the conventional many-body perturbation theory[15]. However, there remains some questions concerning the processes connecting initial and final states as the nuclear field treatment yields more states than a shell model diagonalization which uses properly antisymmetrized states. For a schematic model it has been shown in ref.[8] that besides reproducing the exact results of the shell model diagonalization the nuclear field treatment yields some spurious states. Fortunately, these states can be well isolated from the physical states, at least for the schematic model considered in ref. [8] Further, in ref. [9] it has been demonstrated that the spurious states appear because some classes of diagrams are neglected in the NFT. The aim of the present paper is to investigate these deep and unanswered questions connected with the treatment of the initial and final states in the NFT. For this purpose we present a field theoretical derivation of the NFT for a Fermi system interacting through a general two-body interaction. By using path integral methods [10,11] the NFT Lagrangian is derived. (A strict non-perturbation theoretical derivation of the NFT Hamiltonian has been given up to now only for some schematic models [12,13]). The graphical rules of the nuclear field treatment come out quite naturally. Further, we find out a unique prescription how the Feynman diagrams of the fermion treatment can be converted into the corresponding Mottelson diagrams of the NFT. In this way we establish the relation between the NFT and the usual Feynman diagrammatic many-body perturbation theory. From our investigations some general statements concerning the appearance of spurious states in the NFT can be made.

The paper is organized as follows: In Sect. 2 by introducing collective variables a new effective action is obtained from which the equations of motion of the quasiparticle and collective excitations follow. In particular, there arises a modified perturbation expansion in terms of collective fields in the form of a loop expansion. In Sect. 3 we derive the NFT Lagrangian. Finally, in Sect. 4 we establish the relation between the NFT and the ordinary Feynman diagrammatic many-body perturbation theory for processes connecting initial and final states.

2. FIELD THEORY OF COLLECTIVE NUCLEAR MOTION

By using functional methods we present in this section a field theoretical description of collective nuclear excitations. We start with the following nucleon Lagrangian

$$\mathcal{L}(t) = \sum_{\alpha\gamma} a^+_\alpha(t)(i\partial_t \delta_{\alpha\gamma} - e_{\alpha\gamma})a_\gamma(t) - \frac{1}{2}\sum_{\alpha\beta\gamma\delta} V_{\alpha\gamma,\beta\delta}\, a^+_\alpha(t)a^+_\beta(t)a_\delta(t)a_\gamma(t). \tag{2.1}$$

Here $e_{\alpha\gamma}$ denotes the matrix elements of the single particle Hamiltonian and $a_\alpha(t)$, $a^+_\alpha(t)$ are the fermion operators in the interaction representation. The matrix elements of the two-body interaction have been antisymmetrized

$$V_{\alpha\gamma,\beta\delta} = \frac{1}{2}[\langle\alpha\beta|V|\gamma\delta\rangle - \langle\alpha\beta|V|\delta\gamma\rangle],$$

$$\langle\alpha\beta|V|\gamma\delta\rangle = \int d\underset{\sim}{r}_1 d\underset{\sim}{r}_2 \phi^*_\alpha(\underset{\sim}{r}_1)\phi^*_\beta(\underset{\sim}{r}_2) V(\underset{\sim}{r}_1,\underset{\sim}{r}_2)\phi_\gamma(\underset{\sim}{r}_1)\phi_\delta(\underset{\sim}{r}_2).$$

The nucleon Green functions can be obtained from the generating functional

$$Z[\eta,\eta^+,Q] = \mathcal{N}\int da\,da^+ \exp i\int dt\,\{\mathcal{L}(t) + \sum_\alpha(\eta^+_\alpha(t)a_\alpha(t) + a^+_\alpha(t)\eta_\alpha(t)) + \sum_{\alpha\gamma} Q_{\alpha\gamma}(t)a^+_\alpha(t)a_\gamma(t)\} \tag{2.2}$$

by differentiating with respect to the external sources η, η^+. For convenience we have also introduced a bilocal source Q. Under the functional integral (2.2) the fermion operators are considered anti-commuting (Grassman) variables *. The normalization

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*For this it is important that the matrix elements $V_{\alpha\gamma,\beta\delta}$ have been antisymmetrized.

constant $\mathfrak{N}$ is fixed by the requirement $Z[0,0,0]=1$. Making use of the functional relation* **

$$\exp(-\frac{i}{2}a^+a\,V a^+a) = (\det V^{-1})^{1/2} \times$$

$$\times \int d\Phi \exp i\{\frac{1}{2}\Phi V^{-1}\Phi + \Phi a^+a\}, \qquad (2.3)$$

where Φ is a hermitian Bose field

$$\Phi^+_{\alpha\beta} = \Phi_{\beta\alpha}, \qquad (2.4)$$

the generating functional (2.2) may be written as

$$Z[\eta, \eta^+, Q] = \mathfrak{N} \int da\, da^+ d\Phi \times$$

$$\times \exp i\{\mathcal{L}_c + \eta^+ a + a^+\eta + Q a^+ a\}. \qquad (2.5)$$

with the new effective Lagrangian

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* If confusion is exluded we drop the fermion indices α, β, γ, etc., which have to be summed over. Further, the matrix multiplication is understood in the functional sense what implies integration over intermediate times. In this convention we have $V(t,t') = \delta(t,t')V$. In addition, it is convenient to introduce the following convention: if in an <u>exponent</u> a time variable t is not explicitly indicated it has to be integrated over, e.g., $\exp(i\mathcal{L}) \equiv \exp(i\int \mathcal{L}(t)dt)$.

