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FUNCTIONAL APPROACH TO THE NUCLEAR FIELD THEORY: A SCHEMATIC MODEL WITH PAIRING AND PARTICLE-HOLE FORCES

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FUNCTIONAL APPROACH TO THE NUCLEAR FIELD THEORY: A SCHEMATIC MODEL WITH PAIRING AND PARTICLE-HOLE FORCES

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Функциональный подход к ядерной теории поля: схематическая модель с парными и частично-дырочными силами

Применяется метод континуального интеграла в коллективных переменных к схематической модели с монопольными парными и частично-дырочными силами. Обсуждаются одночастичные и коллективные возбуждения для различных фазовых переходов. Формулируется модифицированная теория возмущения (разложение по петлям), с помощью которого получается ядерная теория поля (ЯТП). Строго выведен лагранжиан ЯТП. Автоматически получаются диаграммные правила ЯТП.

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Functional Approach to the Nuclear Field Theory: a Schematic Model with Pairing and Particle-Hole Forces

Path integral techniques in collective variables are applied to a schematic model with monopole pairing and particle-hole forces. The single particle and collective excitation modes of the system for various kinds of phase transitions are discussed. We formulate a modified perturbation theory (loop expansion) from which, finally, the Nuclear Field Theory (NFT) is obtained. The NFT Lagrangian is strictly derived. The graphical rules of the NFT expansion come out automatically.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

The complexity of the nuclear many-body systems may be considerably reduced by introducing collective coordinates as, for example, in the random phase approximation (RPA) [1]. Then a great deal of the residual interaction between the nucleons is already included in the collective modes. However, in such an approach one is faced with the overcompleteness of the basis involving both single particle and collective states and with the identity of the nucleons appearing in the collective modes as well as in the single particle degrees of freedom. In ref. [2] a "Nuclear Field Theory" (NFT) has been developed which treats correctly both the overcompleteness of the basis and the Pauli principle. The relation of the field theoretical approach to the conventional Feanman diagrammatic many-body perturbation theory [3] has been established in refs. [4,5] by using partial summation techniques, i.e., by comparing the corresponding diagrams of the field and the fermion treatment. However, the nuclear field Hamiltonian has been derived only in a heuristic way [2,6]. One introduces collective modes (e.g., in the RPA) and adds to the full fermion Hamiltonian

$$H = H_{sp} + H_{tb}$$

the free phonon term

$$H_{ph} = \sum_n \omega_n c_n^+ c_n .$$

Further, a coupling term is included

$$H_{PV} = \sum_n (\Lambda^n_{\alpha\beta} c_n + \Lambda^n_{\alpha\beta} c^+_n) a^+_\alpha a_\beta ,$$

where the particle phonon vertices $\Lambda^n_{\alpha\beta}$ are calculated by taking matrix elements of the residual interaction H_{tb} between the phonon state $c^+_n|0\rangle$ and the relevant fermion state $a^+_\alpha a_\beta|0\rangle$

$$\Lambda^n_{\alpha\beta} = \langle 0| a^+_\beta a_\alpha H_{tb} c^+_n |0\rangle .$$

Then the NFT Hamiltonian reads

$$H_{NFT} = H + H_{ph} + H_{PV} .$$

Obviously, in the thus obtained Hamiltonian some correlations are doubly counted. To remove this double counting definite restrictions on the diagrammatic perturbation treatment of H_{NFT} have to be postulated [2]. Clearly, such a heuristic derivation of the NFT, although it may prove correctly, is not satisfactory from the theoretical point of view. Previously [7] an attempt has been made to derive the NFT Hamiltonian H_{NFT} via the canonical transformation method. However H_{NFT} could be obtained there only in the lowest order perturbation theory. The aim of the present paper is to give a rigorous, non-perturbation theoretical derivation of the NFT Hamiltonian together with the corresponding diagrammatic rules via path integral methods [8,9]. Further we want to study explicitly the nuclear field treatment for such systems which have undergone a phase transition, e.g., into a superfluid ground state (systems with spontaneous breaking of symmetry have been considered in the NFT in ref. [10]). To avoid unnecessary complications we develop our field theoretical approach for a schematic two-level model which includes both pairing and particle-hole forces [11]. The present paper has also a pedagogical aim. We want to demonstrate the powerfulness of path integral techniques in the study of nuclear structure.

We start from the path integral representation for the generating functional of the fermion Green functions. The integrations over the fermion variables can be performed by linearizing the (fermion) residual interaction with the help of collective fields describing the particle-particle and particle-hole degrees of freedom, respectively. As a result, a new effective action is obtained from which the equations of motion of the quasi-particle and collective excitations follow. In particular, there arises a modified perturbation theory in form of a loop expansion. Finally, by introducing new collective fields the loop expansion will be converted into the NFT expansion.

We have organized the paper as follows. In Sect. 2 the model is defined and the effective action is derived. In Sect. 3 the single particle and collective excitation spectra are discussed for different cases of phase transitions. The modified perturbation expansion and the derivation of the NFT in the presence of phase transitions are presented in Sect. 4. Some formulae and arguments needed in the text are presented in appendices.

2. MODEL AND METHOD

The model under consideration /11/ consists of N fermions which are distributed over two single particle levels, each of degeneracy $2\Omega = N$. The two levels are separated by an energy $\bar{\epsilon}$. A particle state is characterized by quantum numbers (σ, m), where $\sigma = \pm 1$ designates the upper and lower levels, respectively, and m enumerates the degenerate substates of each shell. The fermions interact pairwise via monopole particle-hole (ph) and particle-particle (pp) interactions *

$$H = H_{sp} + H_{tb} \ , \quad H_{sp} = \frac{\bar{\epsilon}}{2} \sum \sigma a^{+}_{m\sigma} a_{m\sigma} ,$$

$$H_{tb} = - \frac{\kappa}{2} (P^{+} P + P P^{+}) - \rho (A + A^{+})^{2} , \qquad (2.1)$$

where

$$P^{+} = \sum_{m\sigma} a^{+}_{m\sigma} a^{+}_{\bar{m}\sigma} , \quad A^{+} = \sum_{m} a^{+}_{m,1} a_{m,-1} . \qquad (2.2)$$

The operator $a^{+}_{m\sigma}$ $(a_{m\sigma})$ creates (annihilates) a particle in the state (σ, m) The state $(\sigma, \bar{m})$ is related to the state (σ, m) by time reversion. In the ground state $|0\rangle$ the N fermions occupy the 2Ω substates of the lower shell and we have

$$a_{m1} |0\rangle = a^{+}_{m,-1} |0\rangle = 0 . \qquad (2.3)$$

Absorbing the Hartree-Fock self-energy contributions arising from H_{tb} into renormalized single particle energies $\epsilon = ½ (\bar{\epsilon} + 2\rho + \kappa)$ we may rewrite the Hamiltonian as

$$H = \epsilon \sum_{m\sigma} \sigma a^{+}_{m\sigma} a_{m\sigma} - \kappa : P^{+} P : - \kappa : (A + A^{+})^{2} : \qquad (2.4)$$

where the normal product (denoted by : :) is defined with respect to the Hartree-Fock ground state (2.3).