** Note that $V_{\alpha\gamma,\beta\delta}$ is a symmetric matrix with respect to the index pairs $(\alpha\gamma)$ and $(\beta\delta)$.

$$\mathcal{L}_c(t) = \sum_{\alpha\gamma} a^+_\alpha(t)\,(i\partial_t\,\delta_{\alpha\gamma} - e_{\alpha\gamma}\quad)\,a_\gamma(t) +$$

$$+ \sum_{\alpha\gamma}\Phi_{\alpha\gamma}\,(t)\,a^+_\alpha(t)\,a_\gamma(t) + \frac{1}{2}\sum_{\alpha\beta\gamma\delta}\Phi_{\alpha\gamma}(t)\,(V^{-1})_{\alpha\gamma,\beta\delta}\,\Phi_{\beta\delta}(t). \tag{2.6}$$

Although the Φ variable is considered in the functional (2.5) as an independent variable it represents really a composite field. Indeed variation of the action $S_c = \int \mathcal{L}_c(t)dt$ reveals the dependence

$$\Phi_{\alpha\gamma}\,(t) = -\sum_{\beta\delta} V_{\alpha\gamma,\beta\delta}\; a^+_\beta(t)\,a_\delta\,(t)\,.$$

Inserting this relation in eq. (2.6) exhibits the equivalence of both Lagrangians $\mathcal{L}(t)$ (eq. (2.1)) and $\mathcal{L}_c(t)$. However, the Lagrangian $\mathcal{L}_c(t)$ is more favourable for deriving a perturbation expansion which includes collective excitation modes in the unperturbed basis. In addition, $\mathcal{L}_c(t)$ is quadratic in the fermion variables and the integration over these variables can be performed. Defining the total single particle Green function G through

$$G^{-1}_{\alpha\gamma}[\Phi](t,t') = (i\partial_t\,\delta_{\alpha\gamma} - e_{\alpha\gamma} + \Phi_{\alpha\gamma}\,(t))\,\delta\,(t,t') \tag{2.7}$$

the integration over the fermion variables yields

$$Z[\eta,\eta^+,Q] = \mathfrak{N}\int d\Phi\,\exp i\{S[\Phi] +$$

$$+\frac{1}{2}QV^{-1}Q - QV^{-1}\Phi - \eta^+ G[\Phi]\eta\}, \tag{2.8}$$

where the "collective" action $S[\Phi]$ is given by

$$S[\Phi] = \int dt\,\frac{1}{2}\Phi(t)V^{-1}\Phi(t) - i\,\mathrm{tr}\log G^{-1}[\Phi]^*. \tag{2.9}$$

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* Note, according to our convention of matrix multiplication the trace refers also to the time "index" t.

From the least action principle $\delta S[\Phi]/\delta\Phi = 0$ we find the equation of motion of the collective field $\Phi(t)$ (Euler-Lagrange eq.)

$$\Phi_{\alpha\gamma}(t) = i \sum_{\beta\delta} V_{\alpha\gamma,\beta\delta} \; G_{\delta\beta}[\Phi](t,t')\big|_{t=t'-0} \qquad (2.10)$$

which is equivalent to Dyson's equation. In the static approximation $\Phi(t) \to \Phi^{\circ}$ eq. (2.10) becomes the ordinary Hartree-Fock equation. Suppose, that such a static solution Φ° has been found. Then, in general $\Phi^{\circ}_{\alpha\gamma}$ will not be diagonal and may produce, for example, a stable shape deformation. It is then convenient to introduce a new single particle basis $\{\lambda, \mu, \nu, \ldots\}$, in which the propagator of the nucleons in the "external" static field $G^{\circ} = G[\Phi = \Phi^{\circ}]$ becomes diagonal

$$G^{\circ}_{\lambda\mu} = \delta_{\lambda\mu} G^{\circ}_{\lambda} \; . \qquad (2.11a)$$

In the following, it is convenient to adopt a particle-hole picture. Let i, j (k, ℓ) denote the Hartree-Fock states below (above) the Fermi surface. Then the single particle Green function in the (self-consistent) static field approximation reads

$$iG^{\circ}_{\lambda}(\tau) = \begin{cases} \theta(\tau)\, e^{-i\epsilon_{\lambda}\tau} & \text{for } \lambda = k \\ -\theta(-\tau)\, e^{-i\epsilon_{\lambda}\tau} & \text{for } \lambda = i \, , \end{cases} \qquad (2.11b)$$

where ϵ_{λ} denotes the Hartree-Fock single particle energies. Generally, the collective field $\Phi(t)$ may oscillate around its static value Φ°. For small amplitudes $\Phi' = \Phi - \Phi^{\circ}$ we can expand the (full) single particle Green function $G[\Phi = \Phi^{\circ} + \Phi']$ (eq. (2.7)) in powers of $\Phi'(t)$. (The Prime at Φ' is dropped in the following)

$$G_{\lambda\mu}[\Phi](t,t') = \delta_{\lambda\mu} G^{\circ}_{\lambda}(t,t') - \int dt_1\, G^{\circ}_{\lambda}(t,t_1)\Phi_{\lambda\mu}(t_1) G^{\circ}_{\mu}(t_1,t') +$$

$$+\sum_{\nu} \int dt_1 dt_2 G^{\circ}_{\lambda}(t,t_1)\Phi_{\lambda\nu}(t_1)G^{\circ}_{\nu}(t_1,t_2)\Phi_{\nu\mu}(t_2)G_{\mu}(t_2,t')_{\mp} \dots \quad (2.12)$$