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* As our system may undergo a phase transition to the superfluid ground state we should include in the Hamiltonian (2.1) a term $-\lambda \hat{N}$ in order to ensure conservation of the particle number N in the average. However, it turns out that in the considered model the chemical potential λ may always be set identical zero (see appendix C).

The generating functional for the fermion Green function of the system is given by the following path-integral

$$Z[\eta,\eta^{+}] = \mathcal{N}\int Da\,Da^{+}\exp i\int df\{\mathcal{L}_{f}(t)+\eta^{+}a+a^{+}\eta\}, \qquad (2.5)$$

where

$$\mathcal{L}_{f}(t)=\sum_{m\sigma} a^{+}_{m\sigma}(t)(i\partial_{t}-\sigma\epsilon)a_{m\sigma}(t)+\kappa:P^{+}P:+\rho:(A+A^{+})^{2}: \quad (2.6)$$

is the Lagrangian corresponding to the Hamiltonian of eq. (2.4) and $\mathcal{N}$ is an irrelevant normalization factor which is fixed by the requirement $Z[0,0]=1$. The fermion operators $a_{m\sigma}(t)$, $a^{+}_{m\sigma}(t)$ and the external sources $\eta_{m\sigma}(t)$ are now considered as anti-commuting (Grassman) variables. The integration over the fermion variables in eq. (2.5) can be easily carried out by linearizing the interaction terms with the help of (real and complex) dynamical variables (collective fields): Using the funational identities

$$\exp i\int dt\,\rho(A+A^{+})^{2} = c_{1}\int D\Phi\exp i\int dt\{-\frac{1}{4\rho}\Phi^{2}(t)+ \qquad (2.7)$$

$$+\Phi(t)(A+A^{+})\},$$

$$\exp i\int dt\,\kappa P^{+}P = c_{2}\int D\Psi D\Psi^{+}\exp i\int dt\{-\frac{1}{\kappa}\Psi^{+}(t)\Psi(t)+ \qquad (2.8)$$

$$+P^{+}\Psi(t)+\Psi^{+}(t)P\},$$

where $\Phi(t)$, $\Psi(t)$ are commuting (Bose) variables, the generating functional $Z[\eta,\eta^{+}]$ takes the form:

$$Z[\eta,\eta^{+}] = \mathcal{N}_{1}\int Dh\,Dh^{+}\int D\Phi\int D\Psi D\Psi^{+} \times$$

$$\times\exp i\int dt\{h^{+}(t)G^{-1}h(t)+Q^{+}h(t)+h^{+}(t)Q- \qquad (2.9)$$

$$-\frac{1}{\kappa}\Psi^{+}(t)\Psi(t)-\frac{1}{4\rho}\Phi^{2}(t)\}.$$

Here we have, for convenience, introduced the matrix notation

$$h^{+}_{m\sigma} = (a^{+}_{m\sigma}, a_{\bar{m}\sigma}),$$
$$Q^{+}_{m\sigma} = (\eta^{+}_{m\sigma}, -\eta_{\bar{m}\sigma}),$$
$$G^{-1}_{\sigma\sigma'}(m;t,t') = G^{-1}_{\sigma\sigma'}(m;t)\delta(t-t'), \tag{2.10}$$

$$G^{-1}_{\sigma\sigma'}(m,t) = \begin{pmatrix} (i\partial_t - \sigma\epsilon)\delta_{\sigma\sigma'} + (1-\delta_{\sigma\sigma'})\Phi(t); & \Psi(t)\delta_{\sigma\sigma'} \\ \Psi^{+}(t)\delta_{\sigma\sigma'}; & (i\partial_t + \sigma\epsilon)\delta_{\sigma\sigma'} - (1-\delta_{\sigma\sigma'})\Phi(t). \end{pmatrix}$$

Performing in eq. (2.9) the integration over the fermion variables yields

$$Z[Q,Q^{+}] = \mathfrak{N}_2 \int D\Phi \int D\Psi D\Psi^{+} \exp i\{S[\Phi,\Psi,\Psi^{+}] - Q^{+}GQ\}, \tag{2.11}$$

where the new effective action S depends only on the collective variables Φ, Ψ, and is given by

$$S[\Phi,\Psi,\Psi^{+}] = \int dt\{-\frac{1}{\kappa}\Psi^{+}(t)\Psi(t) - \frac{1}{4\rho}\Phi^{2}(t) - \tag{2.12}$$
$$- i\Omega\,\mathrm{tr}(\log G^{-1})(t,t)\}.$$

The quantity G represents the Green function of a fermion moving in the collective fields $\Phi(t), \Psi(t)$. The explicit expression for G can be found in appendix A. The equations of motions of the collective fields follow by variation of S

$$\Phi_0(t) = -i2\Omega\rho\,\mathrm{tr}\{\begin{pmatrix} \hat{1} & 0 \\ 0 & -\hat{1} \end{pmatrix} G(t,t')\}_{t'=t+0}, \tag{2.13}$$

$$\Psi_0(t) = -i\Omega\kappa\, \mathrm{tr}\{\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} G(t,t')\}_{t'=t+0}\,, \qquad (2.14)$$

$$\hat{I} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}:$$

Let us suppose that there exist nontrivial solutions of eqs. (2.13), (2.14). We may then formulate a modified perturbation theory that uses the Green function $G_0 = G(\phi_0, \psi_0, \psi_0^+)$ as unperturbed propagator. For this purpose we expand the integrand of the generating functional in eq. (2.11) around the solution $\underset{\sim}{\Phi}_0(t)$.