This yields for the second term in eq. (2.9)

$$-i\,tr\,(\log G^{-1}[\Phi]) = = -i\,tr(\log G^{\circ -1}) + \sum_{n=1}^{\infty} L_n[\Phi], \quad (2.13)$$

where

$$L_n\,[\Phi] = -i\,tr\{\frac{(-)^{n+1}}{n}(G^{\circ}\Phi)^n\}. \quad (2.14)$$

Collecting the terms of a given power of $\Phi(t)$ yields

$$S[\Phi] = S_0 + S_1\,[\Phi] + S_2[\Phi] + S_{int}\,[\Phi], \quad (2.15)$$

where

$$S_0 = const,$$

$$S_1\,[\Phi] = \int dt\,\Phi^{\circ}\,V^{-1}\,\Phi(t) + L_1[\Phi], \quad (2.16)$$

$$S_2\,[\Phi] = \int dt\,\frac{1}{2}\,\Phi(t)V^{-1}\Phi(t) + L_2\,[\Phi],$$

$$S_{int}\,[\Phi] = \sum_{n\geq 3} L_n[\Phi]. \quad (2.17)$$

The irrelevant constant S_0 can be dropped. Further, the term linear in $\Phi(t)$, $S_1[\Phi]$ vanishes due to the equation of motion (2.10). The quadratic part of the action, $S_2[\Phi]$, is used for defining the (unperturbed) collective excitation modes (free phonons). From the least action principle, $\delta S_2[\Phi]/\delta\Phi = 0$, we find the corresponding free equation of motion of the collective field

$$\Phi_{\lambda\mu}(t) = V_{\lambda\mu,\lambda'\mu'} \int dt' \Gamma_{\lambda'\mu',\lambda''\mu''}(t,t')\Phi_{\lambda''\mu''}(t') \quad (2.18)$$

where the propagator function $\Gamma(t,t')$ is defined by

$$\Gamma(t,t') = \int \frac{d\omega}{2\pi} e^{-i\omega(t-t')} \Gamma(\omega),*$$

with

$$i\Gamma_{\lambda\mu,\lambda'\mu'}(\omega) = \delta_{\lambda\mu'}\delta_{\mu\lambda'} \int \frac{d\epsilon}{2\pi} G^{\circ}_{\mu}(\omega+\epsilon)G^{\circ}_{\lambda}(\epsilon).$$

Its only non-vanishing matrix elements read

$$\Gamma_{ik,ki}(\omega) = \frac{1}{\omega-(\epsilon_k-\epsilon_i)+i\delta}, \Gamma_{ki,ik}(\omega) = \frac{-1}{\omega+\epsilon_k-\epsilon_i-i\delta}.$$

Equation (2.18) is nothing else but the (homogeneous) Bethe-Salpeter equation in the Random Phase approximation (RPA). The quadratic part of the collective action $S_2[\Phi]$ can be more compactly written as

$$S_2[\Phi] = \frac{1}{2} \int dt\, dt' \Phi(t) T^{-1}(t,t') \Phi(t') \quad (2.19)$$

where the free propagator of the collective field, $T(t,t')$, which after Fourier transformation reads

$$T^{-1}(\omega) = V^{-1} - \Gamma(\omega) \quad (2.20)$$

is easily recognized as the particle-hole T-matrix in the RPA (cf. ref. [2]). We should mention that although we have used antisymmetrized matrix elements $V_{\lambda\mu,\lambda'\mu'}$, the whole T-matrix is antisymmetric only in the first order (Born approximation).

As a result of our previous considerations we are left with the following generating functional

* This definition of Fourier transformation is used througout the paper.

$$Z[\eta,\eta^+,Q] = \mathcal{N} \int d\Phi \exp i\{ \frac{1}{2}\Phi T^{-1}\Phi + S_{int}[\Phi] - \qquad (2.21)$$

$$- Q V^{-1}\Phi + \frac{1}{2} Q V^{-1} Q - \eta^+ G[\Phi]\eta \},$$

where now the fermions appear only as external sources. The expansion of the total single particle Green function (eq. (2.12)) shows that an external fermion may emit or absorb an arbitrary number of collective Φ lines (see fig. 1a). No closed fermion loop appears in this expansion. The fermion loops are fully involved in the collective action $S[\Phi]$ (eq. (2.15)) (see fig. 1b). The second order loops (bubble diagrams), $L_2[\Phi]$, have been included in the definition of the collective modes (see above). The higher order fermion loops, $L_n[\Phi]$, $n > 2$, produce effective Φ^n interactions (see fig. 1b). Thus the original fermion interaction (eq. (2.1)) has been transformed into self-interactions of the collective field $\Phi(t)$, which generate anharmonicities in the vibrational spectra.

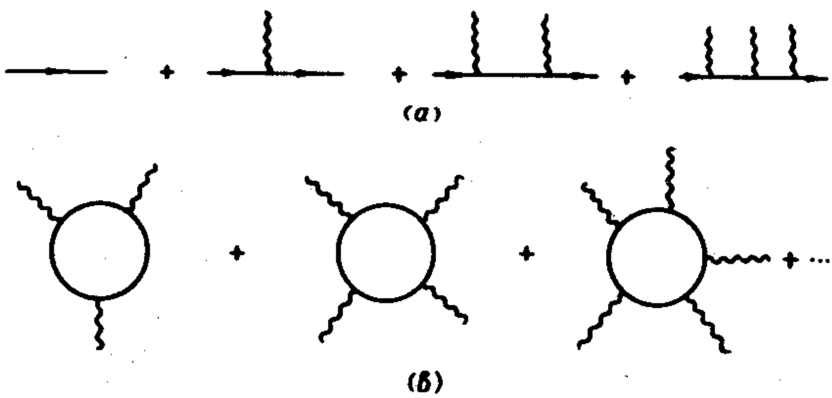

Fig. 1 a. Diagrammatic representation of the expansion of the single particle Green function $G[\Phi]$ (Eq. (2.12)) in powers of the collective field $\Phi(t)$. A full line represents the "free" single particle Green function G^o, a wavy line the collective field $\Phi(t)$. b. Loop expansion of the interaction term $S_{int}[\Phi]$ (eq. (2.17)).