$$\underset{\sim}{\Phi}(t) = \underset{\sim}{\Phi}_0(t) + \underset{\sim}{\Phi}'(t), \quad \underset{\sim}{\Phi}(t) = \begin{pmatrix} \phi(t) \\ \Psi(t) \\ \Psi^+(t) \end{pmatrix} . \qquad (2.15)$$

This brings the third term of eq. (2.12) into the form (the prime at $\underset{\sim}{\Phi}'(t)$ will be omitted in the following)

$$-i\Omega \int dt\, \mathrm{tr}(\log G^{-1})(t,t) = -i\Omega \int dt\, \mathrm{tr}(\log G_0^{-1})(t,t) + \sum_n L_n[\underset{\sim}{\Phi}]\,, \qquad (2.16)$$

where the term

$$L_n[\underset{\sim}{\Phi}] = -i\Omega \int dt\, \mathrm{tr}\{\frac{(-)^{n+1}}{n}[G_0 \begin{pmatrix} \phi\hat{I} & \Psi \\ \Psi^+ & -\phi\hat{I} \end{pmatrix}]^n\}(t,t) \qquad (2.17)$$

represents a closed fermion loop emitting or absorbing n collective lines Φ, Ψ. The bubble processes given by the term $L_2[\underset{\sim}{\Phi}]$ are, as usual, included into the free action.

---

* Matrix multiplication implies here integration over intermediate times.

$$S_{free}[\underset{\sim}{\Phi}] = \int dt\{-\frac{1}{\kappa}\Psi^{+}(t)\Psi(t) - \frac{1}{4\rho}\Phi^{2}(t)\} + L_{2}[\underset{\sim}{\Phi}]\}. \tag{2.18}$$

Thus we have

$$S[\underset{\sim}{\Phi}] = S_{free}[\underset{\sim}{\Phi}] + S_{int}[\underset{\sim}{\Phi}];\ S_{int}[\underset{\sim}{\Phi}] = \sum_{n=3}^{\infty} L_{n}[\underset{\sim}{\Phi}]. \tag{2.19}$$

The free action may be cast into the form

$$S_{free}[\underset{\sim}{\Phi}] = \tfrac{1}{2}\int dt\,dt'\underset{\sim}{\Phi}(t)\hat{T}^{-1}(t,t')\underset{\sim}{\Phi}(t'), \tag{2.20}$$

where the propagator of the collective field

$$\hat{T}(t,t') = \langle \underset{\sim}{\Phi}(t)\underset{\sim}{\Phi}(t')\rangle_{S_{free}}$$

is defined by the following 3x3 matrix ($\langle \dots \rangle_{S_{free}}$ denotes the functional average with the weight factor $\exp iS_{free}$)

$$\hat{T} = -\tfrac{1}{2}[1 + \hat{V}\hat{B}]^{-1}\,\hat{V}. \tag{2.21}$$

Here

$$\hat{V} = \begin{pmatrix} 4\rho & 0 & 0 \\ 0 & 0 & 2\kappa \\ 0 & 2\kappa & 0 \end{pmatrix} \tag{2.22}$$

is the coupling matrix and $\hat{B}$ is a matrix whose elements are given by bubble graphs composed of normal and/or anomalous Green's functions, respectively (for definitions, see appendix B). The collective propagator $\hat{T}$ coincides with the two-channel scattering amplitude for particle-particle (pp) and particle-hole (ph) scattering in the ladder approximation. This may easily be recognized by rewritting eq. (2.21) as an inhomogeneous Bethe-Salpeter equation

$$\hat{T} = -\tfrac{1}{2}\hat{V} - \hat{V}\hat{B}\hat{T}. \tag{2.23}$$

The equation of motion (Euler-Lagrange equation) for the free collective field $\Phi(t)$ follows by variation of the free action (2.20):

$$\underset{\sim}{\Phi}(t) = -\hat{V}\int dt'\hat{B}(t,t')\underset{\sim}{\Phi}(t'). \tag{2.24}$$

It coincides with the homogeneous BS-equation.

3. THE EXCITATION MODES

3.1. Single Particle Excitations

In this section we want to solve the coupled equations (2.13), (2.14). For simplicity we confine ourselves to static solutions $\underset{\sim}{\Phi}_0$. Using the explicit expressions for the Green function $G(t,t')$ given in appendix A equations (2.13), (2.14) take the form

$$\Phi_0 = \Phi_0\frac{\tilde{\rho}}{E}, \quad \Psi_0 = \Psi_0\frac{\tilde{\kappa}}{E}, \qquad (\tilde{\rho} = 4\Omega\rho, \quad \tilde{\kappa} = \Omega\kappa), \tag{3.1}$$

where the quasi-particle energy E is given by

$$E = \sqrt{\epsilon^2 + \Phi_0^2 + |\Psi_0|^2}. \tag{3.2}$$

Now Φ_0, Ψ_0 are easily recognized as energy gaps arising from phase transitions in the ph and/or pp-channels, respectively. Depending on the values of the coupling constants $\tilde{\rho}, \tilde{\kappa}$ we may distinguish the following cases

i) $\tilde{\rho} < \epsilon, \quad \tilde{\kappa} < \epsilon$:

$$\Phi_0 = \Psi_0 = 0;$$

The static field configuration $\Phi_0 = \Psi_0 = 0$ is stable.

ii) $\tilde{\rho} = E > \epsilon$, $\tilde{\kappa} < E$,

$\Phi_0 \neq 0$. $\Psi_0 = 0$.

There exists a phase transition in the ph -channel. The field configuration $\Phi_0 = \Psi_0 = 0$ becomes unstable.

iii) $\tilde{\kappa} = E > \epsilon$, $\tilde{\rho} < E$

$\Phi_0 = 0$, $\Psi_0 \neq 0$.

There exists a phase transition in the pp -channel of superconducting type.

iv) $\tilde{\kappa} = \tilde{\rho} = E > \epsilon$

$\Phi_0^2 + |\psi_0|^2 > 0$.

The system undergoes a simultaneous phase transition in both channels.

In appendix D we show that these solutions realize a minimum of the collective action $S[\underset{\sim}{\Phi}]$.

3.2. Collective Excitation Modes

Let us now discuss the boson excitation spectrum. The eigenfrequencies of the collective modes, ω_n, are obtained by solving the homogeneous BS-equation (2.24) which after Fourier transormation yields the following eigenvalue equation

$$\det[1 + \hat{V}\hat{B}(\omega)] = 0 . \qquad (3.3)$$

where

$$\hat{B}(\omega) = \frac{1}{\omega^2 - 4E^2}\hat{b} \qquad (3.4)$$

and the matrix $\hat{b}$ is defined as (cf. appendix B)

$$
\hat{b} = \frac{\Omega}{2E} \begin{pmatrix} 8(\epsilon^2 + |\Psi_0|^2) & -4\Psi_0^+\Phi_0 & -4\Psi_0\Phi_0 \\ -4\Psi_0^+\Phi_0 & -2\Psi_0^{+2} & 2(E^2+\epsilon^2+\Phi_0^2) \\ -4\Psi_0\Phi_0 & 2(E^2+\epsilon^2+\Phi_0^2) & -2\Psi_0^2 \end{pmatrix} . \qquad (3.5)
$$

Eq. (3.3) has the following roots:

$$
\omega_1 = 2\sqrt{E^2 + \frac{x_1}{4E}} \ .
$$

$$
\omega_2 = 2\sqrt{E(E - \tilde{\kappa})} \ ,
$$

$$
\omega_3 = 2\sqrt{E^2 + \frac{x_3}{4E}} \ ,
$$

where

$$
x_{3,1} = -2[\tilde{\rho}(\epsilon^2 + |\Psi_0|^2) + \tilde{\kappa}(\epsilon^2 + \Phi_0^2)] \pm
$$

$$
\pm 2\sqrt{[\tilde{\rho}(\epsilon^2 + |\Psi_0|^2) + \tilde{\kappa}(\epsilon^2 + \Phi_0^2)]^2 - 4\tilde{\rho}\tilde{\kappa}\epsilon^2 E^2} \ , \ (x_{3,1} < 0):
$$

In dependence on the different possibilities of phase transitions discussed in Sect. 3.1 we get the following eigenfrequencies:

i) $\omega_1 = 2\sqrt{\epsilon(\epsilon - \tilde{\rho})}$ (surface vibration)

$\omega_2 = \omega_3 = 2\sqrt{\epsilon(\epsilon - \tilde{\kappa})}$ (pairing vibration),

ii) $\omega_1 = 2|\Phi \ |$,

$\omega_2 = \omega_3 = 2\sqrt{E(E - \tilde{\kappa})}$,

iii) $\omega_1 = 2\sqrt{E(E-\tilde{\rho})}$,

$\omega_2 = 0$,

$\omega_3 = 2|\Psi_0|$,

iv) $\omega_1 = \omega_2 = 0$,

$\omega_3 = 2\sqrt{\Phi_0^2 + |\Psi_0|^2}$.

Case i) provides us just with the well-known RPA modes (phonons) describing surface and pairing vibrations of a system with a normal ground state. Note that, if $\tilde{\rho} > \epsilon$ and/or $\tilde{\kappa} > \epsilon$, the frequencies ω_n become imaginary leading to an exponential blow-up of the wave function ($e^{-i\omega_n t} \xrightarrow[t \to \infty]{} \infty$). The probability for a phase transition of the system into a new ground state containing Cooper pairs and/or particle-hole pairs thus infinitely increases and the normal ground state becomes unstable. We find this picture consistent with the simultaneous appearance of the gap in the single particle energy if $\tilde{\rho} > \epsilon$ and/or $\tilde{\kappa} > \epsilon$. Case ii) corresponds to a phase transition in the ph-channel. The frequency of the surface phonons is now twice the value of the gap $|\Phi_0|$. Note, that the phase transition in the ph-channel diminishes the ratio $\tilde{\kappa}/E$ ($\tilde{\kappa}/E < \tilde{\kappa}/\epsilon$) characterizing the collectivity of the pairing phonons. Thus, a phase-transition in the ph-channel takes away collectivity from the pp-channel. Similar results hold also for a phase transition in the pp-channel (case iii)). However, there appears now a zero frequency solution $\omega = 0$ (Goldstone boson) corresponding to a zero-energy azimuthal motion in the complex Ψ_0-plane: $|\Psi_0|^2 = R^2$. As it is well known such a solution indicates a spontaneous breakdown of a symmetry in the new ground state (in the present case: violation of the particle number conservation). In the case iv), finally, we observe a phase transition in both ph- and pp-channels what leads to zero-energy azimuthal motions on the sphere $\Phi_0^2 + |\Psi_0|^2 = R^2$.

4. MODIFIED PERTURBATION EXPANSIONS

4.1. Loop Expansion

We are now able to formulate a modified perturbation theory that uses the collective propagator $\hat{T}$ given by eq. (2.21) and the "quasi-particle" propagator G_0 (cf. appendix A) as the "free" propagators of the theory. For this purpose, it is convenient to introduce a source term $i\tilde{j}\tilde{\Phi}$ into the exponent of eq. (2.11) and to write the generating functional as

$$Z[Q,Q^+] = \mathfrak{N}_3 \exp i[S_{int}[\frac{\delta}{i\delta \tilde{j}^+}] - Q^+ G(\tilde{\Phi}_0 + \frac{\delta}{i\delta \tilde{j}^t})Q] \times$$

$$\times \int D\tilde{\Phi} \exp i[S_{free}[\tilde{\Phi}] + \tilde{j}^t \tilde{\Phi}]|_{\tilde{j}=0} .$$

Then the $\tilde{\Phi}$ integral is Gaussian and can be performed yielding

$$Z[Q,Q^+] = \mathfrak{N}_4 \exp i[S_{int}[\frac{\delta}{i\delta \tilde{j}^t}] - Q^+ G(\tilde{\Phi}_0 + \frac{\delta}{i\delta \tilde{j}^t})Q] \times \qquad (4.1)$$

$$\times \exp -\frac{i}{2}\tilde{j}^t \hat{T}\tilde{j}|_{\tilde{j}=0} .$$

Using the identity

$$F(-i\frac{\partial}{\partial x})G(x) = G(-i\frac{\partial}{\partial y})F(y)e^{ixy}|_{y=0},$$

where F, G are any two functions the generating functional can, finally, be cast in the form

$$Z[Q,Q^+] = \mathfrak{N}_4 e^{\frac{i}{2}\frac{\delta}{\partial \tilde{M}^+}T\frac{\delta}{\partial \tilde{M}}} e^{-i\frac{\delta}{\delta N}G(\tilde{\Phi}_0+\tilde{M})\frac{\delta}{\delta N^+}} \times \qquad (4.2)$$

$$\times \exp i\{S_{int}(\tilde{M}) + Q^+N + N^+Q\}\Big|_{\substack{\tilde{M}=0 \\ N=N^+=0}} .$$