For the time being let us forget about the bilocal fermion source Q (i.e., we put $Q=0$) and introduce a new source term $-iq\Phi$. Then we can perform the integration over the Φ variable

$$Z[\eta,\eta^+,0] = \mathcal{N} \exp i\{S_{int}[i\frac{d}{dq}] - \eta^+ G[i\frac{d}{dq}]\eta\} \times \\ \times \exp(-\frac{i}{2} q T q)|_{q=0} \quad . \tag{2.22}$$

Eq. (2.2 2) defines a modified parturbation theory (loop expansion) which is completely equivalent to the usual Feynman diagrammatic perturbation theory. On the other hand, like the NFT it represents a perturbation expansion in powers of the T-matrix. However, the collective field employed in the loop expansion is not yet the phonon field of the NFT. The relationship of the loop expansion (2.22) to the Nuclear Field Theory will be established in the next section.

3. THE NFT FOR PROCESSES CONNECTING INTERMEDIATE STATES. DERIVATION OF THE NFT LAGRANGIAN

In the modified perturbation theory based on the generating functional (2.22) (loop expansion) the fermion variables are completely removed. The fermions appear only as external sources. On the other hand, the NFT starts from an effective Lagrangian involving both fermions and collective (phonon) fields, including their coupling. In the following we shall show how we can obtain from the generating functional (2.21) the NFT expansion. We shall derive the NFT Lagrangian and simultaneously obtain the corresponding graphical rules. For this purpose we express in eq. (2.21) the interaction of the collective field $S_{int}[\Phi]$ by an integral over new fermion variables c_λ, c_λ^+

$$Z[\eta,\eta^+,Q] = \mathcal{N} \int d\Phi \exp(-iL_1[\Phi] - iL_2[\Phi]) \int dc\, dc^+ \times$$

$$\times \exp i\Big(\int dt \Big\{ \sum_\lambda c_\lambda^+(t)(i\partial_t - \epsilon_\lambda)c_\lambda(t) + \tag{3.1}$$

$$+ \sum_{\lambda\mu} \Phi_{\lambda\mu}(t) c_\lambda^+(t) c_\mu(t) \Big\} + \frac{1}{2}\Phi T^{-1}\Phi - QV^{-1}\Phi + \frac{1}{2}QV^{-1}Q +$$

$$+ \eta^+ c + c^+\eta \Big).$$

The operators c_λ, c_λ^+ are the Hartree-Fock quasiparticle operators corresponding to the single particle basis in which the Green function $G^0 = G[\Phi^0]$ is diagonal (see eq. (2.11)). Expressing the Φ variable in $L_1[\Phi], L_2[\Phi]$ by the source Q the integration over this variable can be performed

$$Z[\eta,\eta^+,Q] = \mathcal{N} \exp\Big(-iL_1\Big[iV\frac{\delta}{\delta Q} + Q\Big] - iL_2\Big[iV\frac{\delta}{\delta Q} + Q\Big]\Big) \times$$

$$\times \int dc\, dc^+ \exp i\Big\{ \mathcal{L} + Qc^+c + \eta^+ c + c^+\eta - \tag{3.2}$$

$$- \frac{1}{2}(c^+c - QV^{-1}) T_c (c^+c - V^{-1}Q)\Big\}.$$

Here $\mathcal{L}(t)$ is the fermion Lagrangian in the Hartree-Fock quasiparticle basis:

$$\mathcal{L}(t) = \sum_\lambda c_\lambda^+(t)(i\partial_t - \epsilon_\lambda)c_\lambda(t) - \tag{3.3}$$

$$- \frac{1}{2} \sum_{\lambda\mu\lambda'\mu'} V_{\lambda\mu\lambda'\mu'} c_\lambda^+(t) c_{\lambda'}^+(t) c_{\mu'}(t) c_\mu(t).$$

Further, we have splitted up from the T-matrix the contact term

$$T = V + T_c ,$$

where

$$T_c = VF^{-1}V, \qquad F(\omega) = \Gamma^{-1}(\omega) - V. \tag{3.4}$$

The term involving $T_c(t,t')$ in eq. (3.2) is now linearized by means of a new collective field $\phi(t)$ in analogy to eq. (2.3). This yields

$$Z[\eta,\eta^+,Q] = \mathcal{N}\exp(-iL_1[iV\frac{\delta}{\delta Q}+Q]-iL_2[iV\frac{\delta}{\delta Q}+Q])\times$$
$$\times\int d\phi\, dc\, dc^+ \exp i\{\mathcal{L}_{nf}-\phi V^{-1}Q+Qc^+c+\eta^+c+c^+\eta\}, \qquad (3.5)$$

where

$$\mathcal{L}_{nf} = \mathcal{L}+\mathcal{L}_{ph}+\mathcal{L}_{pv} \qquad (3.6)$$

is the NFT Lagrangian which contains besides the full fermion Lagrangian $\mathcal{L}$, the free Lagrangian of the collective field

$$\mathcal{L}_{ph} = \frac{1}{2}\phi T_c^{-1}\phi \qquad (3.7)$$

and the coupling between the fermion and the collective fields

$$\mathcal{L}_{pv} = \phi c^+c \ . \qquad (3.8)$$

The free equation of motion of the new collective field reads

$$\sum_{\lambda'\mu'}\int dt'(T_c^{-1}(tt'))_{\lambda\mu\lambda'\mu'}\ \phi_{\lambda'\mu'}(t') = 0, \qquad (3.9)$$

where the propagator T_c has been defined in eq. (3.4). Quantization of the collective field $\phi(t)$ yields

$$\phi_{\lambda\mu}(t) = \sum_n(\Lambda^n_{\lambda\mu}B_n(t)+\Lambda^{n+}_{\lambda\mu}B^+_n(t)), \qquad (3.10)$$

where $B_n(t)$, $B_n^+(t)$ are phonon operators in the interaction picture. Further, the particle-phonon vertices Λ^n have to satisfy the free equation of motion (RPA eq.)

$$F(\omega)V^{-1}\Lambda^{n} = F(\omega)\chi^{n} = 0,$$
$$F(\omega)V^{-1}\Lambda^{n+} = F(\omega)\chi^{n+} = 0, \qquad (3.11)$$

yielding the eigenfrequencies $\omega = \omega_n$ and $\omega = -\omega_n$, respectively. The eigenvectors

$$\chi^{n} = V^{-1}\Lambda^{n}, \quad \chi^{n+} = V^{-1}\Lambda^{n+} \qquad (3.12)$$

are given by the usual RPA amplitudes

$$\chi^{n}_{ki} = r^{n}_{ki}, \quad \chi^{n}_{ik} = s^{n}_{ki}, \qquad (3.13)$$

which satisfy the following orthogonality relations

$$\sum_{ki}(s^{n}_{ki}r^{m}_{ki} - r^{n}_{ki}s^{m}_{ki}) = 0$$
$$\sum_{ki}(r^{n*}_{ki}r^{m}_{ki} - s^{n*}_{ki}s^{m}_{ki}) = \delta_{nm} . \qquad (3.14)$$

We may then rewrite the NFT Lagrangian in terms of the phonon operators B_n, B^{+}_{n} . The free phonon term $\mathcal{L}_{ph}$ for instance, takes the form (cf. eq.(3.7))

$$\mathcal{L}_{ph}(t) = \sum_{n} B^{+}_{n}(t)(i\partial_t - \omega_n)B_n(t),$$

where we have used the definition of the amplitudes χ^{n} , eq. (3.12), and the relations (3.14). Further, the collective interaction part T_c can be expressed by the propagators of the phonons

$$D^{\pm}_{n}(\omega) = \frac{1}{\pm\omega - \omega_n + i\delta} .$$

This yields (cf. ref. /2/)

$$T_c(\omega) = \sum_{n}(\Lambda^{n^+}D^{+}_{n}(\omega)\Lambda^{n} + \Lambda^{n}D^{-}_{n}(\omega)\Lambda^{n^+}) . \qquad (3.15)$$

The generating functional (3.5) which for convenience we rewrite as

$$Z[\eta,\eta^+,Q] = \mathfrak{N}\exp(-iL_1[\phi - Vc^+c + Q])\times$$

$$\times\exp(-iL_2[\phi - Vc^+c + Q])\times \tag{3.16}$$

$$\times\int d\phi\, dc\, dc^+ \exp i\{\mathcal{L}_{nf} - \phi V^{-1}Q + Qc^+c + \eta^+c + c^+\eta\}$$

defines a modified perturbation theory which for processes connecting intermediate states coincides with the NFT. The first two exponents in eq. (3.16), $\exp(-iL_1)$ and $\exp(-iL_2)$, project out from the perturbation expansion the Hartree-Fock selfenergy insertions (see fig. 2a,b,c)) and the bubble diagrams (see fig. 2d), respectively. These graphs have been

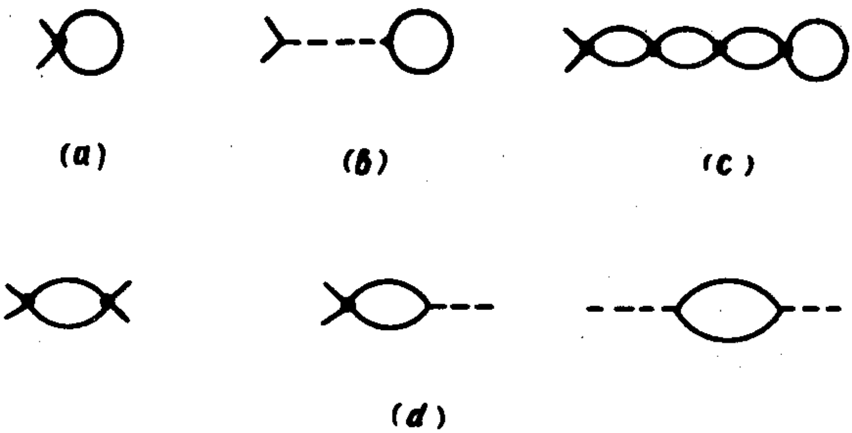

Fig. 2 a. Lowest order Hartree-Fock self-energy diagram. b. Tadpole diagram which has also to be excluded from the NFT expansion. c. Typical Hartree-Fock graph of the fermion treatment involved in the tadpole diagram shown in (b). d. Bubble diagrams. A fat dot stands for the antisymmetrized matrix element of the two-body interaction and a dashed line represents a phonon.

already included in the definition of the single particle and the collective excitation modes, respectively. More precisely, the term $- Vc^+c$ in L_1 cancels the Hartree-Fock insertions shown in fig. 2a while the term ϕ in L_1 eliminates the so-called "tadpole" diagrams (fig. 2b). The fact that the tadpole diagrams have to be excluded from the NFT expansion if Hartree-Fock single particle states are used has up to now not been mentioned in literature. Using the definition of L_n (eq. (2.14)) and the single particle Green function (eq. (2.11)) one finds for L_1 the expression:

$$L_1 [\phi - Vc^+c + Q] = \int dt \{ \sum_i (\phi_{ii}(t) -$$

$$- \sum_{\nu\mu} V_{ii,\mu\nu} \; c^+_\mu(t) c_\nu(t) + Q_{ii}(t)) \}.$$

In the subsequent section we shall use the source Q only for generating external particle-hole states (ki). Therefore we can drop the quantity Q_{ii} in L_1.

In concluding this section we should stress that in the perturbation theory based on the generating functional (3.16) the external states are purely antisymmetrized fermion states while in the NFT they involve both particles and collective modes (phonons) but not any particle configuration that can be replaced by a combination of collective modes. Of course, for practical applications the use of the NFT is more advantageous since its basis states give already a good zero order description of the nuclear excitation modes. However, as mentioned in the introduction, in the NFT there remain some problems concerning its relation to the ordinary Feynman diagrammatic perturbation theory for processes connecting initial and final (external) states. These questions are investigated in the next section.