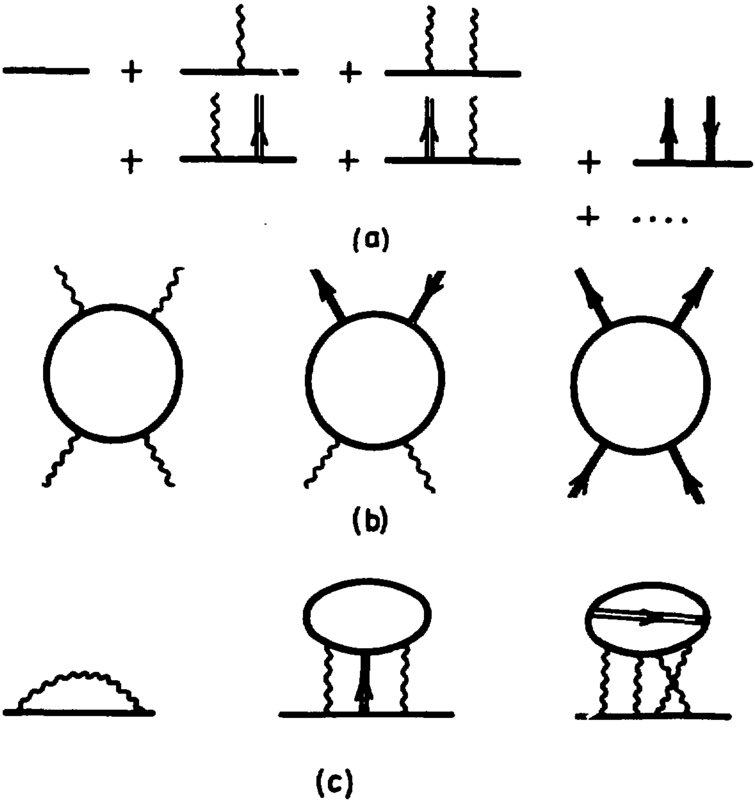
Eq. (4.2) expresses in a compact way the Feynman rules of the loop expansion discussed. In this expansion $G(\tilde{\Phi}_0+\tilde{M})$ and $S_{int}(\tilde{M})$ are given by open and closed fermion lines, respectively, which emit or absorb collective lines to be contracted with the collective propagator $\hat{T}$ (cf. figure). Note that in this picture all fermion loops of the type of a self-energy correction are absent. There remain only the fermion loops of S_{int} which represent effective $\tilde{\Phi}^n$ -interactions of the collective fields. Such integration terms lead to anharmonic effects in the collective excitation spectrum.

It is worth remarking that the phonon frequencies and the gap values contain only the scaled coupling constants $\tilde{\kappa} = \Omega\kappa$, $\tilde{\rho} = 4\Omega\rho$ which are considered to be fixed. If we take into account the fact that the residuum of the collective propagator at the pole $\omega = \omega_n$ contains a factor Ω (see eqs. (3.5), (4.1)) we find that the effective coupling constant of the particle-phonon vertex is of order $\Omega^{-1/2}$. Further, for each closed fermion loop there is an additional Ω factor arising from the trace over the index m. Thus, we get a perturbation expansion in $\frac{1}{\Omega}$, which for sufficiently large values of the level degeneracy Ω (or, equivalently, the fermion number $N = 2\Omega$) converges much better than the original perturbation expansion in the interaction strength's κ and ρ.

4.2. Nuclear Field Theory

In the following we shall prove the equivalence between the modified perturbation expansion (4.2) and the perturbation expansion of the nuclear Field Theory (NFT). To this end we rewrite the collective propagator as

$$\hat{T} = \frac{1}{2}\hat{V} + \hat{T}_c, \tag{4.3}$$



a) Diagrammatic representation of the expansion of the single particle GF in terms of the collective fields $\Phi(t), \Psi(t)$. A full line stands for the "free" GF $G_0 = G(\Phi_0, \Psi_0, \Psi_0^+)$. The collective fields $\Phi(t)$ and $\Psi(t)$ are represented by a wavy line and a double line arrowed, respectively. b) Some typical loop graphs of order $1/\Omega$ arising in the expansion of $\exp iS_{int}$. c) Characteristic diagrams contributing to the total single particle GF.

where *

$$\hat{T}_c = K^t \hat{D} K, \quad K = \frac{1}{\sqrt{2}} U^t \sqrt{\hat{b}}\, \hat{V} \tag{4.4}$$

$$\hat{D} = \begin{pmatrix} \frac{1}{\omega^2 - \omega_1^2} & 0 & 0 \\ 0 & \frac{1}{\omega^2 - \omega_2^2} & 0 \\ 0 & 0 & \frac{1}{\omega^2 - \omega_3^2} \end{pmatrix} \tag{4.5}$$

Here U denotes the orthogonal matrix built up from the eigensolutions $\underset{\sim}{\Phi}_n$ of the homogeneous BS-equation (2.24) written here in the more symmetric form:

$$[\omega_n^2 - 4E^2 + \sqrt{\hat{b}}\, \hat{V} \sqrt{\hat{b}}\,] \underset{\sim}{\Phi}_n = 0, \quad (n = 1,2,3)$$
$$U = (\underset{\sim}{\Phi}_1, \underset{\sim}{\Phi}_2, \underset{\sim}{\Phi}_3). \tag{4.6}$$

In the above discussed loop expansion (see eq. (4.2)) the fermion fields are completely removed from the theory. Our aim is, however, to obtain an effective Lagrangian involving both fermion and collective fields. For this purpose, we express $\exp iS_{int}$ in eqs. (2.11), (2.19) again by an integral over fermion variables. This yields

$$Z[Q, Q^+] = \mathfrak{N}_5 e^{-i \sum_{i=1}^{2} L_i [\frac{1}{i} \frac{\delta}{\delta j^t}]} \int D\underset{\sim}{\Phi} \int D a D a^+ \times$$
$$\times \exp i \int dt \{ h^+ G_0^{-1} h + Q^+ h + h^+ Q + $$
$$+ \frac{1}{2} \underset{\sim}{\Phi}^t \hat{T}^{-1} \underset{\sim}{\Phi} + (\underset{\sim}{j} + \underset{\sim}{P})^t \underset{\sim}{\Phi} \} \Big|_{\underset{\sim}{j}=0} \tag{4.7}$$