4. THE RELATION BETWEEN THE FERMION AND THE NUCLEAR FIELD TREATMENT OF THE RESIDUAL INTERACTION

From our above given discussion it is clear that a one-to-one correspondence between the NFT and the usual Feynman diagrammatic many-body perturbation theory can exist only for processes connecting intermediate states. For these processes there exists a holomorphic mapping of the bubble strings onto the RPA phonon lines (cf. refs. [2,14]). But we cannot generate the external antisymmetric fermion states by phonons. In the following we want to study how the NFT "translates" the antisymmetric fermion states in the phonon picture. Especially, we wish to know which approximations are involved in this step. For this purpose we shall below modify the generating functional (3.16) in such a way that it yields the NFT expansion. In doing so, a unique prescription is obtained, by means of which the Feynman diagrams of the fermion treatment are transformed into the Mottelson diagrams of the NFT. To this aim we change in the generating functional (3.16) the bilocal fermion source by

$$J = V^{-1} Q \tag{4.1}$$

Inserting this expression into eq. (3.16) the generating functional takes form

$$\begin{aligned}\tilde{Z}[\eta^+, \eta, J] = \mathfrak{N} \int d\phi\, dc\, dc^+ \exp(iJVc^+c\ )\times \\ \times\exp(-iL_1[\phi - Vc^+c])\exp(-iL_2[\phi - Vc^+c + VJ])\times \\ \times\exp i\{\mathcal{L}_{nf} - \phi J + \eta^+c + c^+\eta\}.\end{aligned} \tag{4.2}$$

What the source transformation (4.1) means in the form $\tilde{Z}$ resulting perturbation expansion can be read off from our starting functional of the fermion treatment, $Z[\eta, \eta^+, Q]$ given by eq. (2.2). Let us, for instance, consider the two-particle-one-hole Green function K. It follows from $Z[\eta,\eta^+,Q]$ by functional differentiation with respect to the external sources*

$$K(ki\ell,k'i'\ell')\Big|_{\substack{t_k=t_i\\ t_{k'}=t_{i'}}} = \frac{\delta}{i\delta Q_{ik}} \cdot \frac{\delta}{i\delta\eta^+_\ell} \frac{\delta}{(-i)\delta\eta_{\ell'}} \frac{\delta}{i\delta Q_{k'i'}} \times \tag{4.3}$$

$$\times Z[\eta,\eta^+,Q]\Big|_{\eta=\eta^+=Q=0}$$

$$= \int dc\,dc^+\{c_i^+ c_k c_\ell c_{\ell'}^+ c_{k'}^+ c_{i'}\}\, e^{i\mathcal{L}(c,c^+)},$$

where we have used the short notation $c_k \equiv c_k(t_k)$, $Q_{ki} \equiv Q_{ki}(t_k = t_i)$, etc. On the other hand variation with respect to the new source $J = V^{-1}Q$ yields

$$\mathcal{J}_f'(ki\ell,k'i'\ell')\Big|_{\substack{t_k=t_i\\ t_{k'}=t_{i'}}} = \frac{\delta}{i\delta J_{ik}} \frac{\delta}{i\delta\eta_\ell^+} \frac{\delta}{(-i)\delta\eta_{\ell'}} \frac{\delta}{i\delta Q_{k'i'}} \times$$

$$\times \tilde{Z}[\eta,\eta^+,J]\Big|_{\eta^+=\eta=J=0} \tag{4.4}$$

$$= \sum_{\lambda\mu\lambda'\mu'} V_{ik\mu\lambda} V_{\lambda'\mu'k'i'} K(\lambda\mu\ell,\lambda'\mu'\ell')\Big|_{\substack{t_\lambda=t_\mu=t_k\\ t_{\lambda'}=t_{\mu'}=t_{k'}}}.$$

From comparison of eqs. (4.3) and (4.4) it is seen that variation with respect to the source $J = V^{-1}Q$ instead to Q implies an additional interaction of the particle-hole pairs of the original external states $(ki),(k'i')$ (see eq. (4.3)) which as a result of the source transformation (4.1) become intermediate states $(\lambda\mu),(\lambda'\mu')$ (see eq. (4.4)), which have to be

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* We suppose the starting functional $Z[\eta,\eta^+,Q]$ eq. (2.2)) to have been rewritten in the Hartree-Fock single particle basis $\{\lambda,\mu,\nu,\ldots\}$ defined by eq. (2.11).

summed over both particle and hole states in the sense of the Feynman convention. In general, the obtained quantity $\mathcal{J}_f'$, called herafter improper NFT interaction part, does not depend on the choice of the particle hole pairs (ki), (k′i′) in the many-body Green function K of the fermion treatment, as K is total antisymmetric in the fermion indices.

Now, when varying the generating functional $\tilde{Z}[\eta^+, \eta, J]$ with respect to J_{ik} ($J_{k'i'}$) the first exponent in eq. (4.2) leads to Feynman diagrams where the particle-hole pair of the initial (final) state (ki)((k′i′)) interacts once via $V_{ik,\mu\lambda}$ ($V_{\lambda'\mu', k'i'}$) but is not followed by a bubble string. This means that in these diagrams the fermion lines $\mu(\lambda')$ and $\lambda(\mu')$ (see eq. (4.4)) are subsequently annihilated at different times and hence by different interaction processes V. Such particle-hole states cannot be expressed by phonon lines and the corresponding diagrams are neglected in the NFT (see fig. 3). Neglecting the exponent $\exp(iJ\,Vc^+c)$ in eq. (4.2) we have also to exclude in L_2 the term JV that compensates just the bubble strings

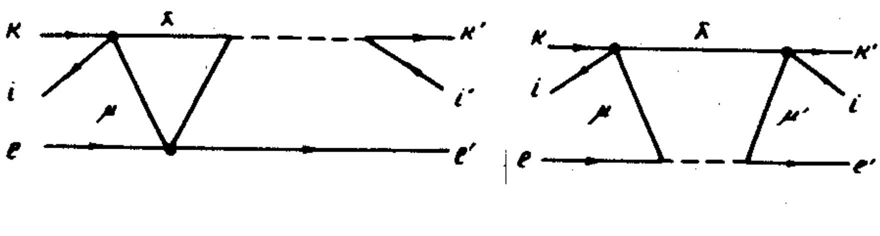

Fig. 3. Typical Feynman diagrams which would arise from the first exponent in eq. (4.2) and which are neglected in the NFT.