---

* The square root of the matrix $\hat{b}$ is defined as usual by $\sqrt{\hat{b}} = W^t \sqrt{\hat{b}_{diag}}\, W$, where W is the orthogonal matrix diagonalizing $\hat{b}$. The letter "t" denotes transposed quantities.

with

$$P = \begin{pmatrix} A + A^+ \\ P^+ \\ P \end{pmatrix} .$$

After performing the $\underset{\sim}{\Phi}$ integration we get

$$Z[Q,Q^+] = \mathfrak{N}_6 e^{-i\sum_{i=1}^{2} L_i \left[\frac{1}{i} \frac{\delta}{\delta \underset{\sim}{j}^t}\right]} \int DaDa^+ \times$$

$$\times \exp i \int dt \{ \mathcal{L}_f + \frac{1}{4} \underset{\sim}{j}^t \hat{V} \underset{\sim}{j} + \frac{1}{2} \underset{\sim}{j}^t \hat{V} \underset{\sim}{P} + Q^+ h + \tag{4.8}$$

$$+ h^+ Q - \frac{1}{2} [(\underset{\sim}{j} + \underset{\sim}{P})^t K^t] \hat{D} [K(\underset{\sim}{j} + \underset{\sim}{P})] \} _{\underset{\sim}{j} = 0} ,$$

where

$$\mathcal{L}_f = h^+ G_0^{-1} h + \frac{1}{4} \underset{\sim}{P}^t \hat{V} \underset{\sim}{P} \tag{4.9}$$

is the Lagrangian of the fermions. In order to linearize the last term in the exponent of eq. (4.8) we may now, in analogy to eq. (2.7), introduce a new collective field $\phi(t)$ the propagator of which is D (cf. eq. (4.5)). This yields

$$Z[Q,Q^+] = \mathfrak{N}_7 e^{-i\sum_{i}^{2} L_i \left[\frac{1}{i} \frac{\delta}{\delta j^t}\right]} \int D\underset{\sim}{\phi} \int DaDa^+ \times$$

$$\times \exp i \int dt \{ \mathcal{L}_{NFT} + \frac{1}{4} \underset{\sim}{j}^t \hat{V} \underset{\sim}{j} + \underset{\sim}{j}^t (\frac{\hat{V}}{2} \underset{\sim}{P} + K^t \underset{\sim}{\phi}) +$$

$$+ Q^+ h + h^+ Q \} _{\underset{\sim}{j} = 0}$$

or equivalently,

$$Z[Q,Q^+] = \bar{\mathfrak{N}}\, e^{-i\sum_{i=1}^{2} L_i \left[\frac{1}{2} \hat{V} \underset{\sim}{\bar{P}} + \frac{1}{i} \frac{\delta}{\delta \underset{\sim}{j}^t}\right]} \int D\underset{\sim}{\phi} \int DaDa^+ \times$$

$$\times \exp i \int dt \{ \mathcal{L}_{NFT} + Q^{+}h + h^{+}Q + j^{t}K^{t}\phi \}_{j=0} , \qquad (4.10)$$

where

$$\mathcal{L}_{NFT} = \mathcal{L}_{f} + \mathcal{L}_{b} + \mathcal{L}_{PV} \qquad (4.11)$$

is just the effective Lagrangian of the Nuclear Field Theory. It comprises besides the full fermion Lagrangian $\mathcal{L}_f$ (defined by eq. (4.9) or in the absence of phase transitions by eq. (2.6)) a free boson Lagrangian $\mathcal{L}_b$ and an interaction part $\mathcal{L}_{PV}$

$$\mathcal{L}_{b} = \frac{1}{2} \phi^{t} \hat{D}^{-1} \phi . \qquad (4.12)$$

$$\mathcal{L}_{PV} = P^{t} K^{t} \phi . \qquad (4.13)$$

respectively. The expression $\bar{P}$ appearing in the argument of $L_i[...]$ in eq.(4.10) is obtained from P by replacing the variables a, a^{+} by the functional derivatives $\frac{\delta}{i\delta\eta^{+}}$, $\frac{\delta}{i\delta\eta}$. K is recognized as the particle-vibration vertex. The functional derivation of the NFT-Lagrangian given above yields simultaneously the corresponding graphical rules for a diagrammatic perturbation theory bases on this Lagrangian: The factor in front of the integrals in eq. (4.10) eliminates the bubble diagrams from the NFT-expansion. This completes our proof of equivalence between the usual fermion treatment, the loop expansion and the NFT treatment for a system with phase transitions*

---

*Due to the condition $j = 0$ the generating functional (4.10) describes bosons appearing only in intermediate states. A natural generalization of eq. (4.10) including external boson states is obtained by rejecting the requirement $j = 0$.

5. SUMMARY AND CONCLUSIONS

In the present paper we have applied path integral techniques to the nuclear many-body system. Within a schematic model different kinds of many-body effects, as, e.g., phase transitions and related phenomena, have been studied. Moreover, the considered model reveals some interesting features concerning the mutual interplay between different kinds of collective excitation modes in the case of phase transitions.

By using the path integral method we could derive modified perturbation theories (the closed loop expansion, the NFT expansion) which show equivalence to the usual Feynman diagrammatic many-body perturbation theory, but which use another expansion parameter (the inverse of the effective shell degeneracy Ω), and may therefore faster converge. Especially, we have derived the NFT-Lagrangian in the presence of phase transitions in a non-perturbation theoretical way. The corresponding graphical rules of the Nuclear Field Theory naturally comes out.

The investigations performed in the present paper show the powerfulness of functional methods in the study of nuclear structure phenomena. In a further publication these techniques are used for investigating some interesting effects of the mutual interweaving of single particle and collective degrees of freedom in spherical nuclei.

APPENDIX A

The Single Particle Green Function

In this appendix some formulae for the Green functions used in the text are collected. For generality, we consider a Hamiltonian $H' = H - \lambda\hat{N}$ where $\hat{N} = \sum_{m\sigma} a^+_{m\sigma} a_{m\sigma}$ is the particle number operator. The Lagrange parameter λ (the chemical potential) has been as usual introduced to guarantee the conservation of N in the average. Let us introduce the following operators and Green functions ($\epsilon_{\pm} = \epsilon \mp \lambda$)

$$
\begin{aligned}
&G_{\pm}^{-1} = (i\partial_t \mp \epsilon_{\pm})\delta(t-t'),\\
&iG_{\pm}(t,t') = \pm\theta[\pm(t-t')]\, e^{\mp i\epsilon_{\pm}(t-t')},\\
&\bar{G}_{\pm}^{-1}(t,t') = (i\partial \pm \epsilon_{\pm})\delta(t-t'),\\
&i\bar{G}_{\pm}(t,t') = -iG_{\pm}(t',t),
\end{aligned}
\tag{A1}
$$

where $\theta(t)$ is the step function. By definition, we have

$$\int dx\, G_{(\ldots)}^{-1}(t,x)\, G_{(\ldots)}(x,t') = \delta(t-t').$$

For convenience, we rewrite the inverse Green function defined by eq. (2.10) as

$$G^{-1}(m;t,t') = \begin{pmatrix} G_a^{-1}(t,t') & \Psi(t)\delta(t-t') \\ \Psi^{+}(t)\delta(t-t') & G_b^{-1}(t,t') \end{pmatrix} \tag{A2}$$

with

$$G_a^{-1}(t,t') = \begin{pmatrix} G_{+}^{-1}(t,t') & \Phi(t)\delta(t-t') \\ \Phi(t)\delta(t-t') & G_{-}^{-1}(t,t') \end{pmatrix} \tag{A3}$$

$$G_b^{-1}(t,t') = \begin{pmatrix} \bar{G}_{+}^{-1}(t,t') & -\Phi(t)\delta(t-t') \\ -\Phi(t)\delta(t-t') & \bar{G}_{-}^{-1}(t,t') \end{pmatrix} \tag{A4}$$

Inverting the matrix operator G^{-1} yields

$$G(m;t,t') = \begin{pmatrix} G_N(t,t') & \bar{G}_A(t,t') \\ G_A(t,t') & \bar{G}_N(t,t') \end{pmatrix} \tag{A5}$$

where the normal and anomalous Green functions are defined by *

$$\begin{aligned} G_N &= G_a(1 - \Psi G_b \Psi^+ G_a)^{-1}, \\ \bar{G}_N &= G_b(1 - \Psi^+ G_a \Psi G_b)^{-1}, \\ \bar{G}_A &= -G_b \Psi^+ G_a (1 - \Psi G_b \Psi^+ G_a)^{-1}, \\ \bar{G}_A &= -G_a \Psi G_b (1 - \Psi^+ G_a \Psi G_b)^{-1}, \end{aligned} \tag{A6}$$

with

$$G_a = \begin{pmatrix} G_+(1-\Phi G_-\Phi G_+)^{-1} & -G_+\Phi G_-(1-\Phi G_+\Phi G_-)^{-1} \\ -G_-\Phi G_+(1-\Phi G_-\Phi G_+)^{-1} & G_-(1-\Phi G_+\Phi G_-)^{-1} \end{pmatrix} \tag{A7}$$

$$G_b = \begin{pmatrix} \bar{G}_+(1-\Phi\bar{G}_-\Phi\bar{G}_+)^{-1} & \bar{G}_+\Phi\bar{G}_-(1-\Phi\bar{G}_+\Phi\bar{G}_-)^{-1} \\ \bar{G}_-\Phi\bar{G}_+(1-\Phi\bar{G}_-\Phi\bar{G}_+)^{-1} & \bar{G}_-(1-\Phi\bar{G}_+\Phi\bar{G}_-)^{-1} \end{pmatrix} \tag{A8}$$

---

*Matrix multiplication implies here integration over intermediate times.

The Green functions for static field configurations Φ_0, Ψ_0 may now easily be calculated from eqs. (A6–A8). With the abbreviation $\tau = t - t'$ we obtain for the normal Green function the expression

$$G_N(\tau) = -i\{\theta(\tau)[\hat{a}(E_\lambda^+)e^{-iE_\lambda^+\tau} + \hat{a}(E_\lambda^-)e^{-iE_\lambda^-\tau}] -$$

$$-\theta(\tau)[\hat{a}(-E_\lambda^-)e^{iE_\lambda^-\tau} + \hat{a}(-E_\lambda^+)e^{iE_\lambda^+\tau}]\}, \qquad (A9)$$

where the matrix $\hat{a}(\omega)$ is given by

$$\hat{a}(\omega) \; f(\omega)\begin{pmatrix} (\omega+\epsilon_+)(\omega^2-\epsilon_-^2-R^2)-2\Phi_0^2\lambda \; ; & -\Phi_0(\omega^2-\epsilon_+^2-R^2)+2\Phi_0\lambda(\omega+\epsilon_+) \\ -\Phi_0(\omega^2-\epsilon_-^2-R^2)+(\omega-\epsilon_-)2\Phi_0\lambda \; ; & (\omega-\epsilon_-)(\omega^2-\epsilon_+^2-R^2)-2\Phi_0^2\lambda \end{pmatrix}$$

with

$$f(\omega) = \begin{cases} [\pm 2E_\lambda^+(E_\lambda^{+2}-E_\lambda^{-2})]^{-1} \; , & \omega = \pm E_\lambda^+ \\ [\mp 2E_\lambda^-(E_\lambda^{+2}-E_\lambda^{-2})]^{-1} \; , & \omega = \pm E^- \end{cases}$$

and

$$R^2 = \Phi_0^2 + |\Psi_0|^2, \qquad (A10)$$

$$E_\lambda^\pm = \sqrt{(\sqrt{\epsilon^2+\Phi_0^2} \mp \lambda)^2 + |\Psi_0|^2} \; .$$

Analogously, we find for the anomalous Green function

$$G_A(\tau) = i\Psi_0^+\{\theta(\tau)[\hat{b}(E_\lambda^+)e^{-iE_\lambda^+\tau} + \hat{b}(E_\lambda^-)e^{-iE_\lambda^-\tau}] - \qquad (A11)$$

$$-\theta(-\tau)[\hat{b}(-E_\lambda^-)e^{iE_\lambda^-\tau} + \hat{b}(-E_\lambda^+)e^{iE_\lambda^+\tau}]\},$$

where

$$\hat{b}(\omega) = f(\omega) \begin{pmatrix} \omega^2 - \epsilon_-^2 - R^2 ; & 2\Phi_0 \lambda \\ 2\Phi_0 \lambda ; & \omega^2 - \epsilon_+^2 - R^2 \end{pmatrix}$$

The Green functions satisfy the following symmetry relations

$$\begin{aligned} \bar{G}_A(\tau) &= -G_N(-\tau) , \\ \bar{G}_A(\tau)/\Psi_0 &= G_A(\tau)/\Psi_0^+ . \end{aligned} \qquad \text{(A12)}$$