(involving, e.g., the interaction $V_{ik,\lambda\mu}$ ($V_{\lambda'\mu',k'i'}$) which would arise from the exponent $\exp(iJVc^+c)$). We are then left with a generating functional

$$\tilde{\tilde{Z}}[\eta,\eta^+,J] = \mathfrak{N}\int d\phi\, dc\, dc^+ \exp(-iL_1[\phi - Vc^+c]) \times \qquad (4.5)$$

$$\times \exp(-iL_2[\phi - Vc^+c])\exp i\{\mathcal{L}_{nf} - \phi J + \eta^+ c + c^+\eta\}$$

which yields by functional differentiation with respect to the external sources the (proper) NFT interaction part $\mathcal{F}_f$. From $\mathcal{F}_f$ we obtain the corresponding NFT Green functions by cutting off the external particle-phonon vertices (see fig. 5, cf. also ref. /9/). This can be achieved by an additional differentiation with respect to the particle-phonon vertices Λ^n (see eq. (3.10)) or more conveniently by introducing new sources

$$q_n(t) = \sum_{\lambda\mu}\Lambda^n_{\lambda\mu} J_{\lambda\mu}(t), \quad q^+_n(t) = \sum_{\lambda\mu}\Lambda^{n+}_{\lambda\mu} J_{\lambda\mu}(t), \qquad (4.6)$$

Then, we, finally, arrive at the generating functional of the NFT

$$Z_{nf}[\eta,\eta^+,q,q^+] = \mathfrak{N}\int dc\, dc^+ dB\, dB^+ \times$$

$$\times \exp(-iL_1[\phi(B,B^+) - Vc^+c])\exp(-iL_2[\phi(B,B^+) - Vc^+c]) \qquad (4.7)$$

$$\times \exp i\{\mathcal{L}_{nf} + q^+B + B^+q + \eta^+c + c^+\eta\}$$

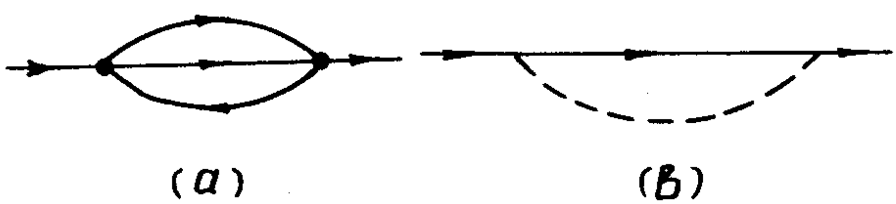

Fig. 4 . a. Part of a Feynman diagrams with two equivalent fermion lines, b. the corresponding Mottelson diagram of the NFT, in which the diagram shown in (a) is doubly counted.

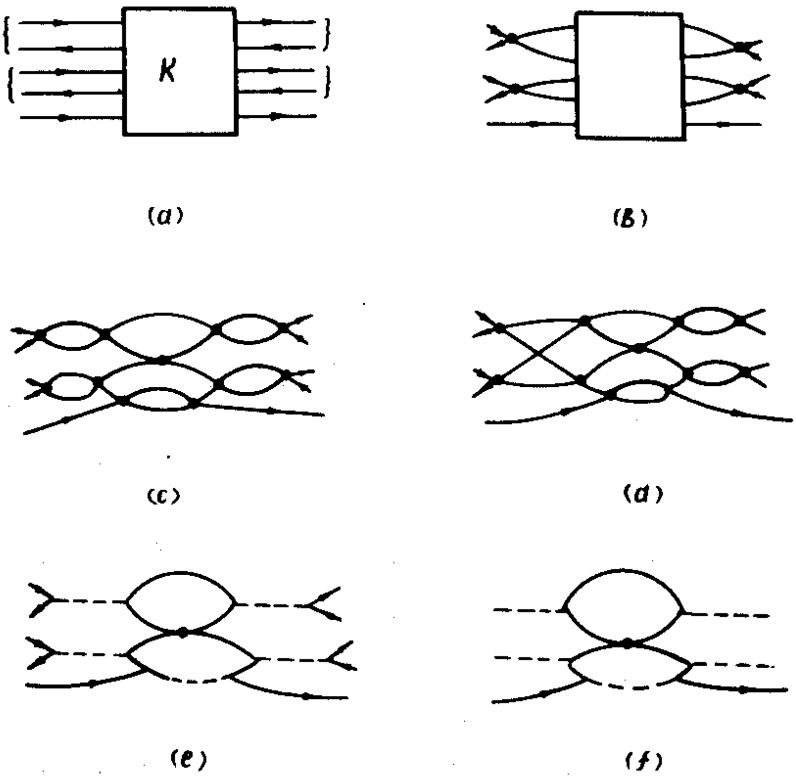

Fig. 5. a. Diagrammatic representation of a many-body Green function K, where the external lines have been grouped in particle-hole pairs. b. The particle hole pairs have been supplied with an additional two-body interaction. c. Typical diagram included in the NFT interaction part $\mathfrak{J}_f$. d. Typical diagram neglected in the NFT. e. Equivalent representation of the diagram shown in (c) in the NFT after partial summation over the involved bubble graphs has been performed, f. the corresponding Mottelson diagram of the NFT Green function.