For completeness, we quote also the corresponding expressions for $\lambda = 0$ used in the text

$$\begin{aligned} G_N(\tau) &= -i\{\theta(\tau)\hat{a}(E)e^{-iE\tau} - \theta(-\tau)\hat{a}(-E)e^{iE\tau}\} \\ \bar{G}_A(\tau) &= i\Psi_0\{\theta(\tau)\hat{b}(E)e^{-iE\tau} - \theta(-\tau)\hat{b}(-E)e^{iE\tau}\}, \end{aligned} \qquad \text{(A13)}$$

where now

$$\hat{a}(\pm E) = \pm \frac{1}{2E} \begin{pmatrix} \pm E + \epsilon , & -\Phi_0 \\ -\Phi_0 , & \pm E - \epsilon \end{pmatrix}$$

$$\hat{b}(\pm E) = \pm \frac{1}{2E} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} .$$

APPENDIX B

Definition of the Matrix $\hat{B}$

The elements of the "bubble" matrix $\hat{B}$ (cf. eq. (2.21)) are defined as follows, $\hat{B} = -i\frac{i\Omega}{2}\hat{A}$, with

$$\begin{aligned} A_{11}(t,t') = \mathrm{tr}\{&\hat{G}_N(t,t')\hat{G}_N(t',t) + \hat{\bar{G}}_N(t,t')\hat{\bar{G}}_N(t',t) - \\ &-[\hat{\bar{G}}_A(t,t')\hat{G}_A(t',t) + \hat{G}_A(t,t')\hat{\bar{G}}_A(t',t)]\}, \end{aligned}$$

$$A_{12}(t,t') = \mathrm{tr}\{G_N(t,t')\hat{G}_A(t',t) - G_A(t,t')\hat{\bar{G}}_N(t',t)\},$$

$$A_{13}(t,t') = \mathrm{tr}\{\bar{G}_A(t,t')\hat{G}_N(t',t) - \bar{G}_N(t,t')\hat{\bar{G}}_A(t',t)\},$$

$$A_{21}(t,t') = \mathrm{tr}\{\hat{G}_A(t,t')G_N(t',t) - \hat{\bar{G}}_N(t,t')G_A(t',t)\},$$

$$A_{22}(t,t') = \mathrm{tr}\{G_A(t,t')G_A(t',t)\}, \qquad (B1)$$

$$A_{23}(t,t') = \mathrm{tr}\{\bar{G}_N(t,t')G_N(t',t)\},$$

$$A_{31}(t,t') = \mathrm{tr}\{\hat{G}_N(t,t')\bar{G}_A(t',t) - \hat{\bar{G}}_A(t,t')\bar{G}_N(t',t)\},$$

$$A_{32}(t,t') = \mathrm{tr}\{G_N(t,t')\bar{G}_N(t',t)\},$$

$$A_{33}(t,t') = \mathrm{tr}\{\bar{G}_A(t,t')\bar{G}_A(t',t)\},$$

where we have used the short notation

$$\hat{G}(t,t') = G(t,t')\hat{I},!$$

$$\hat{I} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \text{etc.}$$

APPENDIX C

Calculation of the Chemical Potential

In this appendix we prove that the chemical potential may be fixed to the value $\lambda = 0$ for static field solutions. The chemical potential is determined by the requirement

$$N = \langle \hat{N} \rangle_{av} ,$$

where the average number is given by

$$\langle \hat{N} \rangle_{av} = \langle \sum_{m\sigma} a^{+}_{m\sigma} a_{m\sigma} \rangle_{av} =$$

$$= \sum_{m\sigma} i \frac{\delta}{\delta \eta_{m\sigma}(t')} \frac{\delta}{\delta \eta^{+}_{m\sigma}(t)} Z[\eta, \eta^{+}] \Big|_{\substack{\eta = \eta^{+} = 0 \\ t' = t + 0}} . \tag{C1}$$

Using for the generating functional $Z[\eta, \eta^{+}]$ the representation (2.11) we find after differentiation

$$N = \langle -i2\Omega \, \mathrm{tr}\, G_N(t-t' = -0) \rangle_{av} =$$

$$= \frac{\int D\underset{\sim}{\Phi}\, e^{i[S_{free} + S_{int}]} (-i)\, 2\Omega \, \mathrm{tr}\, G_N(-0)}{\int D\underset{\sim}{\Phi}\, e^{i[S_{free} + S_{int}]}} = \tag{C2}$$

$$= (-i) 2\Omega \, \mathrm{tr}\, G_N(-0) \Big|_{\underset{\sim}{\Phi} = \underset{\sim}{\Phi}_0} .$$

Inserting (A9) into (C2) we obtain, finally

$$N = 2\Omega \left[1 - \frac{1}{2} \frac{\sqrt{\epsilon^2 + \Phi_0^2} - \lambda}{E^{+}_{\lambda}} + \frac{1}{2} \frac{\sqrt{\epsilon^2 + \Phi_0^2} + \lambda}{E^{-}_{\lambda}} \right] . \tag{C3}$$

Obviously, for $\Psi_0 \neq 0$, the requirement $N = 2\Omega$ can be fulfilled only for $\lambda = 0$ (see eq. (A.10)). For $\Psi_0 = 0$ eq. (C3) is independent of λ, so we may choose again $\lambda = 0$. This proves our statement.

APPENDIX D

Stability Condition

A necessary and sufficient condition for the static gap solutions Φ_0, Ψ_0 presented in Sect. 3.1 realize a minimum of the collective action $S[\underset{\sim}{\Phi}]$ is

$$\delta^2 S = \sum_{i,j=1}^{3} \frac{\delta^2 S}{\delta\Phi_i \, \delta\Phi_j}\Big|_{\underset{\sim}{\Phi}_0} \Delta\Phi_{0i} \, \Delta\Phi_{0j} =$$

$$= -\mathrm{const} \sum_{i,j=1}^{3} \hat{T}^{-1}_{ij}(\omega = 0) \Delta\Phi_{0i} \, \Delta\Phi_{0j} > 0 . \tag{D1}$$

By diagonalizing the quadratic form (D1) one finds that the eigenvalues μ_n of the inverse propagator $\hat{T}^{-1}(\omega = 0)$ are all non-positive $(\mu_n \le 0)$ with at least one $\mu_n \neq 0$. Thus, $\delta^2 S > 0$ is guaranteed.

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