from which by variation with respect to the sources q , q^+ , η , η^+ the needed particle-phonon Green functions of the NFT follow, which do not contain particle-hole pairs in the external states, which can be replaced by a combination of collective modes. Further, the first exponent in eq. (4.7) eliminates the Hartree-Fock insertions (see fig. 2a,b,c) while the second exponent projects out from the NFT expansion the bubble graphs (see fig. 2d). However, we should keep in mind that we have used antisymmetrized interaction matrix elements $V_{\lambda\mu,\lambda'\mu'}$. In the expansion of the third exponent in eq. (4.7), $\exp(i\mathcal{L}_{nf})$, bubble diagrams may appear with two equivalent fermion lines (see fig. 4a). But two equivalent fermion lines contribute an additional factor 1/2 to the diagram (see. ref.[15]). On the other hand, the second exponent yields the negative of these bubble diagrams but without the factor 1/2 arising from two equivalent fermion lines. Thus, effectively, these bubble diagrams (including the factor 1/2) of the fermion treatment are subtracted in the NFT expansion*. This is in agreement with the rule stated in ref.[4] : If a fermion line is in parallel with a phonon line (see fig. 4b) the corresponding second order bubble diagram has to be subtracted in order to avoid double counting. This completes our derivation of the graphical rules of the NFT. In summarizing our results we can formulate the following unique prescription for obtaining the Mottelson diagrams of the NFT from the Feynman diagrams of the fermion treatment:

i) Take the Feynman diagrams of the (total antisymmetric) many-body Green function K and select out in the initial and final state, separately, as much as possible particle-hole pairs (see fig. 4a).

*This result is immediately obtained by expanding in eq. (4.7) $\exp(-iL_2)$ up to the first order and $\exp(i\mathcal{L}_{nf})$ up to the second order.

ii) The thus obtained particle-hole pairs are supplied with an additional two-body interaction (see fig. 4b). As a result they become intermediate states, which have to be free summed over.

iii) Neglect such diagrams where not all the (new) external particle-hole pairs are followed by a bubble string (see figs. 4c,d). The remaining diagrams form the NFT interaction part $\mathcal{T}_f$ (cf. eq. (4.4)).

iv) Perform partial summation over the bubble diagrams what yields the collective interaction part T_c, which has to be expressed by the phonon propagators $D_n^{\pm}$ (see eq. (3.15) and fig. 4e).

v) Cut off the particle-phonon vertices Λ_{ki}^{n} of the external particle-hole pairs (see fig. 4f). The diagrams left are the Mottelson diagrams of the NFT.

In this way we have by now established the relation between the NFT and the usual Feynman diagrammatic many-body perturbation theory.

As the NFT is not completely equivalent to the fermion treatment, for the time being we cannot expect that the NFT yields the exact excitation energies of interacting Fermi systems. On the other hand, for the schematic models considered so far in the NFT (see, e.g., refs. /1,4/) the exact results following from a shell model diagonalization could be reproduced. In addition the nuclear field treatment yields, however, also some spurious states /8,9/. Therefore a question arises: Can we make some general statements about the excitation energies following from the nuclear field treatment in comparison with the excitation energies which would follow from the fermion treatment? To get some information about the excitation energies obtained in the NFT we have to recapitulate what diagrammatic modifications have led to the NFT (see i)-v)). The excitation energies are given by the poles of the (antisymmetric) many-body Green functions $K(\omega)$ in the energy representation defined by

$$K(t,t') = \int \frac{d\omega}{2\pi} e^{-i\omega(t-t')} K(\omega),$$

where $K(t,t')$ is the Green function in which all fermion operators of the initial and final state have been taken at equal times t and t′, respectively. Obviously, by the steps i) and ii) the pole behaviour cannot be changed if we exclude an accident cancellation of any pole due to the summation over the external states of K, implied in step ii). Thus poles of $K(\omega)$ will also lead to poles of $\mathcal{T}'_f$ (see eq. (4.4)). Further, the diagrams neglected in step iii) differ from the ones left in the NFT only by the exchange of some external fermion lines of $K(\omega)$. Hence the in the NFT neglected and included diagrams of $\mathcal{T}'_f$ have the same poles but may yield contributions of opposite sign. Therefore some poles present in both direct and exchange diagrams may cancel in their sum, and hence are absent in the total Green function $K(\omega)$. By neglecting the exchange diagrams of K in the NFT, this cancellation is annuled and, as a consequence, the NFT amplitude $\mathcal{T}_f$ or equivalently the NFT Green function K_f may have some additional poles (not present in $K(\omega)$) corresponding to spurious states (cf. also ref. /9/).

We are led to the following conclusion:

Although some definite (exchange) diagrams are neglected in the NFT, it provides us with the correct excitation energies. However, in addition, some spurious states may arise in the nuclear field treatment of the residual interaction.

5. SUMMARY AND CONCLUSION

By using path integral methods the Nuclear Field Theory has been developed for a Fermi system interacting via a general two-body force. The NFT Lagrangian has been derived, the corresponding graphical rules have been obtained. Besides, showing the equivalence between the NFT and the usual Feynman diagrammatic perturbation theory for intermediate processes, we have established the relation between both theories for processes connecting initial and final states, too. We have found a unique prescription how the Mottelson diagrams of the NFT can be obtained from the corresponding Feynman diagrams of the fermion treatment. This prescription enables us to make definite statements about the results obtained in the NFT as compared to the exact results of a shell model diagonalization. The nuclear field treatment yields the correct excitation energies although, as we could show, it neglects definite exchange diagrams. This neglect, which corresponds to a non-proper treating of the Pauli principle in the external states, results in the appearance of spurious states in the Nuclear Field Theory.

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